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POTENTIAL AND PERCEIVED MEASURES OF WEIGHTED FUZZY SETS

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ABSTRACT. Clustering similar type of large number of objects is a difficult task in real life. In this paper we have introduced potential measures between weighted fuzzy sets and between weighted fuzzy elements and also illustrated with examples. Often reported perceptual opinions do not reflect the exact value or reality. To overcome this situation we have introduced perceived measures of weighted fuzzy sets to find the perceived value of the weighted fuzzy sets and given a numerical example.

1. INTRODUCTION

To approach uncertain situations Zaheh [6] introduced the concept of fuzzy set. Fuzzy set is used to find the appropriate solutions where classical sets fails. Also J. M. Mendel and R. B. John [4] developed Type-2 fuzzy set concept where the membership of fuzzy sets itself is fuzzy. Later Atanassov [5] established intuitionistic fuzzy sets, which explains membership of an element and non-membership of an element. Because when we encounter real life situations both membership and non-membership play major role to make decisions. T. Pathinathan and E. Mike Dison [2] have introduced rotational fuzzy set model and their properties to represent the ambiguity in terms of angle. T. Pathinathan and P. Mahimairaj [7,8] developed fuzzy sets into weighted fuzzy sets and verified their properties.

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Weighted fuzzy sets explain the membership of elements and impact of each element over the sets.

In real life situations grouping 'similar' types of objects, data, information, report, ect... is a complex task. Similarity measures is an important tool to group the similar kind of objects together. Hyung Lee-Kwang, Yoon-Seon Song and Keon - Myung Lee [3] introduced the similarity measures between fuzzy sets and fuzzy elements. Wen – June Wang [9] developed the similarity between fuzzy sets and fuzzy elements and gave numerical examples. Zhizhen Liang and Pengfei Shi [11] have found the similarity measures for intuitionistic fuzzy sets. W. L. Hung and M. S. Yang [10] introduced the similarity measures between type-2 fuzzy sets. Also Chengyi Zhang and Haiyan Fu [1] have found the similarity measures between elements and between sets for intuitionistic fuzzy sets, fuzzy rough sets and rough fuzzy sets.

Through this paper, we have introduced a new concept namely potential measures between weighted fuzzy sets and between weighted fuzzy elements, for numerical example, we have taken Indian and New Zealand ODI team (https:// www.cricbuzz.com/ cricket-series /2697/icc- cricket-world-cup-2019 /squads). Also we have introduced the perceived measures of weighted fuzzy sets and illustrated with numerical example.

This paper is organized as follows. Section two provides basic definitions of fuzzy sets. Section three provides potential measures of weighted fuzzy sets, application with numerical examples and result and interpretations. Section four provides perceived measures of weighted fuzzy sets with graphical representation and numerical examples. Final section provides the conclusion.

1.1. **Basic Definition.**

Definition 1.1. ***Weighted Fuzzy sets [8]:*** *Let X be a universe of discouse, then a weighted fuzzy subset* $W(\underset{\sim}{A})$ *in X is given by*

$$W(\underset{\sim}{A}) = \left\{ \left\langle a, \mu_{(W(\underset{\sim}{A}))}(a), \eta_{a\in W(\underset{\sim}{A})} \right\rangle / a \in \underset{\sim}{A} \right\},$$

where, $\mu_{W(\underset{\sim}{A})} : X \to [0,1]$ *denotes the degree of presence of an elements* $\eta_{a\in W(\underset{\sim}{A})} : W(\underset{\sim}{A}) \to [0,1]$ *denote the degree of impact of the set corresponding to each element presence in the set. Then, a weighted fuzzy subset* $\underset{\sim}{A}$ *is defined*

$$W(\underset{\sim}{A}) = \left\{ \begin{array}{l} 1, if \sum_{i=1}^{n} \frac{\eta_{a \in W(\underset{\sim}{A})}}{n} \geq 0.5 \\ 0, if \sum_{i=1}^{n} \frac{\eta_{a \in W(\underset{\sim}{A})}}{n} \prec 0.5 \end{array} \right\}$$

2. Potential Measures

In this section, we have introduced Potencial measure of weighted fuzzy set and Potential measures between weighted fuzzy elements.

