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FUZZY LINEAR PROGRAMMING PROBLEM WITH GENERALIZED TRAPEZOIDAL FUZZY NUMBERS

S. MOHANASELVI[1], K. PRABAKARAN, AND K. SUJA

ABSTRACT. In this paper, a solution procedure for the fuzzy optimal solution of fuzzy linear programming problem with generalized trapezoidal fuzzy number is presented. A new ranking function and arithmetic operations of generalized fuzzy numbers are proposed in parametric form for solving fuzzy linear programming problem by not converting it into deterministic model. The efficacy of the proposed method is reveled through an illustrative example and a comparison study has been made with existing methods available in literature.

1. INTRODUCTION

Fuzzy modeling had made the decision makers feel convenient in making proper decisions as it tackles with the uncertainties present in the real world applications. The idea of fuzzy decision making was first introduced by Bellman and Zadeh [1]. In literature several different approaches have been proposed to solve Fuzzy linear programming problem(FLPP). One important method for solving FLPP is by using ranking function. Another class of methods is based on parametric form in which the original fuzzy linear programming problem is transformed into equivalent problem in crisp environment [2-4]. Ganesan and Veeramani [5] introduced a new for solving linear programming problem with symmetric trapezoidal fuzzy numbers by not converting them to deterministic model. Chen [6] in 1985 pointed

[1]*corresponding author*

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out that in many real life situations it is very difficult to frame the membership function in the normal form and hence introduced the idea of generalized fuzzy numbers. From then the study on generalized fuzzy numbers has been active. Several authors have introduced different ranking function and arithmetic operations for generalized fuzzy numbers [1], [7-12]. Amit kumar et al [13] has explained about disadvantage of normalization process and used fuzzy big-M and two phase method for solving FLPP involving generalized fuzzy numbers. In this paper we propose a new ranking function of generalized trapezoidal fuzzy number (GTrFN) and applied in primal simplex algorithm in solving FLPP by not converting to deterministic model. The basic terminologies about GTrFN are introduced in section 2 and ranking of GTrFN and its arithmetic operations are proposed in the same section. The mathematical model of FLPP with GTrFNs is presented and related theorems are proved in section 3. In section 4, fuzzy version of simplex algorithm is proposed and obtained the fuzzy optimal solution by not converting to deterministic model. In next section a numerical example is discussed to illustrate the theory proposed in this paper.

2. Preliminaries

Definition 2.1. *A fuzzy set A defined on the set of real numbers R is said to be a generalized trapezoidal fuzzy number if its membership function $[\mu_A^{\mathrm{w}} : R \to [0,1]]$ has the following characteristics:*

(i) $\mu_{\tilde{A}}^{\mathrm{w}}(\mathrm{x}) = 0,\ -\infty < \mathrm{x} \leq \mathrm{a}_1.$

(ii) $\mu_{\tilde{A}}^{\mathrm{w}}(\mathrm{x})$ *is strictly increasing on* $[\mathrm{a}_1, \mathrm{b}_1]$.

(iii) $\mu_{\tilde{A}}^{\mathrm{w}}(\mathrm{x}) = \mathrm{w}$ *in* $[\mathrm{b}_1, \mathrm{c}_1]$.

(iv) $\mu_{\tilde{A}}^{\mathrm{w}}(\mathrm{x})$ *is strictly decreasing on* $[c_1, \mathrm{d}_1]$.

(v) $\mu_{\tilde{A}}^{\mathrm{w}}(\mathrm{x}) = 0,\ \ \mathrm{d}_1 \leq \mathrm{x} < \infty.$

and is denoted by $A = (\mathrm{a}_1, \mathrm{b}_1, \mathrm{c}_1, \mathrm{d}_1; \mathrm{w})$*,* $0 < \mathrm{w} \leq 1$*. The membership function of the generalized trapezoidal fuzzy number is defined as*

$$\mu_A^{\mathrm{w}}(\mathrm{x}) = \begin{cases} \mathrm{w}\left(\frac{\mathrm{x}-\mathrm{a}_1}{\mathrm{b}_1-\mathrm{a}_1}\right); & \mathrm{a}_1 \leq \mathrm{x} \leq \mathrm{b}_1 \\ \mathrm{w}; & b_1 \leq \mathrm{x} \leq c_1 \\ \mathrm{w}\left(\frac{\mathrm{d}_1-\mathrm{x}}{d_1-\mathrm{c}_1}\right); & c_1 \leq \mathrm{x} \leq d_1 \end{cases}.$$

