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INFLUENCE OF RADIATION ON MHD FREE CONVECTIVE FLOW OF VISCOUS FLUID IN A VERTICAL CHANNEL IN THE PRESENCE OF VARIABLE PROPERTIES

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ABSTRACT. A semi analytical approach has been carried out to analyze the combined effects of radiation and magnetic on the flow of viscous fluid between two parallel plates. It is assumed that the viscosity of the fluid and the thermal conductivity linearly varies with temperature. The dimensionless transport equations for the flow and heat transfer are solved using Differential Transform Method (DTM). The effects of physical pertinent parameters on the flow field are presented through graphs. In addition, the skin friction and the Nusselt number are evaluated. The present solutions are compared with the numerical one by Runge-Kutta Fourth-Fifth order along with shooting technique and it was in good agreement.

1. INTRODUCTION

The differential transform is a efficient method for analyzing and evaluating the transport equations to the problem. One dimensional DTM has been introduced first by Zhou [1] with Taylor's expansion to solve IVP that come out in electrical-circuits. He has given exact values of the nth derivative of an exact function values at a point in terms of boundary conditions (known/un-known) in a fast manner. Chen and Ho [2] have used two dimensional DTM to solve PDE problems. The new

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inductive reasoning of the two-dimensional D T M will protract the application of the method to linear PDE of fractional order introduced by Odibat et al. [3]. Ayaz [4] applied this method for the system of differential equations. Arikoglu and Ozkol [5,6] have solved both integro-differential equation systems and fractional integro-differential equations by utilizing differential transform method.

In the last few decades, several researchers have investigated the channel flow problems using DTM for finding semi-analytical solutions. To list a few, Rashidi et al. [7] have investigated a semi-analytical solution of micropolar flow between porous plates with mass injection. Chen et al. [8] examined the entropy generation with viscous dissipation effects in a parallel-plate vertical channel. The convection in a porous medium for a laminar, non-Darcian model flow in an inclined channel has been investigated by Komurgoz et al. [9].

Many papers concerned with the problem of natural convective flow in a vertical parallel channel have been published in the literature but only few papers have examined the variable properties on free convection flow in a parallel channel. To list a few, Umavathi and Shekar [10] investigated the conflated variations of viscosity and conductivity in a vertical channel using DTM. Recently, the impact of temperature dependence viscosity and conductivity over vertical parallel-plates was investigated with the inclusion of magnetic field using perturbation technique by Umavathi et al. [11] and with a inclusion of radiation effect using Adomian Decomposition Method by Ajibade and Bichi [12].

Thermal radiation has a very substantial application in space technology. The important industrial applications of radiation are water evaporation from open reservoirs, heating and cooling of chambers, solar power technology solar fans, solar collectors, photo chemical reactors, etc. In view of these applications, several researchers (Bakier [13], Raptis and Perdikis [14], Grosan and Pop [15], Ghosh and Beg [16]) analyzed the effect of radiation on convective flow of fluids.

The objective of the present problem is to investigate the conflated variations of magnetic field and thermal radiation on the fully developed free convective flow in a parallel channel. With the assumption of variable viscosity and conductivity, the transport equations are solved by differential transform method.

2. Mathematical Formulation

Considering a Cartesian coordinate system where $x*$-axis is parallel to the gravitational force in opposite direction and y-axis is perpendicular to the channel walls $y* = -L$ and $y* = +L$ respectively (fig.1). Since the fluid velocity $(u*, v*)$ is assumed to be parallel with the x*-axis, the following relations can be drawn:

$$v* = 0, \; \frac{\partial v*}{\partial y*} = 0, \; \frac{\partial p*}{\partial y*} = 0. \tag{2.1}$$