2.1. **Potential measures of Weighted Fuzzy Sets.** We define potential measures between two weighted fuzzy sets $W(\underset{\sim}{M})$ and $W(\underset{\sim}{N})$ as follows

$$P(W(\underset{\sim}{M}), W(\underset{\sim}{N})) = \left\{ \begin{array}{l} \left[1 - \sum_{i=1}^{K} \frac{\left| \mu_{W(\underset{\sim}{M})}(a_i) - \mu_{W(\underset{\sim}{N})}(a_i) \right|}{K} \right], \\ \qquad \left[1 - \sum_{i=1}^{K} \frac{\left| \eta_{W(\underset{\sim}{M})}(a_i) - \eta_{W(\underset{\sim}{N})}(a_i) \right|}{K} \right] \end{array} \right\}.$$

Then the potential measures of weighted fuzzy sets satisfies the following properties

$P_1 : P(W(\underset{\sim}{M}), W(\underset{\sim}{N})) = P(W(\underset{\sim}{N}), W(\underset{\sim}{M}))$ where $W(\underset{\sim}{M}), W(\underset{\sim}{N}) \subseteq X$

$P_2 :$

(i) If $\mu_{W(\underset{\sim}{M})}(a_i) \neq \mu_{W(\underset{\sim}{N})}(a_i)$ and $\eta_{W(\underset{\sim}{M})}(a_i) \neq \eta_{W(\underset{\sim}{N})}(a_i)$ then the value of potential measures of weighted fuzzy sets is $0 \prec P(W(\underset{\sim}{M}), W(\underset{\sim}{N})) \prec 1$.

(ii) If $\mu_{W(\underset{\sim}{M})}(a_i) = \mu_{W(\underset{\sim}{N})}(a_i)$ and $\eta_{W(\underset{\sim}{M})}(a_i) \neq \eta_{W(\underset{\sim}{N})}(a_i)$ then the value of potential measures of weighted fuzzy sets is partially similar.

(iii) If$\mu_{W(\underset{\sim}{M})}(a_i) \neq \mu_{W(\underset{\sim}{N})}(a_i)$ and $\eta_{W(\underset{\sim}{M})}(a_i) = \eta_{W(\underset{\sim}{N})}(a_i)$ then the value of potential measures of weighted fuzzy sets is partially similar.

(iv) If $\mu_{W(\underset{\sim}{M})}(a_i) = \mu_{W(\underset{\sim}{N})}(a_i)$ and $\eta_{W(\underset{\sim}{M})}(a_i) = \eta_{W(\underset{\sim}{N})}(a_i)$ then the value of potential measures of weighted fuzzy sets is totally similar.

P_3 : If $W(\underset{\sim}{M}) \subseteq W(\underset{\sim}{N}) \subseteq W(Q)$ for all $W(\underset{\sim}{M}), W(\underset{\sim}{N}), W(Q) \subseteq X$, then $P(W(\underset{\sim}{M}), W(\underset{\sim}{N})) \geq P(W(\underset{\sim}{M}), W(Q))$ and $P(W(\underset{\sim}{N}), W(Q)) \geq P(W(\underset{\sim}{M}), W(Q))$.

It is easy to prove that the potential measures of weighted fuzzy sets satisfies properties P_1 and P_2. Here we are going to verify the property P_3.

