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ON THE Ψ-CONDITIONAL EXPONENTIAL ASYMPTOTIC STABILITY OF LINEAR MATRIX DIFFERENCE EQUATIONS

G. NAGA JYOTHI[1], T. S. RAO, CH. VASAVI, G. SURESH KUMAR, AND T. SRINIVASA RAO

ABSTRACT. In this paper we develop the if and only if condition for Ψ-conditional exponential asymptotic stability of zero solution of the linear matrix difference equation, by using the concept of Kronecker product of matrices.

1. INTRODUCTION

In Applied mathematics area, in Continuous manner, the differential equations concept plays very powerful role, these are studied by many authors like [1], [2], [3], [4] and [5]. In different discrete ares like numerical methods, dynamical and control systems, gathering the information by using signals, theory of oscillations, finite element methods, modeling the physical phenomena problems as equations and graph theory, group theory like the difference equations concept plays very important role. The existence of Ψ -conditional exponential asymptotic stability of non linear matrix differential equations was studied in [4].

The existence of Ψ-boundedness, Ψ-stability, Ψ-asymptotic stability for matrix difference equations was studied by many authors like [6], [7], [8], [9] and [10]. Recently the concepts of Ψ-bounded solutions for Sylvester matrix dynamical systems on time scales was studied by many authors like [11-23].

The existence of Ψ-conditional exponential asymptotic stability of linear matrix difference system was not yet discussed.

[1] *corresponding author*

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In this paper we develop the if and only if condition for Ψ-conditional exponential asymptotic stability of zero solution of the matrix linear difference equation

$$X(n+1) = [U(n) + U1(n)]X + X[V(n) + V1(n)] \tag{1.1}$$

which consider as a perturbed equations of the linear equation

$$X(n+1) = U(n)X + XV(n) \tag{1.2}$$

We develop the rules on mappings U1, V1 and G under the zero solution of (1.1)-(1.4) and conditions on the fundamental matrices of the equations

$$X(n+1) = U(n)X \tag{1.3}$$

$$X(n+1) = XV(n) \tag{1.4}$$

are Ψ-conditionally exponentially asymptotically stable on N. The introduction of Ψ-matrix function gives a mixed asymptotic behavior for the components of solutions.

Here for getting the proofs, we will use the concept of Kronecker product of matrices; these concepts are used very frequently in the fields of graph theory, group theory, matrix theory and dynamical systems.

2. PRELIMINARIES

Here we define some basic definitions, notations and theorems which are useful further. Let R^n is Euclidean n-dimensional space.

Let $\mathrm{x} = (\mathrm{x}_1, \mathrm{x}_2 \dots \mathrm{x}_n)^{\mathrm{T}} \in$ and $\|x\| = \max\{|x_1|, |x_2| \dots |x_n|\}$ be the norm of x, here T gives transpose.

Let $\mathrm{M}_{\mathrm{dxd}}$ be the Linear space of all dxd real valued matrices. Let $\mathrm{U} = (a_{jk}) \in \mathrm{M}_{\mathrm{dxd}}$, we define the norm $|U|$ by $|U| = \sup_{\|x| \leq 1} \|Ux\|$. It is taken that $|U| = \mathrm{Max}_{1 \leq j \leq d} \left\{ \sum_{k=1}^{d} |a_{jk}| \right\}$.

Suppose $G : \mathrm{N} \times \mathrm{M}_{\mathrm{dxd}} \to \mathrm{M}_{\mathrm{dxd}}^{\rightarrow} \mathrm{G}(\mathrm{n}, 0_{\mathrm{d}}) = 0_{\mathrm{d}}$ (Null matrix of order dxd).

Let $\psi_j : N \to (0, \infty), \quad j = 1, 2, \dots, d$ is a function, and $\psi = \mathrm{diag}[\psi_1, \psi_2, \dots, \psi_d]$.

3. Ψ-CONDITIONAL EXPONENTIAL ASYMPTOTIC STABILITY OF LINEAR MATRIX DIFFERENCE EQUATIONS

For the equation:

$$X(n+1) = U(n)X \quad \text{and} \quad X(n+1) = (U(n) + U1(n))X. \tag{3.1}$$

Here we discuss the Ψ-conditional exponential asymptotic stability .

For the equation (1.3), the rules for Ψ-conditional exponential asymptotic stability of can be expressed in terms of solutions or in terms of fundamental matrix for (1.3).

