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EFFICIENT DOMINATION ZERO DIVISOR OF LINE GRAPH

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ABSTRACT. In this paper, we consider for a finite commutative ring $R^{'}$ and let $\Gamma(Z_n^{'})$ be the zero divisor graph of $R^{'}$. Let $L(\Gamma(Z_n^{'}))$ be a line graph of $\Gamma(Z_n^{'})$. Let $\Gamma(Z_n^{'})$ be a zero divisor of line graph, a set $E_d \subseteq V(L(Z_n^{'}))$ $(V(L(Z_n^{'})) = q = E(G))$ is an efficient domination zero divisor of line graph $\Gamma(Z_n^{'})$, if each vertex in $V(L(Z_n^{'}) - E_d$ is adjacent to every vertex in E_d and $\parallel N[e] \cap E_d \parallel = 1$ for all $e \in V(L(Z_n^{'}))$. The minimum Cardinality of edge in said a set is called the efficient domination zero divisor of number $L(\Gamma(Z_n^{'}))$ and is denoted by $\gamma_e(L(\Gamma(Z_n^{'})))$. In this paper many bounds on were found in terms of the elements of $\gamma_e(L(\Gamma(Z_n^{'})))$. Moreover its relations with other domination parameters were obtained.

1. INTRODUCTION

Let R be a commutative ring and let $Z(R)$ be its set of zero divisor. The zero-divisor graph of a ring is the simple graph whose vertex set is the set of non-zero zero divisors and an edge is drawn with two separate and an edge is drawn among two different vertices if their product is zero.

Through this paper, we study the commutative ring by R and zero divisor graph by $\Gamma(Z_n)$. The zero-divisor graph is very useful of determine the algebraic structures and properties of ring. In this paper many bounds on $\gamma_e(L(\Gamma(Z_n)))$ were obtained in terms of the element of $L(\Gamma(Z_n))$ [1], [2].

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In this paper, the entire graph considered here are simple, finite, non-trivial, undirected and connected graph. As common $p = |V|$ and $q = |E|$ denote the number of vertices and edges of a graph respectively.

The line graph of a graph G, denoted by $L(G)$, is a graph whose vertices are the edges of G and two vertices of $L(G)$ are adjacent any where the corresponding edges of G are incident to a common vertex; see [3] $(V(L(G) = q = E(G))$.

A dominating set is a set of vertices such that each vertex of V is either in D or has at least one neighbour in D. The minimum cardinality if such a set is called the domination number of the graph G and it is denoted by $\gamma(G)$.

A set $S \subseteq E(G)$ is said to be independent if two edges in S are adjacent in G. The set S is called a maximal independent set provided it is not a proper subset of some other independent set.

The maximum cardinality of an edge independent set of G is called the edge independent number or matching number of G and is denoted by $\beta^{'}$.

A dominating set S of vertices is called an efficient dominating set, if for every vertex $u \in V, |N(u) \cap S| = 1$ An edge uv is said to dominate any edge ux or vx where x in V, including the edge uv itself.

An edge subset $E^{'} \subseteq E$ is an efficient edge dominating set for G if each edge in E is dominated by exactly one edge in $E^{'}$. Efficient edge dominating sets correspond to efficient dominating sets in the line graph $\gamma(L(G))$.

The domination theory is very useful in communication network, traffic signal, computer science and so on.

2. Efficient domination zero divisor in line graph

Definition 2.1. *Let $L(\Gamma(Z_n^{'}))$ be a zero divisor of line graph. A set is $E_d \subseteq V(L(\Gamma(Z_n^{'})))$ an efficient domination zero divisor of line graph $\Gamma(Z_n^{'})$, if each vertex $V(L(\Gamma(Z_n^{'}))) - E_d$ in is adjacent to every vertex in E and $|N^{'}[e] \cap E_d| = 1$ for all $e \in V(L(\Gamma(Z_n^{'})))$. The minimum Cardinality of vertex in such a set is called the efficient domination zero divisor of number of $L(\Gamma(Z_n^{'}))$ and is denoted by $\gamma_e(L(\Gamma(Z_n^{'})))$.*

Example 1. *For $L(\Gamma(Z_n^{'}))$. Consider $\Gamma(Z_8^{'})) = \{2, 4, 6\}$ be a zero divisor graph.*
Therefore $\gamma_e(L(\Gamma(Z_n^{'})))$.

