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NUMERICAL SOLUTION OF MAGNETOHYDRODYNAMIC VISCOUS FLOW OVER A SHRINKING SHEET WITH SECOND ORDER SLIP UNDER FUZZY ENVIRONMENT

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ABSTRACT. In this paper, we investigate magnetohydrodynamic viscous flow with second order slip flow model over a permeable shrinking surface under fuzzy environment. The main differential equation can be obtained by using similarity variable technique. Then using Zadeh's extension theorem the same equation can be fuzzified. The effect of magnetic parameter has been investigated in fuzzy environment. It is found that there is a range in which magnetic parameter will work and also the corresponding range will same as in crisp solution.

1. INTRODUCTION

Most of the real life phenomenon are uncertain and imprecise in nature. The motion of rigid bodies or fluids, vehicle, washing machine etc. are uncertain due to uncertainty of equation of motion, the variable and the parameter in mathematical model are also uncertain. Hence there are enough scope to fuzzify variable separated and the equation as a while here are Fuzzified the equations of motion, variable, parameter and the boundary condition of the flow model of the fluid flow due to a moving boundary.

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Sakiadis introduce the concept on boundary layer on a stretching surface with a constant speed. But Wang discovered the exact solution of Navier Stokes equation of the Sakiadis proposed problem with lot of assumption. Subsequently many researchers has done lots of work on the said problem with constrain to make the equation of motion are exact. This assumption leads to some fuzzy to the originally proposed problems. Such problems are models to differential equation having some prescribe boundary condition. The solution and behaviors of such differential equation are fuzzy in nature. So it is important to fuzzify the differential equations appeared in the governing equation and to solve system under fuzzy environment [1]. Chang and Zadeh [2] invent the theory of fuzzy valued function, following Dubois and Prade [3], Puri and Ralescu [4], Kaleva [6], Seikkala [8] and many more has done some work related to it. On the field of fuzzy differential equation research are running now a days see e.g. [7], [8] and [9]. For the solution of such differential equation many researchers studied into the solution of the numerical methods such as Runge-Kutta [10], Fuzzy transform [11], difference methods [12], shooting Method [13] and many more.

Magnetohydrodynamic (MHD) flow is traditionally different from general flow. In recent years lots of research are going on [14]. In this paper we studied MHD viscous flow over a shrinking sheet with second order slip flow. The equations governing the motion are fuzzified and finally, numerical solution is carried out by developing computer codes for the problem. The crisp solution and mid value solution of the triangular fuzzified system of equations is in good agreement. The solution and effect of slip parameter magnetic parameter and suction parameter under fuzzy environment with details through graph.

2. Formulation of the problem

We consider a two dimensional laminar flow of magnetized viscous fluid over a continuously shrinking sheet. The shrinking velocity of the sheet is $U_w = -U_0 x$, where U_0 is a constant and $v_w = v_w(x)$ is wall mass transfer velocity. The $x-$axis is along the shrinking surface in the direction opposite the sheet motion and $y-$axis is perpendicular to it. By these assumption the corresponding Navier-Stoke's equations for the present problem can be summarized by the following set of equation:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{2.1}$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\vartheta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma\frac{B^2 u}{\rho} \tag{2.2}$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\vartheta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{2.3}$$

The boundary conditions of the above problems are given by

$$u\left(x,0\right)=U_0\left(x\right)+U_{slip}$$

$$v\left(x,0\right)=v_w$$

$$u\left(x,0\right)=0. \tag{2.4}$$

Here, u and v are the components of the velocity in the x and y directions respectively, ϑ is the kinematic viscosity,p is the fluid pressure, ρ is the fluid density and U_{slip} is second order velocity slip which is valid for any arbitrary Knudsen number, K_n and is given by

$$\begin{aligned}U_{slip}&=\frac{2}{3}\frac{3-\alpha l^3}{\alpha}+\frac{3}{2}\left(\frac{1-l^2}{K_n}\right)\lambda\frac{\partial u}{\partial y}-\frac{1}{4}\left(l^4+\frac{2}{{K_n}^2}\left(1-l^2\right)\right)\lambda^2\frac{\partial^2 u}{\partial y^2}\\&=A\frac{\partial u}{\partial y}+B\frac{\partial^2 u}{\partial y^2}.\end{aligned} \tag{2.5}$$