Theorem 3.1. *The if and only if condition for the equation (1.3) has a Ψ-unbounded solution on N and a nontrivial solution $X_0(n)$ such that $\psi(n)X_0(n) |\leq Me^{-\mu n}$ for all $n \in N$ where M, μ are positive constants is the equation (1.3) is Ψ-conditionally exponentially asymptotically stable on N.*

Proof. First consider (1.3) is Ψ-conditionally exponentially asymptotically stable on N.

For equation (1.3), take Z(n) fundamental matrix. Z(n) are fundamental matrices for equation (1.3). Thus, the linear equation (1.3) has atleast one Ψ-unbounded solution on N.

Further, there exist a function $(X_t(n))$ of non trivial solutions of (1.3) such that $\underset{t\to\infty}{Lt}\,\psi(n)X_t(n) = 0_d$ uniformly on N and there exist positive constants M, μ such that $|\psi(n)X_t(n)| \leq Me^{-\mu n}$ for all $n \in N, t \in N$.

Suppose non trivial solution $X_0(n)$ of (1.3) such that $|\psi(n)X_0(n)| \leq Me^{-\mu n}$ for all $n \in N$ where M,μ are positive constants and (1.3) has a Ψ-unbounded solution on N.

It gives that the fundamental matrix Z(n) of (1.3) such that $|\psi(n)Z(n)|$ is unbounded on N. Further, the matrix linear difference equation (1.3) is not Ψ-stable on N [see Theorem 3.1, [3]] Further $(\frac{1}{t}X_0(n))$ is a function of nonzero solutions of (1.3) such that $\lim_{t\to\infty}\psi(n)\left(\frac{1}{t}X_0(n)\right) = 0_d$ uniformly on N. And $\left|\psi(n)\left(\frac{1}{t}X_0(n)\right)\right| \leq Me^{-\mu n}$ for all $n \in N$. Thus the equation (1.3) is Ψ-conditionally exponentially asymptotically stable on N. Hence the proof was completed. □

Theorem 3.2. *Let Z(n) are fundamental matrix for equation (1.3). The if and only if conditions for (1.3) are Ψ-conditionally exponentially asymptotically stable on N are taken as follows.*

(a) *We have a projection $Q_1 : R^d \to R^d$ such that $\psi(n)Z(n)Q_1$ is unbounded on N.*

(b) *There exists a projection $Q_2 : R^d \to R^d, Q_2 \neq 0$ and two positive constants $\tilde{M}, \mu$ such that $|\psi(n)Z(n)Q_2| \leq \tilde{M}e^{-\mu n}$ for all $n \in N$.*

Proof. First, suppose the matrix linear difference equation (1.3) is Ψ-conditionally exponentially asymptotically stable on N. Then by definition of Ψ-conditionally exponentially asymptotically stable on N of (1.3) and Theorem 2.1, it follows that is unbounded on N. Further, we have a nontrivial solution $X_0(n)$ of (1.3) such that $|\psi(n)X_0(n)| \leq Me^{-\mu n}$ for all $n \in N$ where M, μ are positive fixed values. Thus we got $L \in 0_d$ satisfies Z(n)L is nonzero solution of (1.3) on N and $|\psi(n)Z(n)L| \leq Me^{-\mu n}$ for all $n \in N$.

Assume column $l_i = (l_{1i}, l_{2i}, \ldots, l_{di})^T \neq 0$ of L. Let $l_{jk} = \|l_j\|$. Let Q_2 be the zero matrix 0_d, here replaced the column 1 with the column $l_{kj}^{-1} l_j$. It is easy to see that $Q_2 \neq 0$is a projection and there exist a positive fixed value $\tilde{M}$ such that $|\psi(n)Z(n)Q_2| \leq \tilde{M}e^{-\mu n}$ for all $n \in N$.

