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IMPLEMENTED MODIFIED DIJKSTRA'S ALGORITHM TO FIND PROJECT COMPLETION TIME

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ABSTRACT. Longest path problems in network analysis provide an important functional method for planning and managing broad projects in the architecture, medical and different sectors. We may use PERT / CPM approaches to calculate the project completion time or the longest path in the diagram in question. Calculation of traditional Dijkstra's algorithm has been commonly used in the shortest path problems. Indeed, it's one of the most referenced. In this paper, traditional PERT compared to Modified Dijkstra's algorithm and calculate earliest and latest times.

1. INTRODUCTION

Lewis [2] describes project management as "the planning, scheduling and controlling of project activities to achieve project objectives-performances, cost and time for a given scope of work". A Project completion on time relies upon a right schedule strategy. There is a breakdown mechanism of the work technique in project management, which divides a project into smaller challenges. Every task has its own span of time; it needs necessary conditions and gives the outcome. Additionally, tasks are utilized in the project management approach, which is determined by the project scale, difficulty, and project duration. The approach must be an efficient and easy procedure. There are a few techniques that follow these

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rules; Gantt Charts and the Critical Path Method (CPM) are two of those. Anyway, Gantt Charts have disadvantages in showing the interdependency of activities, for example, Gantt Charts don't display how every action relies upon others.

critical path approach has been in use since the 1950s [1] and used effectively. This approach is an unusual way of arranging the analysis of individual tasks in a sequence that must be performed on planned time. And activity only needs to be handled after the previously scheduled operation is complete. To reduce the costs of plant shutdown, Du Pont Company in 1957 developed critical path method by a pair of mathematicians.

The Critical Path Method (CPM) [7] is a network-based approach intended to help with project planning, time management, and project monitoring. Two fundamental outcomes gave by CPM are the project duration and the critical path. Each one of the processes for identifying a critical path is (a) calculate the earliest event times by means of a forward pass, (b) determine the latest event times through a backward pass of each occurrence, and (c) distinguish project paths. Program Evaluation and Review Technique (PERT) civilized in the 1950's.The technique [Hajdu] used in Polaris Missile Project. Program Evaluation and Review Technique take the CPM network and adds distributions to express the project activity times.

PERT is a project management method used to evaluate the approximate time needed for a project to be effectively completed. In general, this is a tool to evaluate the activities required in order to complete the project, in particular the time required to fulfill every activity, and to decide the minimal time taken achieve the entire plan. PERT is a basic method, which draws its predictions based on event probabilities. It is a beta distribution system. For PERT with beta distribution, three estimates of time are used to determine the parameter of length of activity distribution.

(1) Pessimistic time (t_a): Assuming everything goes wrong.
(2) Most likely time (t_m): Assuming everything proceeds as normal.
(3) Optimistic time (t_b): Assuming everything proceeds better than is normally expected,

$$\text{Expected time (Mean)} = \frac{t_a + 4t_m + t_b}{6}.$$

The longest duration determines the duration of the project, i.e., critical path.

This estimated duration can be shown in the diagram of the network. If three mean absolute times are defined for the optimistic and pessimistic moments, and there are six standard errors between them to estimate the variance for each completion date of the task, so that the variance is given by $\frac{(t_b - t_a)^2}{36}$.

The variation in the completion period of the project has been determined by adding up the differences in the critical path the activity completion time. Given this uncertainty, one can determine the likelihood of completing the project by a certain date expecting normal distribution of probability for the main path.

PERT/CPM techniques [3] applied in carrying out vaccination drive, the eradication of polio, construction of a new hospital, addition of a new wing in the hospital, Commissioning of Primary Health Centers (PHCs) or Community Health Centers (CHCs), Air-conditioning of Hospitals, Construction and commissioning of an intensive care unit, setting up a medical college, Organizing a Family Planning Camp, Conducting a training program for health workers, establishment of network of Family Planning Welfare Centers, Construction of hostel for nurses, Mass Health education campaign, Organizing a sanitation drive, etc. The activity-on-node network with backward and forward pass [6] suggested by Thulasimani in 2018 and extended to hydraulic impeller output for a multi-stage submersible pump.

2. Analysis of the Model

Here I collected application problem from Network sources presented in Table 1 and Network diagram in Figure 1.

Forward Pass

Earliest start time (ES): An activity can start earliest.

ES= Maximum EF of immediate predecessors

Earliest finish time (EF): The earliest time an activity can be finished.

EF=ES+activity time

Backward Pass

Latest start time (LS): The latest start time in the system that a process will begin without creating a delay.

LS=LF−t

Latest finish time (LF): The latest finish in the system that a process will end without creating any delay.