Definition 2.2. *The α -cut of generalized trapezoidal fuzzy numbers is defined as*

$$\alpha_A = \left[\mathrm{a}_1 + \frac{\alpha(\mathrm{b}_1 - \mathrm{a}_1)}{\mathrm{w}},\ \mathrm{d}_1 - \frac{\alpha(\mathrm{d}_1 - \mathrm{c}_1)}{\mathrm{w}}\right]$$

for all $\alpha \in [0, \mathrm{w}],\ \ 0 < \mathrm{w} \leq 1.$

Definition 2.3. *The parametric form of the generalized fuzzy number is given by a pair* $A = (\mathrm{L(a)}, \mathrm{R(a)})$ *of functions* $\mathrm{L}[\mathrm{a(r)}]$ *and* $\mathrm{R}[\mathrm{a(r)}]$ *where* $\mathrm{L}[\mathrm{a(r)}]$ *is a bounded left continuous and* $\mathrm{R}[\mathrm{a(r)}]$ *is a bounded right continuous functions such that* $\mathrm{L}[\mathrm{a(r)}] \leq R[\mathrm{a(r)}], 0 \leq \mathrm{r} \leq \mathrm{w}$*,* $0 < \mathrm{w} \leq 1$*.*

Definition 2.4. *For an arbitrary generalized trapezoidal fuzzy number* $A = (\mathrm{L(a)}, \mathrm{R(a)})$*, the number* $\mathrm{a}_0 = \left(\frac{L(\mathrm{w})+R(\mathrm{w})}{2}\right)$; $0 < \mathrm{w} \leq 1$ *is said to be core of* A*.The left fuzziness and right fuzziness functions of* A *are given by* $\mathrm{a}_L = (\mathrm{a}_0 - L(a))$ *and* $\mathrm{a}_R = (R(\mathrm{a}) - \mathrm{a}_0)$ *respectively. A GTrFN* $A = (\mathrm{a}_1, \mathrm{b}_1, \mathrm{c}_1, \mathrm{d}_1; \mathrm{w})$ *with height* w *is also represented in parametric form by* $A = (\mathrm{a}_0, \mathrm{a}_L, \mathrm{a}_R; \mathrm{w})$*.*

2.1. **Ranking of Generalized trapezoidal fuzzy numbers.** In this section a new ranking function of GTrFN is proposed. For an arbitrary GTrFN $A = (\mathrm{a}_1, \mathrm{b}_1, \mathrm{c}_1, \mathrm{d}_1; \mathrm{w})$ $= (\mathrm{a}_0, \mathrm{a}_L, \mathrm{a}_R; \mathrm{w})$, we define the magnitude of A by

$$\begin{aligned}\Re(A) &= \frac{1}{2}\left(\int_0^{\mathrm{w}} (\mathrm{L(a)} + R(a) + \mathrm{a}_0)\ \mathrm{f(r)}\ \mathrm{dr}\right)\\ &= \frac{1}{2}\left(\int_0^{\mathrm{w}} (\mathrm{a}_R + 4\mathrm{a}_0 - \mathrm{a}_L)\ \mathrm{f(r)}\ \mathrm{dr}\right),\end{aligned}$$