By employing the Boussinesq and Roseseland approximation, the problem will

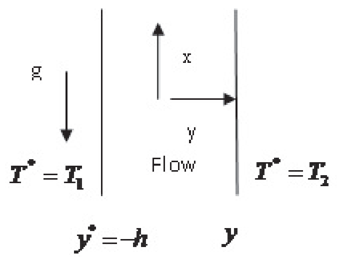

Figure 1. Physical geometry

be analyzed with the combined variations of viscosity and thermal conductivity. Based on the suitable assumptions, the dimensional governing equations are given as

$$\frac{d}{dy^*}\left(\mu \frac{du^*}{dy^*}\right) + \rho_0 g \beta \left(T^* - T_0\right) + \sigma B_0^2 u^* = 0 \tag{2.2}$$

$$\frac{d}{dy^*}\left(K \frac{dT^*}{dy^*}\right) + \mu \left(\frac{du^*}{dy^*}\right)^2 - \frac{dq_r}{dy^*} = 0 \tag{2.3}$$

and the respective boundaries are

$$u^* = 0, \; at \; y^* = \pm L \tag{2.4}$$

$$T^* = T_1, \; and \; y^* = -L, T^* = T_2 \; at \; y^* = +L \tag{2.5}$$

where the viscosity and thermal conductivity variations are assumed to be

$$\mu = \mu_0 e^{-a(T^* - T_0)} \tag{2.6}$$

$$K = K_0 e^{-b(T^* - T_0)} = K_0 \left(1 + b(T_0 - T^*)\right). \tag{2.7}$$

The non-dimensional parameters are given by

$$\left\{ \begin{array}{l} u = \frac{\rho_0}{\mu_0 g \beta L^2 \Delta T} u^*, \theta = \frac{T^* - T_0}{\Delta T}, y = \frac{y^*}{L} \\ N = \frac{\rho_0^2 L^4 g^2 \beta^2 \Delta T}{\mu_0 K_0}, q_r = -\frac{4\sigma *}{3\chi *} \frac{\partial T^4}{\partial y *} \end{array} \right. . \tag{2.8}$$

Using the above non-dimensional parameters, equations (2.2)-(2.6) will become

$$\frac{d^2 u}{dy^2} - b_\nu \frac{d\theta}{dy}\frac{du}{dy} + \theta + b_\nu \theta^2 + M^2 u = 0 \tag{2.9}$$

$$\begin{aligned} \frac{d^2\theta}{dy^2} - b_k \left(\frac{d\theta}{dy}\right)^2 \quad + \quad & N\left(\frac{du}{dy}\right)^2 + (b_k - b_\nu) N \left(\frac{du}{dy}\right)^2 \theta \\ & -b_\nu b_k N \left(\frac{du}{dy}\right)^2 \theta^2 - R\theta = 0 \end{aligned} \tag{2.10}$$

and the respective boundaries

$$u = 0, \ at \ y = \pm 1 \tag{2.11}$$

$$\theta = 1 + m \ at \ y = -1, \ \theta = 1 \ at \ y = 1 \tag{2.12}$$

where $b_\nu = a\Delta T$ is the viscosity parameter, $b_k = b\Delta T$ is the conductivity parameter, N is the buoyancy ratio and $m = \frac{T_1 - T_2}{\Delta T}$ is the wall temperature ratio.

3. Solution

Semi-Analytical Solution (DTM)

The D T M function u(y) is defined as the following

$$\bar{U}(\kappa) = \frac{1}{\kappa!}\left[\frac{d^\kappa u(y)}{dy^\kappa}\right]_{y=0}. \tag{3.1}$$

The D T M inverse-transform of is defined as follows:

$$u(y) = \sum_{\kappa=0}^{\infty} \bar{U}(\kappa) y^\kappa. \tag{3.2}$$

Since in real world applications, u(y) has been regarded as finite series, equation (3.2) can be re-written as