The proof for Potential measures P_3 :

Let $W(\underset{\sim}{M}) \subseteq W(\underset{\sim}{N}) \subseteq W(\underset{\sim}{Q})$ for all $W(\underset{\sim}{M}), W(\underset{\sim}{N}), W(\underset{\sim}{Q}) \subseteq X, \mu_{\underset{\sim}{M}}(a_i) \leq \mu_{\underset{\sim}{N}}(a_i) \leq \mu_{\underset{\sim}{Q}}(a_i)$ and $\eta_{\underset{\sim}{M}}(a_i) \leq \eta_{\underset{\sim}{N}}(a_i) \leq \eta_{\underset{\sim}{Q}}(a_i)$ then

$$\begin{aligned}
&1-\left|\mu_{W(\underset{\sim}{O})}(a_i)-\mu_{W(\underset{\sim}{M})}(a_i)\right| = 1-(\mu_{W(\underset{\sim}{O})}(a_i)-\mu_{W(\underset{\sim}{M})}(a_i))\\
=&1-(\mu_{W(\underset{\sim}{O})}(a_i)-\mu_{W(\underset{\sim}{N})}(a_i)+\mu_{W(\underset{\sim}{N})}(a_i)-\mu_{W(\underset{\sim}{M})}(a_i))\\
\leq&1-(\mu_{W(\underset{\sim}{O})}(a_i)-\mu_{W(\underset{\sim}{N})}(a_i))
\end{aligned}$$

$$1-\left|\mu_{W(\underset{\sim}{Q})}(a_i)-\mu_{W(\underset{\sim}{M})}(a_i)\right| = 1-\left|\mu_{W(\underset{\sim}{Q})}(a_i)-\mu_{W(\underset{\sim}{N})}(a_i)\right| \tag{2.1}$$

$$\begin{aligned}
&1-\left|\eta_{W(\underset{\sim}{Q})}(a_i)-\eta_{W(\underset{\sim}{M})}(a_i)\right| = 1-(\eta_{W(\underset{\sim}{Q})}(a_i)-\eta_{W(\underset{\sim}{M})}(a_i))\\
=&\ 1-(\eta_{W(\underset{\sim}{Q})}(a_i)-\eta_{W(\underset{\sim}{N})}(a_i)+\eta_{W(\underset{\sim}{N})}(a_i)-\eta_{W(\underset{\sim}{M})}(a_i))\\
\leq&\ 1-(\eta_{W(\underset{\sim}{Q})}(a_i)-\eta_{W(\underset{\sim}{N})}(a_i))
\end{aligned}$$

$$1-\left|\eta_{W(\underset{\sim}{Q})}(a_i)-\eta_{W(\underset{\sim}{M})}(a_i)\right| = 1-\left|\eta_{W(\underset{\sim}{Q})}(a_i)-\eta_{W(\underset{\sim}{N})}(a_i)\right| \tag{2.2}$$

Adding equation (2.1) and (2.2)

$$\left\{\begin{bmatrix} 1-\left|\mu_{W(\underset{\sim}{Q})}(a_i)-\mu_{W(\underset{\sim}{M})}(a_i)\right| \\ 1-\left|\eta_{W(\underset{\sim}{Q})}(a_i)-\eta_{W(\underset{\sim}{M})}(a_i)\right| \end{bmatrix}+\right\} \leq \left\{\begin{bmatrix} 1-\left|\mu_{W(\underset{\sim}{Q})}(a_i)-\mu_{W(\underset{\sim}{N})}(a_i)\right| \\ 1-\left|\eta_{W(\underset{\sim}{Q})}(a_i)-\eta_{W(\underset{\sim}{N})}(a_i)\right| \end{bmatrix}+\right\}.$$

Similarly we can prove,

$$\left\{\begin{bmatrix} 1-\left|\mu_{W(\underset{\sim}{Q})}(a_i)-\mu_{W(\underset{\sim}{M})}(a_i)\right| \\ 1-\left|\eta_{W(\underset{\sim}{Q})}(a_i)-\eta_{W(\underset{\sim}{M})}(a_i)\right| \end{bmatrix}+\right\} \leq \left\{\begin{bmatrix} 1-\left|\mu_{W(\underset{\sim}{N})}(a_i)-\mu_{W(\underset{\sim}{M})}(a_i)\right| \\ 1-\left|\eta_{W(\underset{\sim}{N})}(a_i)-\eta_{W(\underset{\sim}{M})}(a_i)\right| \end{bmatrix}+\right\}.$$