$0 \leq \mathrm{r} \leq \mathrm{w}$, $0 < \mathrm{w} \leq 1$, where the function $f(r)$ is a non-negative and increasing function on $[0, \mathrm{w}]$ with $f(0) = 0$, $\mathrm{f}(r) = r;\ \ 0 < r \leq \mathrm{w}$ and $\int_0^1 \mathrm{f(r)}\ \mathrm{dr} = \frac{1}{2};\ if\ \mathrm{w} = 1$. The function $f(r)$ is a weighting function and is assumed as $f(r) = r$ in this paper $\Re(A)$ is used to rank fuzzy numbers. For any two generalized trapezoidal fuzzy numbers $A = (\mathrm{a}_0, \mathrm{a}_L, \mathrm{a}_R; \mathrm{w}_A)$ and $B = (b_0, b_L, b_R; \mathrm{w}_B)$ we define the ranking of A and B by comparing the $\Re(A)$ and $\Re(B)$ on R as follows:

(i). $A \succeq B$ if and only if $\Re(A) \geq \Re(B)$,
(ii). $A \preceq B$ if and only if $\Re(A) \leq \Re(B)$,
(iii). $A \approx B$ if and only if $\Re(A) = \Re(B)$.

Example 1. *Consider an example discussed by Ching-Hsue Cheng [14], Sheen [15] and Ha Thi Xuan Chi [16] for comparing normal fuzzy number and generalized fuzzy number* $A = (3, 5, 5, 7; 1)$*,* $B = (3, 5, 5, 7; 0.8)$*. Represent these fuzzy numbers in core, left fuzziness, right fuzziness index functions and height as follows* $A = (5, 2 - 2\mathrm{r}, 2 - 2\mathrm{r}; 1)$*,* $0 < \mathrm{r} \leq 1$ *and* $B = (5, 2 - 2\mathrm{r}, 2 - 2\mathrm{r}; 0.8)$*,* $0 < \mathrm{r} \leq 0.8$*. Then their ranks are given below:*

TABLE 1. Comparison of ranking function

Methods	$\Re(A)$	$\Re(B)$
Ching-Hsue Cheng's Method(1998)	*5.03*	*5.02*
Sheen's method(2006)	*5.044*	*5.018*
Ha Thi Xuan Chi's Method(2018)	*5*	*5*
Proposed method	*3.74*	*2.4*

By the proposed ranking we have $\Re(A) = \frac{1}{2}\int\limits_0^1 15\mathrm{r}\,\mathrm{dr} = 3.74$ *and* $\Re(B) = \frac{1}{2}\int\limits_0^{0.8} 15\mathrm{r}\,\mathrm{dr} = 2.4$. *Thus the ranking order is* $A \succ B$.

2.2. **Arithmetic operations of Generalized trapezoidal fuzzy numbers.** The proposed arithmetic operations on two generalized trapezoidal fuzzy numbers $A = (\mathrm{a}_0, \mathrm{a}_L, \mathrm{a}_R; \mathrm{w}_A)$ and $B = (b_0, b_L, b_R; \mathrm{w}_B)$ is defined as follows:

1. $A + B = (\mathrm{a}_0, \mathrm{a}_L, \mathrm{a}_R; \mathrm{w}_A) + (b_0, b_L, b_R; \mathrm{w}_B) = \mathrm{a}_0 + b_0, \max(\mathrm{a}_L, b_L), \max(\mathrm{a}_R, b_R); \min(\mathrm{w}_A, \mathrm{w}_B))$
2. $A - B = (\mathrm{a}_0, \mathrm{a}_L, \mathrm{a}_R; \mathrm{w}_A) - (b_0, b_L, b_R; \mathrm{w}_B) = (\mathrm{a}_0 - b_0, \max(\mathrm{a}_L, b_L), \max(\mathrm{a}_R, b_R); \min(\mathrm{w}_A, \mathrm{w}_B))$
3. $A \times B = (\mathrm{a}_0, \mathrm{a}_L, \mathrm{a}_R; \mathrm{w}_A) \times (b_0, b_L, b_R; \mathrm{w}_B) = (\mathrm{a}_0 \times b_0, \max(\mathrm{a}_L, b_L), \max(\mathrm{a}_R, b_R); \min(\mathrm{w}_A, \mathrm{w}_B))$
4. $A \div B = (\mathrm{a}_0, \mathrm{a}_L, \mathrm{a}_R; \mathrm{w}_A) \div (b_0, b_L, b_R; \mathrm{w}_B) = (\mathrm{a}_0 \div b_0, \max(\mathrm{a}_L, b_L), \max(\mathrm{a}_R, b_R); \min(\mathrm{w}_A, \mathrm{w}_B))$