Here, $xl=\min\left[\frac{1}{K_n}-1\right],\ 0\le\alpha\le 1$ is the momentum accommodation coefficient and $\lambda>0$ is the molecular mean free path.according to the definition of l it is observed that $0\le l\le 1$, for any value of K_n. The stream function and similarity variable would be of the following form

$$\psi\left(x,y\right)=f\left(\eta\right)x\sqrt{vU_0},\eta\left(x,y\right)=y\sqrt{\frac{U_0}{v}}. \tag{2.6}$$

With this definition the components of velocities are

$$u=f'\left(\eta\right)xU_0,v=-f\left(\eta\right)\sqrt{vU_0}. \tag{2.7}$$

The wall mass transfer velocity becomes

$$v_w=f\left(0\right)\sqrt{vU_0}. \tag{2.8}$$

Using Equation (2.7) and (2.8) in (2.3),

$$\frac{p}{\rho}=v\frac{\partial v}{\partial y}-\frac{v^2}{2}+constant. \tag{2.9}$$

Also by using equations (2.7)-(2.9) in equation (2.2), we have

$$f''' + ff'' - f'^2 - M^2 f' = 0, \tag{2.10}$$

where $M^2 = \frac{\sigma B^2}{\rho \mu_0}$. And the boundary conditions are

$$f'(0) = -1 + \gamma f''(0) + \delta f'''(0), \tag{2.11}$$

$$f(0) = s, \tag{2.12}$$

$$f(1) = 0. \tag{2.13}$$

Here, s is the mass transfer parameter, γ is the first order velocity slip parameter, $\gamma = A\sqrt{\frac{U_0}{v}}$ and $\delta = \frac{BU_0}{v} < 0$ is the second order velocity parameter. The pressure term can be obtained from equation (2.9). All the equation(2.1)-(2.13), all the in crisp form, now we have use the Zadeh extension principle to convert the equation (2.10) into fuzzy differential equation we have,

$$\widehat{f'''} + \hat{f}\widehat{f''} - \hat{f}'^2 - \widehat{M^2}\hat{f}' = 0.$$

The boundary conditions in Fuzzified form are as follows

$$\widehat{f'(0)} = \widehat{-1} + \hat{\gamma}\widehat{f'''(0)} + +\hat{\delta}\widehat{f'''(0)},$$
$$\widehat{f'(0)} = \widehat{s}$$
$$\widehat{f'(1)} = \widehat{0}$$

Considering the fuzzy number are triangular then we have the equation (2.6) and (2.7) are as follows

$$\begin{aligned}
&[\underline{f'''}, f''', \overline{f'''}] + [\underline{f'}, f', \overline{f'}][\underline{f''}, f'', \overline{f''}] - [\underline{f'^2} + f'^2 + \overline{f'^2}] \\
&-[\underline{M'^2} + M'^2 + \overline{M'^2}][\underline{f'} + f' + \overline{f'}] = [\underline{0} + 0 + \overline{0}] \\
\Longrightarrow &[\underline{f'''}, f''', \overline{f'''}] + [minT, T_0, maxT] - [\underline{f'^2} + f'^2 + \overline{f'^2}] - [minK, K_0, maxK] \\
&= [\underline{0} + 0 + \overline{0}] \\
\Longrightarrow &[\underline{f'''} + minT, f''' + T_0, \overline{f'''} + maxT] - [\underline{f'^2} + minK, f'^2 + K_0, \overline{f'^2} + maxK] \\
&= [\underline{0} + 0 + \overline{0}] \\
\Longrightarrow &[\underline{f'''} - \underline{f'^2} - maxK, f''' - \overline{f'^2} + minT - maxK, \underline{f'^2} - \overline{f'''} + minK - maxT \\
&= [\underline{0} + 0 + \overline{0}].
\end{aligned}$$

Here,

$$T = \overline{ff''}, \overline{f}\underline{f''}, \underline{f}\overline{f''}, \quad \text{and} \quad T_0 = ff''$$

$$K = \overline{M^2}\overline{f'}, \overline{M^2}\underline{f'}, \underline{M^2}\overline{f'}, \underline{M^2}\underline{f'} \quad \text{and} \quad K_0 = M^2 f''$$