By above condition (a) and Theorem 2.1, [6], it follows that equation (1.3) is not Ψ-stable on N. Assume $X_0(n)$ be a nonzero solution on N of the equation (1.3). Let (μ_t) be satisfies $\mu_t \in R - \{1\}$, $\mathrm{Lt}_{t\to\infty} \mu_t = 1$ and $(X_t(n))$ defined as $X_t(n) = Z(n)Q_2Z^{-1}(0)\left(\mu_i X_0(0)\right) + Z(n)\left(I - Q_2\right)Z^{-1}(0)X_0(0)$ for all $n \in N$. Clearly $X_t(n)$ are satisfies the equation (1.3). For $t \in N$ and $n \geq 0$, we got

$$
\begin{aligned}
&|\psi(n)X_i(n) - \psi(n)X_0(n)| = |\psi(n)Z(n)Q_2Z^1(0)\left(\mu_t X_0(0)\right) \\
&\quad + \psi(n)Z(n)\left(I - Q_2\right)Z^1(0)X_0(0) - \psi(n)X_0(n)| \\
&= |\psi(n)Z(n)Q_2Z^{-1}(0)\mu_t X_0(0) + \psi(n)Z(n)Z^{-1}(0)X_0(0) - \psi(n) \\
&Z(n)Q_2Z^{-1}(0)X_0(0) - \psi(n)X_0(n)| \\
&= |\psi(n)Z(n)Q_2Z^{-1}(0)\left(\left[\mu_i - 1\right]X_0(0)\right)| \\
&\leq |\psi(n)Z(n)Q_2 \left\|\mu_t - 1\right\| Z^{-1}(0)X_0(0)| \\
&\leq \tilde{M}e^{-\mu n}\left|\mu_t - 1\right|\left|Z^{-1}(0)X_0(0)\right|,
\end{aligned}
$$

$$\lim_{t\to\infty} \psi(n)X_i(n) - \psi(n)X_0(n) \mid= 0.$$

Thus, $\lim_{t\to\infty} \psi(n)X_t(n) = \psi(n)X_0(n)$ uniformly on N and $\psi(n)(X_i(n) - X_0(n)) \leq \tilde{M}e^{-\mu n}$ for all $n \in N, t \in N$. Hence the equation (1.3) is Ψ-conditionally exponentially asymptotically stable on N. □

Theorem 3.3. *Let Z(n) be a fundamental matrix for equation (1.3). Then there exist a projection $Q : R^d \to R^d, Q \neq 0_d$ and two positive constants $\tilde{M}, \mu$ such that*

(a) $\psi(n)Z(n)(I-Q)$ *is unbounded on* N*;*
(b) $\psi(n)Z(n)Q \mid\leq \tilde{M}e^{-\mu n}$ *for all* $n \in N$.

Proof. By using Theorem 3.2, clearly it follows. ☐

Theorem 3.4. *The matrix linear difference equation (1.3) is* Ψ*-conditionally exponentially asymptotically stable on N if there are two supplementary projections* $Q_i : R^d \to R^d, Q_1 \neq 0, Q_2 \neq 0$ *and a constant N>0 such that Z(n) for the equation (4) satisfies the following condition*

$$\sum_{s=0}^{n} \left|\psi(n)Z(n)Q_1Z^{-1}(s)\psi^{-1}(s)\right| + \sum_{s=n}^{\infty} \left|\psi(n)Z(n)Q_2Z^{-1}(s)\psi^{-1}(s)\right| \leq N$$

for all $n \in N$.

Proof. We have fixed non negative value M such that $|\psi(n)Z(n)Q_1| \leq \tilde{M}e^{-N^{-1}n}$ for all $n \in N$ and the matrix function $\psi(n)Z(n)Q_2$ is unbounded on N. Now, by applying Theorem 3.2, clearly we got the proof. ☐

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DEPARTMENT OF MATHEMATICS
KONERU LAKSHMAIAH EDUCATION FOUNDATION
VADDESWARAM
ANDHRA PRADESH, INDIA
Email address: g.jyothi2403@gmail.com

DEPARTMENT OF MATHEMATICS
KONERU LAKSHMAIAH EDUCATION FOUNDATION
VADDESWARAM, GUNTUR DIST
ANDHRA PRADESH, INDIA
Email address: tagallamudi_me@kluniversity.in

DEPARTMENT OF MATHEMATICS
KONERU LAKSHMAIAH EDUCATION FOUNDATION
VADDESWARAM, GUNTUR DIST
ANDHRA PRADESH, INDIA
Email address: vasavi.klu@gmail.com

DEPARTMENT OF MATHEMATICS
KONERU LAKSHMAIAH EDUCATION FOUNDATION
VADDESWARAM, GUNTUR DIST
ANDHRA PRADESH, INDIA
Email address: drgsk006@kluniversity.in

DEPARTMENT OF MATHEMATICS
KONERU LAKSHMAIAH EDUCATION FOUNDATION
VADDESWARAM, GUNTUR DIST
ANDHRA PRADESH, INDIA
Email address: tsr_2505@kluniversity.in