3. LINEAR PROGRAMMING PROBLEM WITH GENERALIZED FUZZY NUMBER

Let $\Im(R)$ be set of all GTrFNs. A fuzzy linear programming problem with all parameters considered as generalized trapezoidal fuzzy numbers is defined as follows:

$$\max \mathrm{Z} \approx CX \text{ subject to } \mathrm{A} \preceq Xb, X \succeq 0. \tag{3.1}$$

Definition 3.1. *Any vector* $X = (\mathrm{x}_1, \mathrm{x}_2, \ldots\ldots, \mathrm{x}_\mathrm{n})^T \in \Im^n(R)$ *is said to be a fuzzy feasible solution of (3.1) if and only if satisfies all the constraints and non-negativity restrictions of (3.1).*

Definition 3.2. *Let $S = \{\mathrm{X} \in \Im^n(R); A\mathrm{X} \preceq \mathrm{b}; \mathrm{X} \succeq 0\}$ be the set of all feasible solution of (3.1). A fuzzy feasible solution $X^* \in S$ is said to be a fuzzy optimal solution to (3.1) if $\mathrm{C}X^* \succeq \mathrm{C}X$ for all $\mathrm{X} \in S$.*

Definition 3.3. *Any fuzzy linear programming problem can be converted to its standard form by introducing necessary fuzzy slack variables and fuzzy surplus variables. The standard form of fuzzy linear programming problem is defined as*

$$\mathrm{A}X \approx b\,;\; X \succeq 0.$$

Consider the system $\mathrm{A}X \approx b$,where A is $\mathrm{m} \times \mathrm{n}$ fuzzy matrix and rank of A is m. Let B be any $m \times m$ fuzzy matrix framed by m linearly independent columns of A. That is

$$\begin{aligned} \mathrm{A}X &\approx b \\ (\mathrm{B};\mathrm{N})\begin{pmatrix} X_B \\ X_N \end{pmatrix} &\approx b. \end{aligned}$$

Then we have $X_B \approx \mathrm{B}^{-1}b = \left(\mathrm{x}_{B_1}, \mathrm{x}_{B_2}, \mathrm{x}_{B_3}, \ldots.., \mathrm{x}_{B_m}\right)^T$ and $X_N \approx 0$. Then $X = \left(\mathrm{x}_{B_1}, \mathrm{x}_{B_2}, \mathrm{x}_{B_3}, \ldots, \mathrm{x}_{B_m}, 0, 0, \ldots, 0\right)^T$ is called the fuzzy basic solution. Here X_B is the fuzzy basic solution.

Definition 3.4. *A fuzzy basic solution which satisfies the non negativity restriction $X_B \succeq 0$ is called fuzzy basic feasible solution of (3.1).*

Definition 3.5. *A fuzzy basic solution to $\mathrm{A}X \approx b$ is degenerate if one or more of the fuzzy basic variables vanishes.*

Definition 3.6. *A fuzzy feasible solution is said to be a fuzzy optimal solution if it optimizes the objective function of the fuzzy linear programming problem.*