The above equation can also be splitting like

$$\underline{f'''} - \overline{f'^2} + minT - maxK = \underline{0}, \tag{2.14}$$

$$f''' - f'^2 + minT - maxK = 0, \tag{2.15}$$

$$\underline{f'^2} - \overline{f'''} + minK - maxT = \overline{0}. \tag{2.16}$$

With the boundary condition can be in the form of

$$\overline{f'(0)} = -1 + maxW + maxU, \tag{2.17}$$

$$f'(0) = \overline{-1} + W_0 + U_0, \tag{2.18}$$

$$\overline{f'(0)} = -1 + minW + minU, \tag{2.19}$$

$$\overline{f(0)} = \overline{s}, \tag{2.20}$$

$$f(0) = s, \tag{2.21}$$

$$\underline{f(0)} = \underline{s}, \tag{2.22}$$

$$\overline{f(1)} = \overline{0}, \tag{2.23}$$

$$f(1) = 0, \tag{2.24}$$

$$\underline{f(1)} = \underline{0}, \tag{2.25}$$

Here,

$$U = \overline{\gamma}\overline{f''(0)}, \quad \overline{\gamma}\underline{f''(0)}, \quad \underline{\gamma}\overline{f''(0)}, \quad \underline{\gamma}\underline{f''(0)}$$

$$V = \overline{\delta}\overline{f'''(0)}, \quad \overline{\delta}\underline{f'''(0)}, \quad \underline{\delta}\underline{f'''(0)}, \quad \underline{\gamma}\overline{f''(0)}.$$

3. Result and Discussion

Equation (2.14)-(2.16) together with boundary condition (2.17)-(2.25) is solved numerically by using finite difference scheme. The discritised Fuzzified equation are solved using an iterative method based on Gauss Seidel method by developing suitable codes in python.

Result are obtain for different values for the parameter $s, M,\ \delta,\ \gamma$ and for different $\alpha - cut$ of the Fuzzified equations.

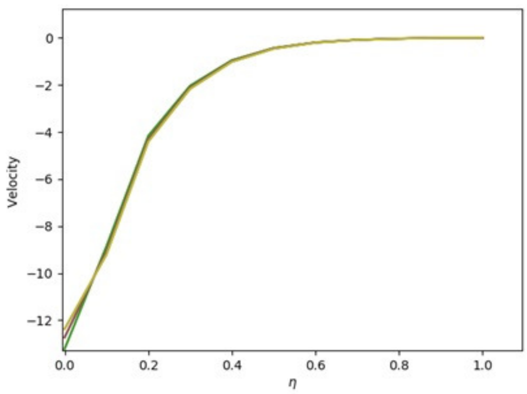

FIGURE 1. Fuzzified velocity profile for $\gamma = \ 0.25\ ,\ M = 0.1,\ s = 1,\ \delta = -0.04$ and $\alpha - cut = 1$

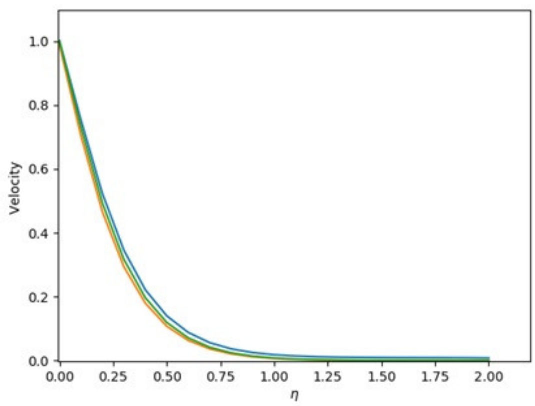

FIGURE 2. Fuzzified velocity profile for for $\gamma = \ 0.25\ ,\ M = 0.25,\ s = 1,\ \delta = -0.04$ and $\alpha - cut = .2$

