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A CODING TECHNIQUE WITH FELICITOUS LABELING AND CATERPILLAR GRAPHS

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ABSTRACT. A caterpillar graph is a felicitous labeled graph. An innovative way of assigning numbers to the English alphabets named 'Alphabets maneuvered', is introduced. A coding technique with a combination of caterpillar graph, felicitous labeling and alphabets maneuvered is presented here.

1. INTRODUCTION

The applications of graph theory to various fields are ever increasing. The study and introduction of new graph labelings are also in the progress. The Researchers seeking an application using graph labelings were motivated by [3] and struck at a new coding technique with a new way of using alphabets, named Alphabets maneuvered. It is named as Alphabets Maneuvered Graph Labeled coding technique.

1.1. **Literature Review.** The Researchers follow the notations and terminology as in [1]. In [3], a new technique called GMJ coding was introduced. A number of graphs such as two and three star graphs, Fibonacci web, doubly duplicated sunflower and gear graphs combined with a variety of graph labelings such as super mean, vertex product, edge cordial and difference cordial labelings are dealt with in [3–6]. Being inspired by [2], the Researchers selected the caterpillar graphs with felicitous labeling,

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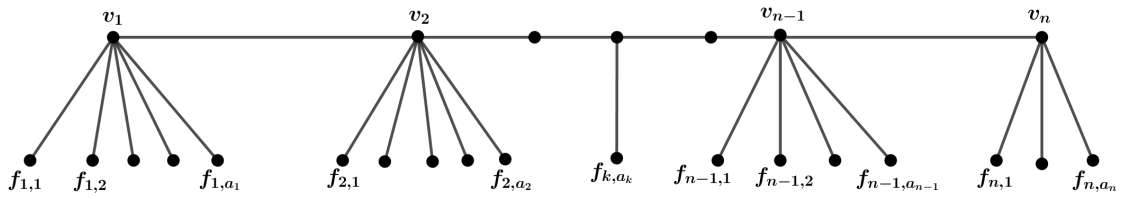
'Alphabets maneuvered' and developed a coding technique which is presented in this paper.

1.2. **Prerequisites.**

Definition 1.1. *A Graph G is called felicitous if there exists an injection $f : (V(G)) \to \{0, 1, 2, \cdots, q\}$ such that the induced function $f^* : (E(G)) \to \{0, 1, 2, \cdots, q-1\}$ defined by $f^*(uv) = (f(u) + f(v))(\mod q)$ $uv \in E(G)$ is a bijection.*

The injection function f is called a felicitous labeling of G. The values $f(u)$ and $f^*(uv)$ are called felicitous labels of vertex u and the edge uv respectively.

Definition 1.2. *A Caterpillar is a tree in which a single path (the spine) incident to (or contains) every edge.*



Note that a spine vertex may not have a leg edge. Also a spine vertex may have more than one leg. A foot vertex is always connected to a spine vertex through a leg edge.

Definition 1.3 (AMGL Coding technique)**.** *A Coding technique is developed by a combination of alphabets maneuvered and a graph which is labeled. This technique is called Alphabets maneuvered graph labelled coding technique (AMGL).*

AMGL also stands for Antony Maria Gabriel, the father of one of the Researchers Gabriel Margaret Joan.

1.3. **Notations.**

The graph taken for discussion is the caterpillar. The path vertices are also known as spine vertices. They are denoted by sv_i. The pendant vertices are called feet vertices and they are denoted by f_i. As the number of legs associated with each spine vertex need not be the same, two suffixes are used for leg vertices. The j^{th} leg vertex connected with the i^{th} spine vertex is denoted by $f_{i,j}$. Similarly the spine edges and the leg edges can be denoted by se_i and le_i. The total number of spine vertices is n and the feet vertices is ℓ such that $n + \ell = p$.

2. DESCRIPTION OF AMGL CODING TECHNIQUE

i. Alphabets Maneuvered
A selection of names of Countries, Leaders, Religions, Mathematicians, Mathematical words, Graphs, Flowers, Trees, Animals and so on, such that each list contains **All alphabets A to Z** is made.
ii. Clues are given to guess the list selected.
iii. Counting the numbers of letters in the words is done
iv. The names of choice is arranged in ascending order according to the number of letters in each name; if the number of letters are equal for a few names, arrange them in the alphabetical order. Then assign numbers 1 to 26 in order to the letters arranged as stated above.
The numbers are ready for the letters A to Z which will be used while coding a message with respect to certain labeling. That is, alphabets are maneuvered.
v. The clue for the alphabets maneuvered:
It is given as an n tuple. The first component contains two alphabets: MN for names of Mathematician, ML for Mathematical words, CY for countries, FR for names of flowers and so on.
If the smallest word contains, say, three letters starting with E, it is denoted as 5_3 ($E - 5^{th}$ alphabet in the natural order, 3 the number of letters.)
The clue is ready which is given below
$(ML, 5_3, 11_4, 16_5, 18_5, \cdots)$
vi. A clue, not necessarily mathematical is given to guess the labeling.
vii. A clue is given to strike at the graph on which labeling is assigned. As the graph is a caterpillar, the number of legs attached to spine vertices in order are provided in the form of an n - tuple $(K_1, K_2, \cdots K_n)$, where K_i is a positive integer. The graph labeling is assigned and the graph is ready for coding.
viii. Now coding is done and the letters are written using the graph.