$$u(y) = \sum_{\kappa=0}^{n} \bar{U}(\kappa) y^\kappa. \tag{3.3}$$

TABLE 1. The operations for D T M

Original	Transformation
$\Upsilon(\varsigma)=\Phi(\varsigma)\pm\Psi(\varsigma)$	$\Upsilon(\varsigma)=\Phi(\varsigma)\pm\Psi(\varsigma)$
$\Upsilon(\varsigma)=\alpha\Phi(\varsigma)$	$\Upsilon(\varsigma)=\alpha\Phi(\varsigma)$
$\Upsilon(\varsigma)=(\varsigma+1)\Phi(\varsigma+1)$	$\Upsilon(\varsigma)=(\varsigma+1)\Phi(\varsigma+1)$
$\Upsilon(\varsigma)=(\varsigma+1)(\varsigma+2)\Phi(\varsigma+2)$	$\Upsilon(\varsigma)=(\varsigma+1)(\varsigma+2)\Phi(\varsigma+2)$
$\Upsilon(\varsigma)=\sum\limits_{l=0}^{\varsigma}\Phi(l)\Theta(\varsigma-l)$	$\Upsilon(\varsigma)=\sum\limits_{l=0}^{\varsigma}\Phi(l)\Theta(\varsigma-l)$
$\Upsilon(\varsigma)=\delta(\varsigma-m)=\begin{cases}1, if\varsigma=m\\ 0, if\varsigma\neq m\end{cases}$	$\Upsilon(\varsigma)=\delta(\varsigma-m)=\begin{cases}1, if\varsigma=m\\ 0, if\varsigma\neq m\end{cases}$

For the sake of brevity, we have listed here the the velocity and temperature profile for the case of combined viscosity and conductivity variations (Case-3 only). According to the Table 1 and using equations (3.1)-(3.3), the governing equations (2.2)-(2.6) have been solved and as follows:

$$\overline{U}(2+\kappa)=\begin{cases}V\ \frac{1}{(1+\kappa)(2+\kappa)}\Big(b_{\upsilon}\sum\limits_{s=0}^{\kappa}(s+1)(\kappa-s+1)\overline{U}(s+1)\overline{\Theta}(\kappa-s+1)\\ \quad -\overline{\Theta}(\kappa)-b_{\upsilon}\sum\limits_{s=0}^{\kappa}\overline{\Theta}(\kappa-s)\overline{\Theta}(r)\Big)+M^2\overline{U}(\kappa)\end{cases}$$

$$\overline{\Theta}(2+\kappa)=$$
$$\begin{cases}\frac{1}{(1+\kappa)(2+\kappa)}\times b_k\sum\limits_{m=0}^{\kappa}(m+1)(\kappa-m+1)\overline{\Theta}(m+1)\overline{\Theta}(\kappa-m+1)\\ -N\sum\limits_{m=0}^{\kappa}(m+1)(\kappa-m+1)\overline{U}(m+1)\overline{U}(\kappa-m+1)\\ -(b_{\kappa}-b_{\nu})N\sum\limits_{m=0}^{\kappa}\sum\limits_{s=0}^{m}(m-s+1)\overline{U}(m-s+1)(\kappa-m+1)\times\overline{U}(\kappa-m+1)\overline{\Theta}(s)\\ +b_{\nu}b_{\kappa}N\sum\limits_{m=0}^{\kappa}\sum\limits_{s=0}^{\kappa}\sum\limits_{t=0}^{\kappa}(\kappa-m+1)(m-s+1)\overline{U}(\kappa-m+1)\overline{U}(m-s+1)\overline{\Theta}(s-t)\overline{\Theta}(t)\\ -R\overline{\Theta}(\kappa)\end{cases}.$$

With the help of transformations, the corresponding boundaries become

$$\overline{U}(0)=a_{1*},\overline{U}(1)=a_{2*},\overline{\Theta}(0)=b_{1*},\overline{\Theta}(1)=b_{2*}.$$

The transformed transport equations for all the cases are solved with the corresponding boundary conditions to find the dimensionless velocity and temperature and the obtained results are illustrated in graphical and tabular forms.

Numerical Solution: In order to compare the semi analytical results, we have solved the non-dimensional governing equations with boundary conditions for all the three cases numerically by Runge Kutta Felhlberg fourth-fifth order with shooting technique.