4. Improving a fuzzy basic feasible solution

Consider the standard form of fuzzy linear programming problem, $\mathrm{A}X \approx b$. Let X_B be a fuzzy basic solution. $X_B = \left(\mathrm{x}_{B_1}, \mathrm{x}_{B_2}, \mathrm{x}_{B_3}, \ldots, \mathrm{x}_{B_m}\right)^T$ where $BX_B \approx b$. Corresponding to any such X_B, we define C_B, called reduced fuzzy cost vector, containing the prices of the basic variables i.e. $C_B = \left(\mathrm{c}_{B_1}, \mathrm{c}_{B_2}, \mathrm{c}_{B_3}, \ldots, \mathrm{c}_{B_m}\right)$. The value of the objective function is given by $\mathrm{Z} \approx C_B^T X_B$. Let $\mathrm{J_N}$ be the set of indices associated with the current non basic variables. For each non basic variables $\mathrm{x_j}, \mathrm{j} \in$

J_N we define $\mathrm{Z_j} \approx C_B^T \mathrm{Y_j} \approx C_B^T \mathrm{B}^{-1}\mathrm{a_j}$. To improve the objective function Z, we have to find other fuzzy basic feasible solution by replacing one of the columns of basis matrix The following theorems help us in obtaining the fuzzy optimal solution [8].

Theorem 4.1 (Improving fuzzy basic solution). *Let X_B be fuzzy basic solution. If for any fuzzy basic feasible solution with basis B the condition $(Z_j - C_j) \prec 0$ holds for some non basic variables $\mathrm{x_j and} Y_j \succ 0$ then it is possible to obtain a new fuzzy basic solution with the objective value $\mathrm{Z_{New}}$ such that $\mathrm{Z} \preceq \mathrm{Z_{New}}$.*

Theorem 4.2 (Unboundedness). *Let X_B be fuzzy basic solution. If for any fuzzy basic feasible solution with basis B the condition $(Z_j - C_j) \prec 0$ holds for some non basic variables $\mathrm{x_j and Y}_j \preceq 0$ then the fuzzy linear programming problem has unbounded solution.*

Theorem 4.3 (Fuzzy optimality). *Let X_B be a fuzzy basic feasible solution to a fuzzy linear programming problem with corresponding objective value $\mathrm{Z} \approx C_B^T X_B$. If $(Z_j - C_j) \succeq 0$ for every $\mathrm{x_j}, 1 \leq \mathrm{j} \leq \mathrm{n}$, then X_B is the fuzzy optimal solution.*

4.1. **Algorithm.** (Fuzzy primal simplex algorithm) [5,16]

(1) Represent each fuzzy data $(\mathrm{a}_1, \mathrm{b}_1, \mathrm{c}_1, \mathrm{d}_1; \mathrm{w})\,; 0 < \mathrm{w} \leq 1$ in the given fuzzy linear programming problem in terms of $(\mathrm{a}_0, \mathrm{a}_L, \mathrm{a}_R; \mathrm{w})$.
(2) Convert the fuzzy linear programming problem to its standard form by introducing necessary fuzzy slack variables and fuzzy surplus variables. Put the coefficients of these fuzzy slack and fuzzy surplus variables equal to zero in the objective function.
(3) For the given standard fuzzy linear programming problem choose an initial basic feasible solution.
(4) Let $X_B \approx \mathrm{B}^{-1}b; X_N \approx 0 \& Z \approx C_B X_B$ and $Z_\mathrm{j} - C_\mathrm{j} \approx C_B \mathrm{B}^{-1} a_\mathrm{j} - c_\mathrm{j}$ calculate for all non basic variables and let $Z_\mathrm{k} - C_\mathrm{k} \approx \max Z_\mathrm{j} - C_\mathrm{j}$.
(5) If $Z_\mathrm{k} - C_\mathrm{k} \succeq 0$ then current fuzzy basic fusible solution fuzzy optimal solution.
(6) If $Y_k \preceq 0$ then the solution is unbounded.
(7) If $Y_k \succ 0$ then $\mathrm{x_k}$ enters the basic and $\mathrm{x}_{B_\mathrm{r}}$ leaves the basis providing that $\frac{(\mathrm{B}^{-1}b)_\mathrm{r}}{\mathrm{y_{rk}}} = \min\limits_{1 \leq \mathrm{i} \leq \mathrm{n}} \left\{ \frac{(\mathrm{B}^{-1}b)_\mathrm{i}}{\mathrm{y_{ik}}}, \mathrm{y_{ik}} \succ 0. \right.$
(8) Update the basis B where a_j replaces $\mathrm{a}_{B_\mathrm{r}}$.