2.1. **A Rule for the Felicitous Labeling.** The first spine vertex sv_1 is given 0 and then the next number 1 onwards are allotted to all the feet of the legs associated with the second spine vertex sv_2. The next number is allotted to third spine vertex and then the feet vertices which are associated with the fourth spine vertex and so on. The procedure is the same for odd and even number of spine vertices.
Case 1: Let the number of spine vertices be even. Then the last spine vertex takes the value q, even if some spine vertices are without legs.

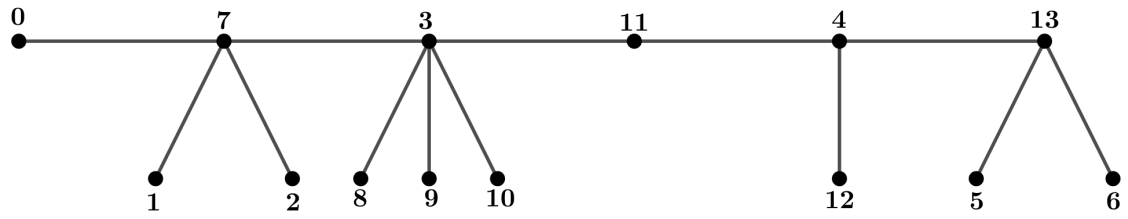


FIGURE 1

Case 2: Let the number of spine vertices be odd. Then if the last spine vertex has legs, then the last leg takes the value q. If the last spine vertex has no leg, the the previous spine vertex sv_{n-1} takes the value q. In this case, after numbering the last spine vertex, the second spine vertex has to be allotted the next number and then proceed alternately spine to leg.

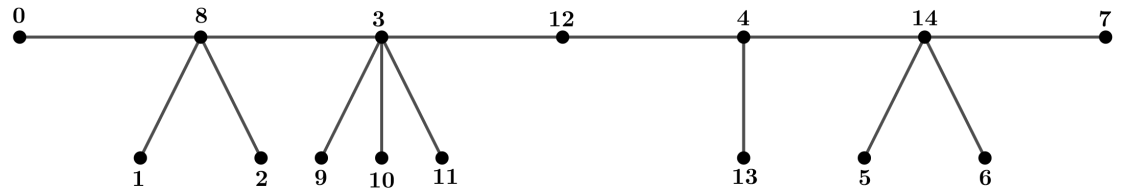


FIGURE 2

Illustration 1
Plain Text: Do not be shocked to know the content of this message
Clues:
Clue for the labeling: Heartiest welcome to you sir!
(Meaning - Felicitous Labeling)
Clue for the graph: Source of silk! (Implying Caterpillar graph)
Clue for Alphabets maneuvered
The list of names of mathematicians taken is given below:

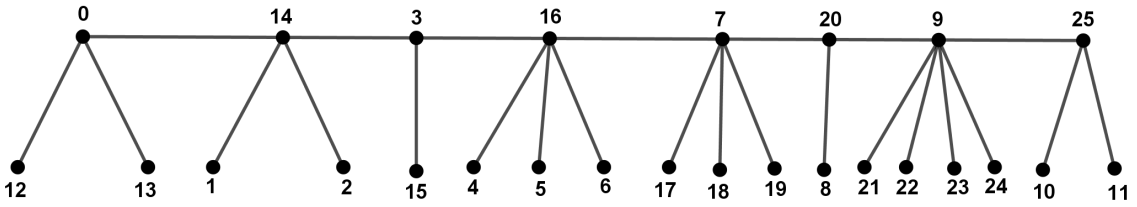
Levi	Gauss	Fermat	Harary	Kepler	Newton	Bolzano	Eudoxus	Jacques
12_4	7_5	6_6	8_6	11_6	14_6	2_7	5_7	10_7

According to rule (v), $(MN, 12_4, 7_5, 6_6, 8_6, 11_6, 14_6, 2_7, 5_7, 10_7)$ is the required clue.
Alphabets Maneuvered

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
6	20	25	22	2	9	5	13	4	24	15	1	11	17	19	16	26	10

S	T	U	V	W	X	Y	Z
8	12	7	3	18	23	14	21

Graph:



According to rule (vii), the clue for the caterpillar graph $(2, 2, 1, 3, 3, 1, 4, 2)$.