4. Results and Discussion

The behavior of velocity (u) and temperature (θ) for various flow variables in graphical form are explored in this segment. The impression of the natural convective flow of fluid with variations in viscosity, thermal conductivity, and conflated variations of both is investigated including magnetic and radiation effect. The default values of the pertinent constants took as N=0.5, m=1, M=1, R=3, b_ν=1, b_k=1 and unless otherwise specified.

Fig. 2(a) and (b) demonstrates the combined effect of magnetic and the viscosity on the velocity and temperature distribution respectively. From Fig. 2, it is noted that a raise in the magnetic number is to reduce the velocity and temperature profiles for both cases. Further, the variations in viscosity lead to change the shape of the temperature distribution from parabolic due to fluctuations in the resistance at the plate. The conflicted effects of radiation and the viscosity variations on the flow-velocity & thermal profile are shown in the Fig 3. It is noted that an increase in the radiation parameter is to increase the velocity and temperature profile for both cases.

Fig. 4(a) and (b) displays the influence of magnetic and conductivity variations on the velocity and thermal profile. It is observed that a raise in the magnetic number is to decrease the velocity profile and increase the temperature distribution. Also, it is noted that the flow rate has been reduced in the case of higher magnetic field, as expected. The velocity and temperature profile with various values of radiation and conductivity parameter have been illustrated in Fig. 5 (a) and (b). It is found that the velocity and thermal distributions are increased when the radiation is applied.

Fig. 6 (a) and (b) displays the variations of the wall-ratio on the flow and thermal profile. An increase in the wall ratio parameter is to increase the velocity and temperature distributions in the presence/absence of radiation and magnetic field. With the presence of combined effects, the temperature of the fluid increased and then decreased at the another plate.

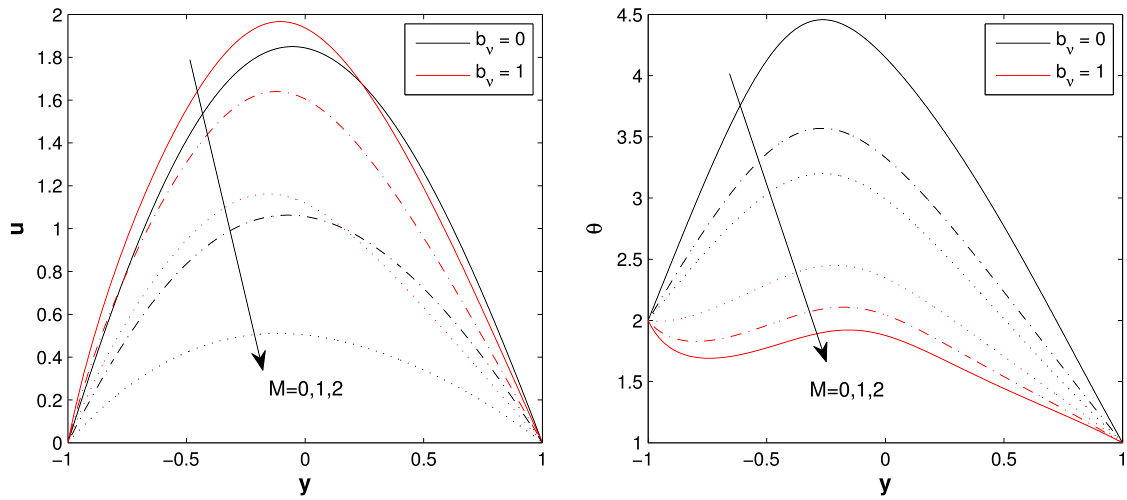

FIGURE 2. Effect of magnetic number on (a) the velocity (b) the temperature with various values of Viscosity parameter.