Example 1. *Let*

$$\begin{aligned}
W(\underset{\sim}{M}) &= \{(a_1,0.9,0.6),(a_2,0.7,0.4),(a_3,0.8,0.7),(a_4,0.9,0.7)\},\\
W(\underset{\sim}{N}) &= \{(a_1,0.7,0.5),(a_2,0.4,0.4),(a_3,1,0.8),(a_4,1,0.9)\}
\end{aligned}$$

are two weighted fuzzy sets then the potential measures of weighted fuzzy sets are as follows

$$\begin{aligned}
P(W(\underset{\sim}{M}),W(\underset{\sim}{N})) = \Bigg\{&\left[\ 1-\sum_{i=1}^{4}\frac{\left|\mu_{W(\underset{\sim}{M})}(a_i)-\mu_{W(\underset{\sim}{N})}(a_i)\right|}{4}\ \right],\\
&\left[\ 1-\sum_{i=1}^{4}\frac{\left|\eta_{W(\underset{\sim}{M})}(a_i)-\eta_{W(\underset{\sim}{N})}(a_i)\right|}{4}\ \right]\Bigg\}
\end{aligned}$$

$$P(W(\underset{\sim}{M}),W(\underset{\sim}{N})) = \left\{ \begin{bmatrix} 1-\frac{|0.9-0.7|+|0.7-0.4|+|0.8-1|+|0.9-1|}{4} \end{bmatrix}, \begin{bmatrix} 1-\frac{|0.6-0.5|+|0.4-0.4|+|0.7-0.8|+|0.7-.09|}{4} \end{bmatrix} \right\},$$

$$P(W(\underset{\sim}{M}),W(\underset{\sim}{N})) = (0.8, 0.9).$$

2.2. **Potential measures between Weighted Fuzzy elements.** We define potential measures between weighted fuzzy sets $W(\underset{\sim}{M_i})$ where $i = 1, 2, 3 \cdots K$ as follows

$$P_e(x,y) = \left\{ \left[1 - \sum_{i=1}^{k} \frac{\left|\mu_{\underset{\sim}{M_i}}(x) - \mu_{\underset{\sim}{M_i}}(y)\right|}{k} \right], \left[1 - \sum_{i=1}^{k} \frac{\left|\eta_{\underset{\sim}{M_i}}(x) - \eta_{\underset{\sim}{M_i}}(y)\right|}{k} \right] \right\}$$

then the potential Measures of weighted fuzzy elements satisfies the following properties

$P_1 : P_e(x, y) = P_e(y, x)$ where $x, y \in W(\underset{\sim}{M_i})$

P_2 :

(i) If $\mu_{\underset{\sim}{M_i}}(x) \neq \mu_{(\underset{\sim}{M_i})}(y)$ and $\eta_{\underset{\sim}{M_i}}(x) \neq \eta_{(\underset{\sim}{M_i})}(y)$ then the value of potential measures of weighted fuzzy elements is $0 \prec P_e(X, Y) \prec 1$.

(ii) If $\mu_{\underset{\sim}{M_i}}(x) = \mu_{(\underset{\sim}{M_i})}(y)$ and $\eta_{\underset{\sim}{M_i}}(x) \neq \eta_{(\underset{\sim}{M_i})}(y)$ then the value of potential measures of weighted fuzzy elements is partially similar.

(iii) If$\mu_{\underset{\sim}{M_i}}(x) \neq \mu_{(\underset{\sim}{M_i})}(y)$ and $\eta_{\underset{\sim}{M_i}}(x) = \eta_{(\underset{\sim}{M_i})}(y)$ then the value of potential measures of weighted fuzzy elements is partially similar.

(iv) If $\mu_{\underset{\sim}{M_i}}(x) = \mu_{(\underset{\sim}{M_i})}(y)$ and $\eta_{\underset{\sim}{M_i}}(x) = \eta_{(\underset{\sim}{M_i})}(y)$ then the value of potential measures of weighted fuzzy elements is totally similar.