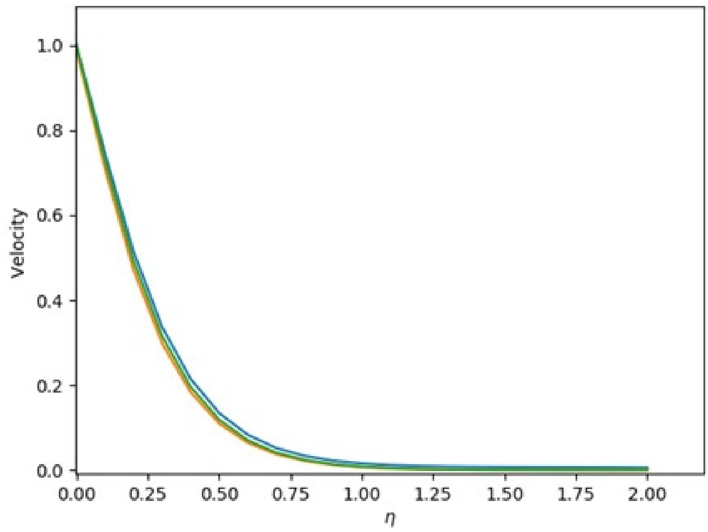

FIGURE 3. Fuzzified velocity profile for for $\gamma = \ 0.25\ ,\ M = 0.25,\ s = 1,\ \delta = -0.04$ and $\alpha - cut = .4$

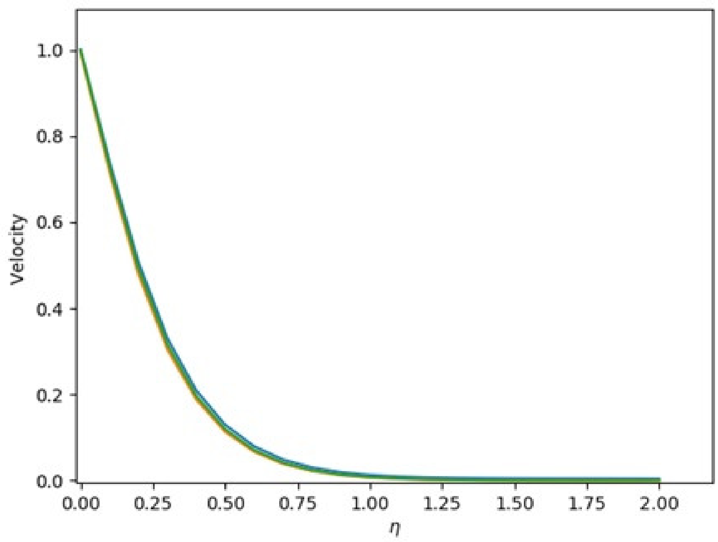

FIGURE 4. Fuzzified velocity profile for for $\gamma = \ 0.25\ ,\ M = 0.25,\ s = 1,\ \delta = -0.04$ and $\alpha - cut = .6$

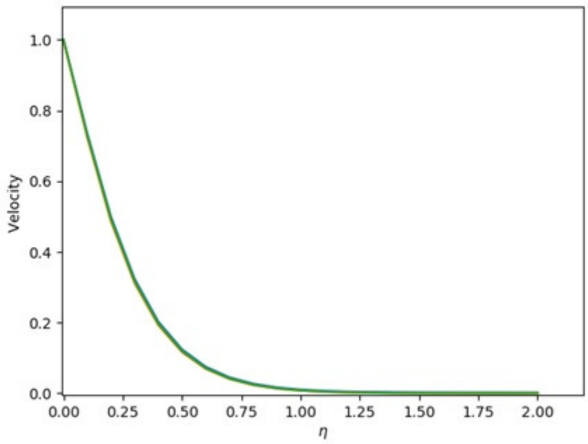

FIGURE 5. Fuzzified velocity profile for for $\gamma = 0.25$, $M = 0.25,\ s = 1,\ \delta = -0.04$ and $\alpha - cut = .8$

From figure 1-5 it is observed that when $\alpha - cut$ increases the right value coincide with the mid value and left value also tends to the mid value. It may be noted that the mid value represent the crisp value coincides with $\alpha - cut = 1$