TABLE 1. Application problem

Activity	Node	Predecessor	t_a	t_m	t_b	ET
1-2	P	-	5	6	7	6
1-3	Q	-	1	3	5	3
1-4	R	-	1	4	7	4
2-5	S	P	1	2	3	2
3-6	T	Q	1	2	9	3
4-6	U	R	1	5	9	5
4-7	V	R	1	5	9	5
6-7	W	T,U	4	4	10	5
5-8	X	S	2	5	8	5
7-8	Y	W,V	2	2	8	3

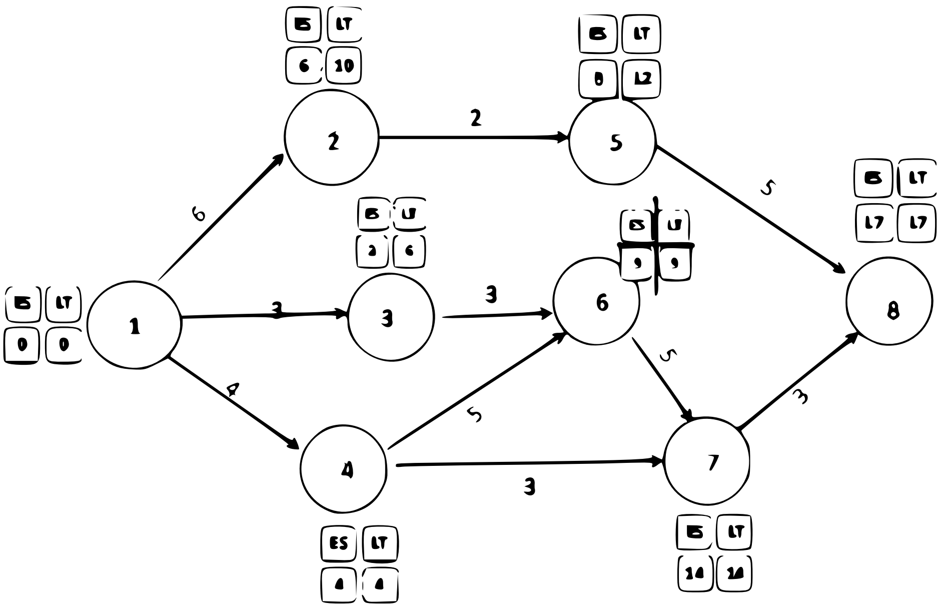

FIGURE 1. Example of a Project Network

Critical Path: Longest path in the Network

(1) ES=LS
(2) EF=LF
(3) EF-ES=LF-LS

TABLE 2. Earliest and Latest Times in PERT Network

Activity	Node	Activity duration	ES	EF	LS	LF	Float
1-2	P	6	0	6	4	10	4
1-3	Q	3	0	3	3	6	3
1-4	R	4	0	4	0	4	0^* Critical activity
2-5	S	2	6	8	10	12	4
3-6	T	3	3	6	6	9	3
4-6	U	5	4	9	4	9	0^* Critical activity
4-7	V	3	4	7	11	14	7
6-7	W	5	8	13	12	17	4
5-8	X	5	9	14	9	14	0^* Critical activity
7-8	Y	3	14	17	14	17	0^*Critical activity

The Critical path is: $1 \ldots 4 \ldots 6 \ldots 7 \ldots 8$.
Therefore Completion of project duration is 17.

3. DIJKSTRA'S ALGORITHM

A graph search algorithm proposed by Dijkstra's in 1959 which can be utilized to resolve the shortest single-source path problem for any network with a weighted edge path. Subsequently, Lee updated this graph search algorithm in 2006 and extended to the traffic guidelines.

Dijkstra's Algorithm [5] based on finding a minimum weight path in a weighted graph connecting two given vertices. Dijkstra's algorithm that can be used in many fields such as telecommunications, cable networks, telephone networks, power supply networks, gas pipe networks, water supply network, railways etc. to determine the shortest path between two or more than two cities.

Modified Dijkstra's Algorithm

Ravi Shankar and Sireesha [4] proposed the algorithm Modified Dijkstra's in 2010. Modified Dijkstra's algorithm is made to find a maximal path in a weighted graph, from starting vertices to ending vertices. Modified Dijkstra's algorithm gives labels to vertices in given weighted graph. Initially gives 0 labels to all vertices.

After that these vertices are having some are permanent labels and some are temporary labels. In Modified Disjkstra's algorithm we can find earliest time by using forward pass algorithm and latest time by using backward pass algorithm.

Forward Pass algorithm

Step 1: In sequence $v_1 = 1, v_2 = 2, \ldots, v_n = n,$ allocate n vertices.