P_3 : If $\mu_{\underset{\sim}{M_i}}(x) \leq \mu_{\underset{\sim}{M_i}}(y) \leq \mu_{\underset{\sim}{M_i}}(z)$ and $\eta_{\underset{\sim}{M_i}}(x) \leq \eta_{\underset{\sim}{M_i}}(y) \leq \eta_{\underset{\sim}{M_i}}(z)$ for all $\mu_{\underset{\sim}{M_i}}(x)$, $\mu_{\underset{\sim}{M_i}}(y), \mu_{\underset{\sim}{M_i}}(z), \eta_{\underset{\sim}{M_i}}(x), \eta_{\underset{\sim}{M_i}}(y), \eta_{\underset{\sim}{M_i}}(z) \in W(M_i)$, then $P_e(x, y) \geq P_e(x, z)$ and $P_e(y, z) \geq P_e(x, z)$.

It is easy to prove that the potential measures of weighted fuzzy elements satisfies properties P_1, P_2 and P_3.

Example 2. *Let*

$$W(\underset{\sim}{M_1}) = \{(x, 0.9, 0.6), (y, 0.7, 0.4), (z, 0.8, 0.7)\},$$

$$W(\underset{\sim}{M_2}) = \{(x, 0.7, 0.5), (y, 0.4, 0.4), (z, 1, 0.8)\}$$

are two weighted fuzzy sets then the potential measures of weighted fuzzy elements are as follows

$$P_e(x,y) = \left\{ \left[1 - \sum_{i=1}^{3} \frac{\left|\mu_{\underset{\sim}{M_i}}(x) - \mu_{\underset{\sim}{M_i}}(y)\right|}{3} \right], \left[1 - \sum_{i=1}^{3} \frac{\left|\eta_{\underset{\sim}{M_i}}(x) - \eta_{\underset{\sim}{M_i}}(y)\right|}{3} \right] \right\}$$

$$\begin{aligned} P_e(x,y) &= \left\{ \left[1 - \frac{|0.9-0.7| + |0.7-0.4|}{2} \right], \left[1 - \frac{|0.6-0.4| + |0.5-0.4|}{2} \right] \right\} \\ &= (0.75, 0.85). \end{aligned}$$

Similarly we can find $P_e(y,z), P_e(x,z)$.

2.3. **Applications.** In this section we calculated the potential measures between two cricket teams and potential measures between certain attributes in the same team. For our example we have taken ODI performance of teams India and New Zealand (https://www.cricbuzz.com/cricket-series/2697/icc-cricket-world-cup-2019/squads) in the year 2019. We have categorized each team with six attributes namely: batting, bowling, all-rounder, fielding, experience and inexperience. The team combinations are always based on batting first or fielding first, home matches or away home matches, batting pitches or bowling pitches and so on.

S.No	Attributes(a_i)	Category
1	Batting(a_1)	Top Order, Middle Order, Finishing, Wicket keeper
2	Bowling(a_2)	Fastbowling , Spin Bowling
3	All rounder(a_3)	Fast bowling-Batsman, Spin bowling-Batsman
4	Fielding(a_4)	Power play-One, Power Play-Two, Power Play-Three
5	Experience(a_5)	Batting, Bowling, Fielding
6	Inexperience(a_6)	Batting, Bowling, Fielding

Here we have taken six attributes. The weighted fuzzy set value of teams India and New Zealand are as follows,

$$\begin{aligned} W(\underset{\sim}{IND}) =& \{(a_1, 0.9, 1), (a_2, 0.8, 0.9), (a_3, 0.8, 0.6), (a_4, 0.7, 0.5), \\ & (a_5, 0.8, 0.7), (a_6, 0.5, 0.5)\} \\ W(\underset{\sim}{NZ}) =& \{(a_1, 0.8, 0.7), (a_2, 0.8, 0.8), (a_3, 0.9, 0.8), (a_4, 0.7, 0.7), \\ & (a_5, 0.9, 0.8), (a_6, 0.4, 0.5)\} \end{aligned}$$