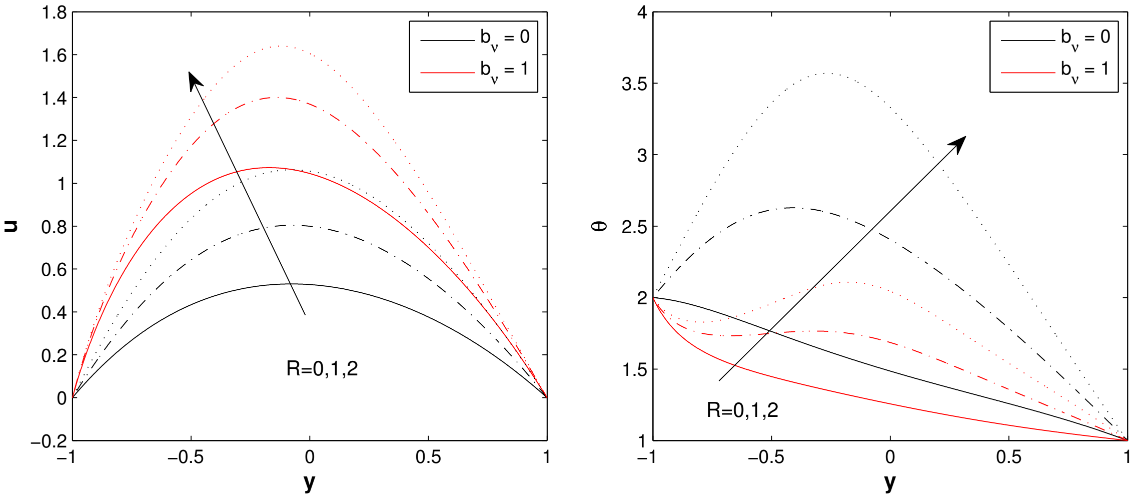

FIGURE 3. Influence of the radiation parameter R on (a) the velocity (b) the temperature with viscosity parameter.

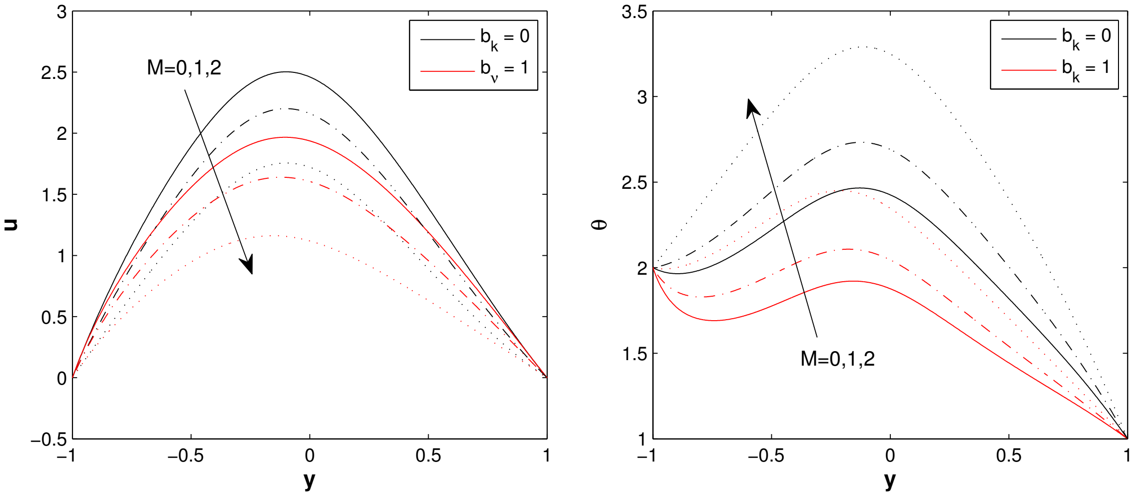

FIGURE 4. Effect of magnetic number on (a) the velocity (b) the temperature with conductivity parameter.