5. Numerical Example

Consider an example discussed by Amit Kumar et al [13].

$$\begin{aligned} &\text{MaxZ} \approx (3,5,8,13;0.6) \otimes x_1 + (4,6,10,16;0.5) \otimes x_2 \\ &\text{such that } 3x_1 \oplus x_2 \preceq (1,3,5,6;0.9) \\ &\qquad 2x_1 \oplus x_2 \preceq (1,2,4,7;0.7) \\ &\qquad x_1, x_2 \succeq 0 \end{aligned}$$

Now representing the above problem in terms location index, left and right fuzziness index function the above problem can be converted

$$\text{MaxZ} \approx \left(6.5, \tfrac{21-20r}{6}, \tfrac{39-50r}{6}; 0.6\right) \otimes x_1 + (8, 4-4r, 8-12r; 0.5) \otimes x_2$$

such that

$$\begin{aligned} &3x_1 \oplus x_2 \preceq \left(4, \tfrac{27-20r}{9}, \tfrac{18-10r}{9}; 0.9\right) \\ &2x_1 \oplus x_2 \preceq \left(3, \tfrac{14-10r}{7}, \tfrac{28-30r}{7}; 0.7\right) \\ &x_1, x_2 \succeq 0 \end{aligned}$$

Convert the above fuzzy linear programming problem to the standard form and obtain the initial basic solution . Using step 5 of the proposed algorithm x_2 enters the basis and s_2 leaves the basis. Using step 7 we obtain the following fuzzy optimal solution as follows:

$$\begin{aligned} &\mathrm{x}_1 \approx 0; \quad \mathrm{x}_2 \approx \left(3, \tfrac{14-10r}{7}, \tfrac{28-30r}{7}; 0.7\right) \quad \textit{and} \\ &Z \approx (24, 4-4r, 8-12r; 0.5) \end{aligned}$$

Comparison between the proposed method and Amit kumar et.al [13] method is given below:

Table 2. Comparison table of proposed method

Methods	Max Z
Amit kumar et.al method(2010)	$Z = (26, 8-8r, 72-72r; 0.5)$
Proposed method	$Z = (24, 4-4r, 8-12r; 0.5)$

It can be noted that the width of the fuzzy optimal solution obtained by the proposed method is narrow when compared with Amit Kumar et al method.

6. CONCLUSION

A new ranking function of generalized trapezoidal fuzzy number is proposed in this paper. A comparison study has been made with the existing methods to validate the proposed ranking function and can be noted that it is computationally easy while using in applications. A generalized simplex algorithm using the proposed ranking function and arithmetic operations of generalized trapezoidal fuzzy number is presented for obtaining the fuzzy optimal solution to a fuzzy linear programming problem.

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DEPARTMENT OF MATHEMATICS, COLLEGE OF ENGINEERING AND TECHNOLOGY
FACULTY OF ENGINEERING AND TECHNOLOGY, SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
SRM NAGAR, KATTANKULATHUR-603203, KANCHIPURAM, CHENNAI, TN, INDIA
Email address: mohanass@srmist.edu.in

DEPARTMENT OF MATHEMATICS, COLLEGE OF ENGINEERING AND TECHNOLOGY
FACULTY OF ENGINEERING AND TECHNOLOGY, SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
SRM NAGAR, KATTANKULATHUR-603203, KANCHIPURAM, CHENNAI, TN, INDIA
Email address: prabakak2@srmist.edu.in

DEPARTMENT OF MATHEMATICS, COLLEGE OF ENGINEERING AND TECHNOLOGY
FACULTY OF ENGINEERING AND TECHNOLOGY, SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
SRM NAGAR, KATTANKULATHUR-603203, KANCHIPURAM, CHENNAI, TN, INDIA
Email address: sujak@srmist.edu.in