2.3.1. *Potential Measures of Two Cricket Teams.*

$$P(W(IN\underset{\sim}{D}),W(N\underset{\sim}{Z})) = \left\{\left[\,1-\sum_{i=1}^{6}\frac{\left|\mu_{W(IN\underset{\sim}{D})}(a_i)-\mu_{W(N\underset{\sim}{Z})}(a_i)\right|}{K}\,\right],\left[\,1-\sum_{i=1}^{6}\frac{\left|\eta_{W(IN\underset{\sim}{D})}(a_i)-\eta_{W(N\underset{\sim}{Z})}(a_i)\right|}{K}\,\right]\right\}$$

$$P(W(IN\underset{\sim}{D}),W(N\underset{\sim}{Z})) = \left\{\begin{array}{l}\left[\,1-\frac{|0.9-0.8|+|0.8-0.8|+|0.8-0.9|+|0.7-0.7|+|0.8-0.9|+|0.5-0.4|}{6}\,\right],\\ \left[\,1-\frac{|1-0.7|+|0.9-0.8|+|0.6-0.8|+|0.5-0.7|+|0.7-0.8|+|0.5-0.5|}{6}\,\right]\end{array}\right\}$$

$$P(W(IN\underset{\sim}{D}),W(N\underset{\sim}{Z})) = (0.96,0.85).$$

The potential measures of teams India and New Zealand ODI cricket team value is $(0.96, 0.85)$.

2.3.2. *Potential Measures between Attributes.*

$$P_e(a_i,a_j) = \left\{\left[\,1-\sum_{i=1}^{2}\frac{\left|\mu_{T_k}(a_i)-\mu_{T_k}(a_j)\right|}{2}\,\right],\left[\,1-\sum_{i=1}^{2}\frac{\left|\eta_{T_k}(a_i)-\eta_{T_k}(a_j)\right|}{2}\,\right]\right\}$$

where $i \neq j$

$$P_e(a_1,a_2) = \left\{\left[1-\frac{|0.9-0.8|+|0.8-0.8|}{2}\right],\left[1-\frac{|1-0.7|+|0.9-0.8|}{2}\right]\right\} = (0.95,0.8).$$

Similarly we can find the following values

$$P_e(a_1,a_3) = (0.9,0.75), P_e(a_1,a_4) = (0.85,0.75), P_e(a_1,a_5) = (0.9,0.8),$$
$$P_e(a_1,a_6) = (0.55,0.65), P_e(a_2,a_3) = (0.95,0.85), P_e(a_2,a_4) = (0.85,0.75),$$
$$P_e(a_2,a_5) = (0.95,0.9), P_e(a_2,a_6) = (0.65,0.65), P_e(a_3,a_4) = (0.85,0.9),$$
$$P_e(a_3,a_5) = (1,0.95), P_e(a_3,a_6) = (0.6,0.8), P_e(a_4,a_5) = (0.85,0.85),$$
$$P_e(a_4,a_6) = (0.75,0.9), P_e(a_5,a_6) = (0.6,0.75)$$

2.3.3. *Result and Interpretations.* Potential measures between teams India and New Zealand are $P(W(IN\underset{\sim}{D}),W(N\underset{\sim}{Z})) = (0.96,0.85)$. The membership value of attributes representing teams India and New Zealand are 0.96 similar. The impact of each attributes over teams India and New Zealand are 0.85 similar. Potential measure of any two attributes between teams India and New Zealand are found. The highest value of potential measure between any two attributes of teams India and New Zealand are $P_e(a_3,a_5) = (1,0.95)$. This value represents the All-rounder

(a_3) and Experience (a_5) of teams India and New Zealand are more similar and are the strength of both the teams. The least value of potential measure between any two attributes of teams India and New Zealand are $P_e(a_1, a_6) = (0.55, 0.65)$. This value represents the Batting (a_1) and Inexperience (a_6) of teams India and New Zealand are less similar and are the weakness of both the teams.