Step 2: Assign permanent label 0 to the primary vertex $v_1 = 1,$ and provisional label 0 to the rest of $n-1$ edges.

Step 3: Every vertex j is not permanently labeled would receive a new provisional label, i.e., $E_j = \max[$old label of $j,$ (old label of $i + t_{ij})]$, where i is permanently labeled with the new vertex and t_{ij} is the duration of activity between vertices i and j. If an edge is not connected to i and j, so $t_{ij} = \infty$.

Step 4: The next vertex turns into the fixed label.

Step 3 and Step 4 repeated until $v_n = n$ gets a fixed label. The $E_j's$ permanently labeled values are the earliest times as $E_1 = 0.$

Backward pass algorithm

Step 1: Set n vertices to $v_n = n, v_{(n-1)} = n-1, \ldots, v_1 = 1.$

Step 2: Allocate fixed label $L_n = E_n$ to the vertex $v_n = n$ and temporary labels to remains $n-1$ vertices.

Step 3: Any node j does not get a constant label, gets a new provisional label, i.e., $L_j = \min[$old label of $i,$ (old label of $j + t_{ij})]$, where j is the fixed labeled with the new vertex t_{ij} is the duration of activity among vertices i and j. If an edge is not connected to i and j, then $t_{ij} = 0.$

Step 4: As per Step 1, the next vertex will become fixed label or permanent label.

Repeated Step 3 and Step 4 until then the initial vertex $v_1 = 1$ gets a fixed label.

4. PROPOSAL METHOD

Activity times in Modified Dijkastra's algorithm

Here we proposed a method to find project completion duration time, Earliest and Latest times using Modified Dijkstra's algorithm with most likely time, Pessimistic time and optimistic time.

Initially assigned each vertex v to $1, 2, 3, \ldots, n$ numbers.

Step 1: Vertex v, which has not yet been permanently labeled, gets a new temporary label, the result of which is provided by $\max[$old label of $i,$ (old label of $j + d_{ij})]$, where the current vertex j is permanently labeled, in the preceding step and d_{ij} is the separation between the two nodes i and j and an edge not connected by i and j, then $d_{ij} = \infty$.

Step 2: To get the longest path, the maximum value of all temporary labels j is to be identified and held in priority queue with connecting list, this will become the fixed label for the respective vertex.

Step 1 and Step 2 are repeated until to get $v = n$ permanent label. This permanent label is the project completion duration time.

From Table 1 and network (Fig.1).

TABLE 3. Earliest times in Modified Dijkstra's Algorithm

Vertex number or activity number								Earliest time
1	2	3	4	5	6	7	8	
0(F)	0	0	0	0	0	0	0	$E_1 = 0$
0(F)	6	3	4	∞	∞	∞	∞	
0(F)	6(F)	3	4	∞	∞	∞	∞	$E_2 = 6$
0(F)	6(F)	3	4	8	∞	∞	∞	
0(F)	6(F)	3(F)	4	8	∞	∞	∞	$E_3 = 3$
0(F)	6(F)	3(F)	4(F)	8	6	∞	∞	
0(F)	6(F)	3(F)	4(F)	8	9(F)	∞	∞	$E_6 = 9$
0(F)	6(F)	3(F)	4(F)	8	9(F)	14	∞	
0(F)	6(F)	3(F)	4(F)	8	9(F)	14(F)	∞	$E_7 = 14$
0(F)	6(F)	3(F)	4(F)	8	9(F)	14(F)	17	
0(F)	6(F)	3(F)	4(F)	8	9(F)	14(F)	17(F)	$E_8 = 17$

Therefore Project completion duration time is 17.

TABLE 4. Latest times in Modified Dijkstra's Algorithm

Vertex number								Latest time
8	7	6	5	4	3	2	1	
17(F)	17	17	17	17	17	17	17	$L_8 = E_8$
17(F)	14	17	12	17	17	17	17	
17(F)	14(F)	17	12	17	17	17	17	$L_7 = 14$
17(F)	14(F)	9	12	11	17	17	17	
17(F)	14(F)	9(F)	12	11	17	17	17	$L_6 = 9$
17(F)	14(F)	9(F)	12	4	6	17	17	
17(F)	14(F)	9(F)	12	4(F)	6	17	17	$L_4 = 4$
17(F)	14(F)	9(F)	12	4(F)	6	17	0	$L_0 = 0$

5. CONCLUSION

This paper gives same calculations in PERT method and Modified Dijkstra's algorithm. So, we can easily apply to Network problems to finding project completion duration time or Longest path in Network diagrams.

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