Cipher Text

D takes the value 22. 22 is alloted to the second foot connected with the 7^{th} spine vertex. Therefore D is denoted by $f_{7,2}$ Similarly for O. O takes the number 19. It is assigned to the third foot of 5^{th} spine vertex. so O is denoted by $f_{5,3}$. Therefore the cipher text for DO is $f_{7,2}f_{5,3}$.

The cipher text is presented in the form of a table which is given below:

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	W_{11}
L_1	$f_{7,2}$	$f_{5,1}$	sv_6	$f_{6,1}$	$f_{1,1}$	$f_{3,1}$	$f_{1,1}$	sv_8	$f_{5,3}$	$f_{1,1}$	$f_{8,2}$
L_2	$f_{5,3}$	$f_{5,3}$	$f_{2,2}$	$f_{1,2}$	$f_{5,3}$	$f_{5,1}$	$f_{1,2}$	$f_{5,3}$	sv_7	$f_{1,2}$	$f_{2,2}$
L_3		$f_{1,1}$		$f_{5,3}$		$f_{5,3}$	$f_{2,2}$	$f_{5,1}$		$f_{4,1}$	$f_{6,1}$
L_4				sv_8		$f_{5,2}$		$f_{1,1}$		$f_{6,1}$	$f_{6,1}$
L_5				$f_{3,1}$				$f_{2,2}$			$f_{4,3}$
L_6				$f_{2,2}$				$f_{5,1}$			$f_{4,2}$
L_7				$f_{7,2}$				$f_{1,1}$			$f_{2,2}$

Illustration 2

The plain text, clue for the labeling and clue for the graph are the same as in Illustration 1.

The names of countries are taken for AMGL. They are:

Fiji	Peru	India	Quatar	Brazil	Kuwait	Mexico	Hungary	Slovenia
6_4	16_4	9_5	17_5	2_6	11_6	13_6	8_7	19_8

Hence the clue for alphabets maneuvered is $(CY, 6_4, 16_4, 9_5, 17_5, 2_6, 11_6, 13_6,$

$8_7, 19_8)$

Alphabets Maneuvered

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
10	13	20	9	5	1	23	22	2	3	16	15	18	8	21	4	11	6
S	T	U	V	W	X	Y	Z										
25	12	7	26	17	19	24	14										

Graph

The clue for the caterpillar graph taken is $(0, 4, 2, 3, 4, 2, 3, 0)$ as explained in Rule (vii). In Illustration 1, the numbers assigned to the spine vertices and the feet vertices only are taken. In Illustration 2, the numbers attached to the spine edges and leg edges are also taken.

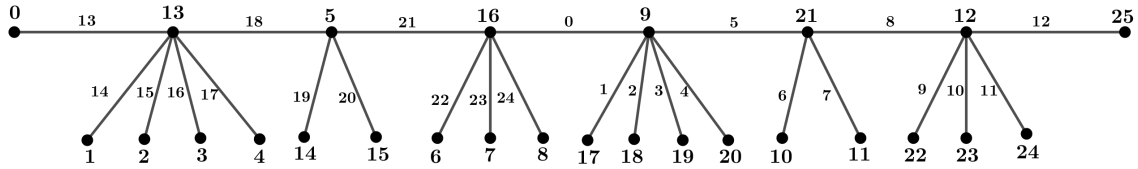


FIGURE 3

Cipher Text

The Cipher text is presented jumbling the words, according to the length.

$sv_5sv_6,\ se_1se_5,\ sv_7sv_6,\ se_3le_{10},\ se_6se_3se_7,\ se_7le_7se_5,\ le_3se_6se_3le_4,\ sv_7f_{7,1}le_{11}sv_8,$
$sv_8f_{7,1}sv_6f_{5,4}sv_4sv_3sv_5,\ f_{5,4}sv_6f_{4,3}sv_7sv_3f_{4,3}se_7,\ se_2se_5sv_8f_{6,1}f_{7,2}sv_3.$

D takes the value 9. The number 9 is allotted to sv_5 and le_{16}. Here sv_5 is taken. With a sound knowledge of graphs, labelings and the way alphabets are maneuvered the cipher texts can be decoded to plain texts.

3. CONCLUSION

The recipient receives the clues for guessing the graph, the labeling and the maneuvering of the alphabets. The recipient is expected to be familiar with the methods which will be employed by the sender. The message can be decoded only by the recipient, that too, not very easily and the decoding is rather too difficult for a third person as a very high degree of secrecy which is very essential for coding is maintained due to the skilful handling of the alphabets. Having introduced a new

coding technique using AMGL the Researchers plan to move forward towards new horizon in the same field.

4. Acknowledgement

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