3. Perceived Measures of Weighted Fuzzy Sets

In this section, we introduce a new concept namely perceived measures of weighted fuzzy sets. Often perceptual opinions reported in the public domain do not reflect the exact reality. Perceived measures of weighted fuzzy sets give the results which is closer to the reality. Finding the outcomes of the sets is one of the difficult assignment to the researchers. We have used mutual association based on the membership values of the element in the set. We have paired the highest membership value with lowest membership values. In the same way we have paired all the elements in the sets. The following figure 1 and 2 explains in details about the perceived measures of weighted fuzzy sets.

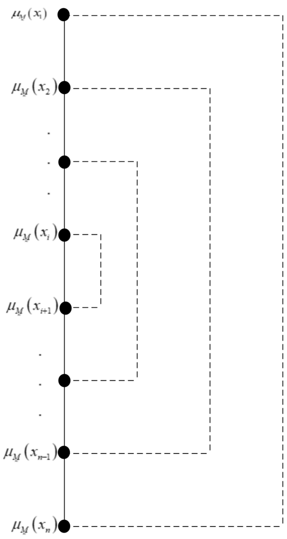

FIGURE 1. When n is even

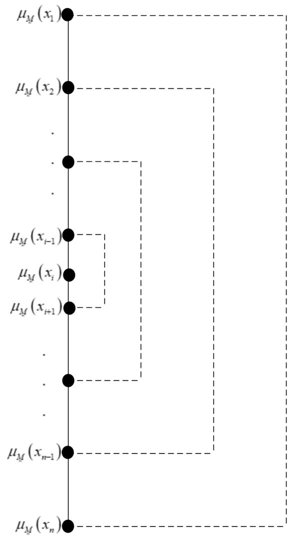

FIGURE 2. When n is odd

We define perceived measures of weighted fuzzy sets $\rho_{\underset{\sim}{M}}$ as follows

Case 1: when n is even

$$\rho_{\underset{\sim}{M}} = \left\{ \begin{array}{l} \left[1 - \left[\begin{array}{r} \frac{|\mu_{\underset{\sim}{M}}(x_1)-\mu_{\underset{\sim}{M}}(x_n)|}{d(\mu_{\underset{\sim}{M}}(x_i),\mu_{\underset{\sim}{M}}(x_n)} + \frac{|\mu_{\underset{\sim}{M}}(x_2)-\mu_{\underset{\sim}{M}}(x_{n-1})|}{d(\mu_{\underset{\sim}{M}}(x_2),\mu_{\underset{\sim}{M}}(x_{n-1})} + \\ \cdots + \frac{|\mu_{\underset{\sim}{M}}(x_i)-\mu_{\underset{\sim}{M}}(x_{i+1})|}{d(\mu_{\underset{\sim}{M}}(x_i),\mu_{\underset{\sim}{M}}(x_{i+1})} \end{array} \right] \right], \\ \qquad \left[1 - \left[\begin{array}{r} \frac{|\eta_{\underset{\sim}{M}}(x_1)-\eta_{\underset{\sim}{M}}(x_n)|}{d(\eta_{\underset{\sim}{M}}(x_i),\eta_{\underset{\sim}{M}}(x_n)} + \frac{|\eta_{\underset{\sim}{M}}(x_2)-\eta_{\underset{\sim}{M}}(x_{n-1})|}{d(\eta_{\underset{\sim}{M}}(x_2),\eta_{\underset{\sim}{M}}(x_{n-1})} + \\ \cdots + \frac{|\eta_{\underset{\sim}{M}}(x_i)-\eta_{\underset{\sim}{M}}(x_{i+1})|}{d(\eta_{\underset{\sim}{M}}(x_i),\eta_{\underset{\sim}{M}}(x_{i+1})} \end{array} \right] \right] \end{array} \right\}$$