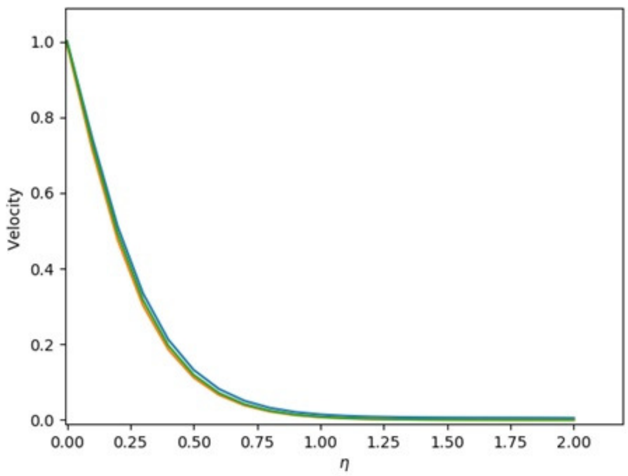

FIGURE 6. Fuzzified velocity profile for $\gamma = 0.25$, $M = 0.25,\ s = 1,\ \delta = -0.04$ and $\alpha - cut = .5$

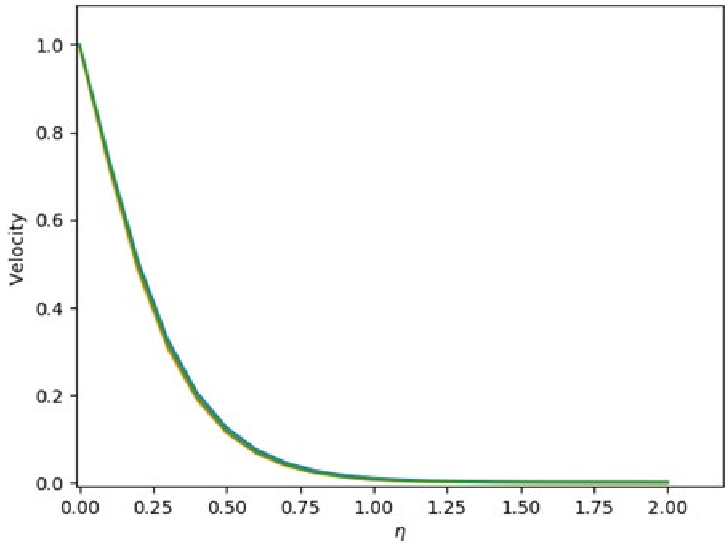

FIGURE 7. Fuzzified velocity profile for $\gamma = 0.25$, $M = 0.25$, $s = 1$, $\delta = -0.04$ and $\alpha - cut = .8$

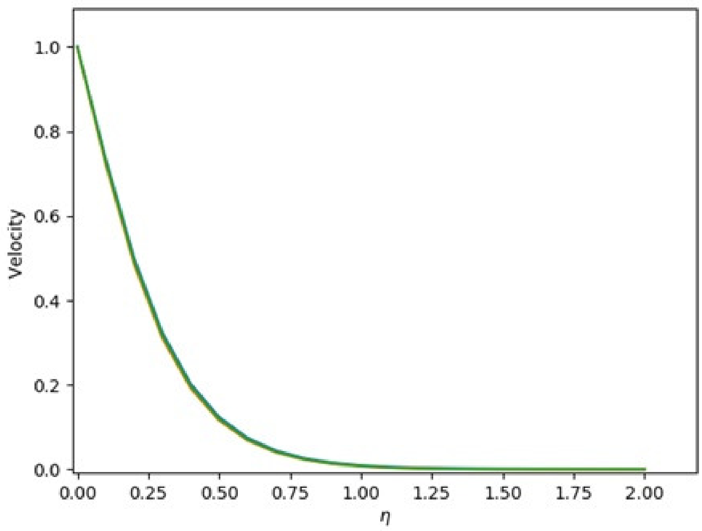

FIGURE 8. Fuzzified velocity profile for $\gamma = 0.25$, $M = 2.25$, $s = 1$, $\delta = -0.04$ and $\alpha - cut = .5$

From figure 5-6 represent the velocity profile for different values of M, the magnetic parameter with the $\alpha - cut = 0.5$.

4. Conclusion

In this paper, we have investigated the fuzzified solution of governing Navier-Stokes equations for the magnetohydrodynamic viscous flow over a shrinking sheet with second order slip. We try to established the fuzzified velocity profile for different values of the parameter of $\gamma,\ \delta,\ s\ and\ M$ with the different $\alpha - cut.$ We observe that the increase of magnetic effect the velocity tends to zero which is same as the crisp solution.

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