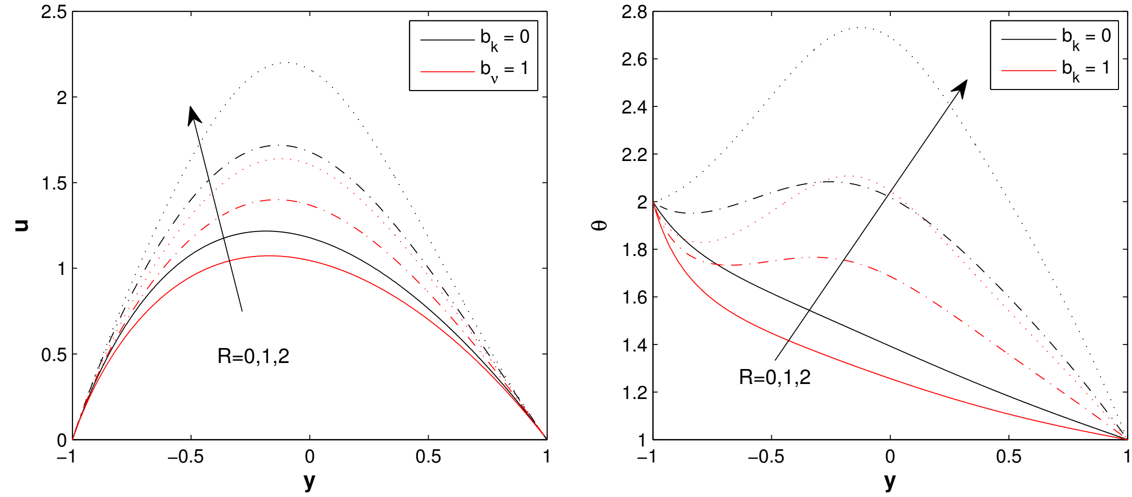

FIGURE 5. Influence of the radiation parameter R on (a) the velocity (b) the temperature with conductivity parameter.

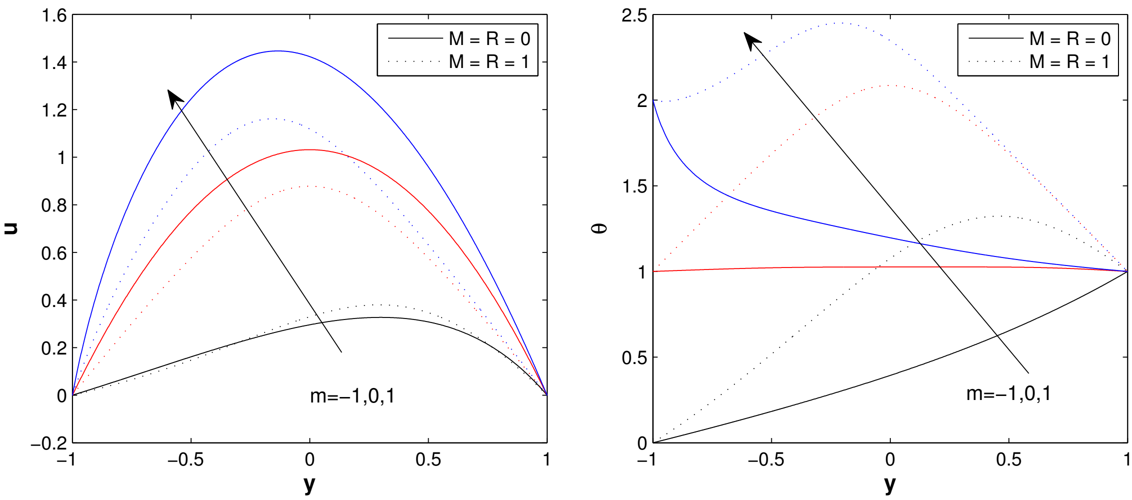

FIGURE 6. (a) Velocity profile and (b) temperature profile for variation of wall temperature ratio

5. CONCLUSION

In this paper, we have investigated the influence of heat generation-absorption on MHD natural convective viscous fluid flow in a parallel channel with inclusion of variable properties. By the approach of DTM, the semi-analytical results have been obtained for the proposed problem. Also, these results have been checked with the numerical one by RKF method with shooting-technique and it was in good agreement.

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