Case 2: when n is odd

$$\rho_{\underset{\sim}{M}} = \left\{ \begin{array}{l} \left[1 - \left[\begin{array}{r} \frac{|\mu_{\underset{\sim}{M}}(x_1)-\mu_{\underset{\sim}{M}}(x_n)|}{d(\mu_{\underset{\sim}{M}}(x_i),\mu_{\underset{\sim}{M}}(x_n)} + \frac{|\mu_{\underset{\sim}{M}}(x_2)-\mu_{\underset{\sim}{M}}(x_{n-1})|}{d(\mu_{\underset{\sim}{M}}(x_2),\mu_{\underset{\sim}{M}}(x_{n-1})} + \\ \cdots + \frac{|\mu_{\underset{\sim}{M}}(x_{i-1})-\mu_{\underset{\sim}{M}}(x_{i+1})|}{d(\mu_{\underset{\sim}{M}}(x_{i-1}),\mu_{\underset{\sim}{M}}(x_{i+1})} + \frac{\mu_{\underset{\sim}{M}}(x_i)}{d(n/2)} \end{array} \right] \right], \\ \qquad \left[1 - \left[\begin{array}{r} \frac{|\eta_{\underset{\sim}{M}}(x_1)-\eta_{\underset{\sim}{M}}(x_n)|}{d(\eta_{\underset{\sim}{M}}(x_i),\eta_{\underset{\sim}{M}}(x_n)} + \frac{|\eta_{\underset{\sim}{M}}(x_2)-\eta_{\underset{\sim}{M}}(x_{n-1})|}{d(\eta_{\underset{\sim}{M}}(x_2),\eta_{\underset{\sim}{M}}(x_{n-1})} + \\ \cdots + \frac{|\eta_{\underset{\sim}{M}}(x_{i-1})-\eta_{\underset{\sim}{M}}(x_{i+1})|}{d(\eta_{\underset{\sim}{M}}(x_{i-1}),\eta_{\underset{\sim}{M}}(x_{i+1})} + \frac{\eta_{\underset{\sim}{M}}(x_i)}{d(n/2)} \end{array} \right] \right] \end{array} \right\}$$

here, $|\mu_{\underset{\sim}{M}}(x_i) - \mu_{\underset{\sim}{M}}(x_j)|$ denotes difference between two membership values and $d(\mu_{\underset{\sim}{M}}(x_i) - \mu_{\underset{\sim}{M}}(x_n))$ denotes the distance in unites, where $i \neq j, i, j = 1, 2, \cdots n$.

Example 3.

Case 1: *When n is even: Let*

$$W(\underset{\sim}{M}) = \{(a_1, 0.9, 1), (a_2, 0.8, 0.9), (a_3, 0.8, 0.6), (a_4, 0.7, 0.5), (a_5, 0.8, 0.7), (a_6, 0.6, 0.6)\}$$

be a weighted fuzzy set, then the perceived measures of weighted fuzzy set is calculated as follows, $\rho_{\underset{\sim}{M}} = \left\{\left[1 - [\frac{0.3}{5} + \frac{0.1}{3} + \frac{0}{1}], [1 - [\frac{0.4}{5} + \frac{0.4}{3} + \frac{0.1}{1}]\right]\right\} = (0.91, 0.69)$. *The perceived measures of given weighted fuzzy sets is* $(0.91, 0.69)$.

Case 2: *When n is odd: Let* $W(\underset{\sim}{M}) = \{(a_1, 0.8, 0.7), (a_2, 0.8, 0.8), (a_3, 0.9, 0.8), (a_4, 0.7, 0.7), (a_5, 0.9, 0.8)\}$ *be a weighted fuzzy set, then the perceived measures of weighted fuzzy set is calculated as follows,*

$$\rho_{\underset{\sim}{M}} = \left\{\left[1 - [\frac{0.1}{4} + \frac{0.1}{2} + \frac{0.9}{2.5}], [1 - [\frac{0.1}{4} + \frac{0.1}{2} + \frac{0.8}{2.5}]\right]\right\} = (0.47, 0.61).$$

The perceived measures of given weighted fuzzy sets is $(0.47, 0.61)$.

4. Conclusion

In this paper we have defined potential measures of weighted fuzzy sets and calculated the potential measures between ODI cricket teams India and New Zealand (2019). In the same way we calculated potential measure of any two attributes of ODI cricket teams India and New Zealand. Using this we concluded the strongest and weakest pair of attributes of team India and team New Zealand. Also we have defined of perceived measures weighted fuzzy sets and given a numerical example.

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