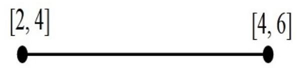

FIGURE 1. $L(\Gamma(Z_n^{'}))$

Theorem 2.1. *If* $L(\Gamma(Z_n^{'}))$ *be an efficient dominating zero divisor set of line graph, thus the cardinality of every efficient dominating zero divisor set equal to the edge domination zero divisor number* $\Gamma(Z_n^{'})$.

Proof. Assume $\{e_1, e_2, \dots, e_k\}$ is an efficient dominating zero divisor set of $\Gamma(Z_n^{'})$. Let $D^{'}$ be any minimum edge dominating zero divisor set. Clearly, $K \geq |D^{'}|$ because $D^{'}$ is a $\gamma(\Gamma(Z_n^{'}))$ set $1 \leq i \leq j \leq k$. So we have $N^{'}[u_i] \cap N^{'}[u_j] = \phi$ and $D^{'} \cap N^{'}[u_i] \neq \phi$, and therefore $|D^{'}| \geq k$. □

Theorem 2.2. *Let* $L(\Gamma(Z_n^{'}))$ *be a line graph of* $\Gamma(Z_n^{'})$. *where* $n = 2p$, *and* p *is an odd prime number. Then* $\gamma_e(L(\Gamma(Z_n^{'}))) = 1$.

Proof. When $n = 2p$, then $\Gamma(Z_n^{'})$ be a star graph. So there is a common vertex which is adjacent to every vertices.

When we draw a line graph of, for $n = 2p$, and let e_1 be the common vertex of $\Gamma(Z_n^{'})$. Then e_1 appears in every vertex of the line graph. $[e_1, f_i]$ in $V(L(\Gamma(Z_n^{'})))$ where $\{i = 2, 2.2, \dots, 2(p-1); p = v_1\}$ forms a complete line graph of $\Gamma(Z_n^{'})$ and here, $[e_1, f_1]$ is adjacent with all other vertices of line graph.

Therefore $\gamma_e(L(\Gamma(Z_n^{'}))) = 1$. □

The following figure explain the above theorem: $Z_{10}^{'} = \{2, 4, 5, 6, 8\}$.

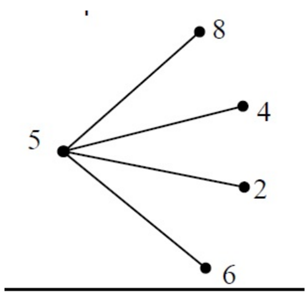

FIGURE 2. $L(\Gamma(Z_n^{'}))$

Let $Z_{10}^{'}$ be a star graph of zero divisor, when we draw a line graph of $\Gamma(Z_n^{'})$, $E = \{v_1, v_2, v_3, v_4\}$.

Efficient dominating set zero divisor of line graph $= A$, $\gamma_e(L(\Gamma(Z_n^{'}))) = 1$.

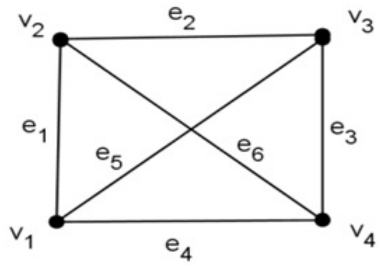

FIGURE 3. $L(\Gamma(Z'_{10}))$

Theorem 2.3. *For any graph $\Gamma(Z'_{2q})$ with $(V(L(\Gamma(Z'_{2q}))) = q)$ vertices and maximum edge degree $\Delta'(\Gamma(Z'_{2q}))$ then $\gamma_e(L(\Gamma(Z'_{2q}))) = q - \Delta'(\Gamma(Z'_{2q}))$.*

Proof. Let e be a vertex with maximum degree $\Delta'(\Gamma(Z'_{2q}))$. If $\Gamma(Z'_{2q})$ is a star, when we draw the line graph of $\Gamma(Z'_{2q})$, then it is a complete graph.

Let $F = \{e_1, e_2, \ldots, e_k\} \subseteq V(L(Z'_{2q}))$ such that $deg(e_i) \geq 2$, $1 \leq i \leq k$. Now, there exist at least one vertex $e \in V(L(\Gamma(Z'_{2q}))$. Hence $V(L(\Gamma(Z'_{2q}))) - N'(e)$ is an edge dominating set of $L(\Gamma(Z_{2q}))$ and $e \in F$ is an edge dominated by exactly one edge in $V(L(\Gamma(Z'_{2q})))$. Then $F = V(L(\Gamma((Z'_{2q})) - N'(e)$. Therefore $\gamma_e(L(\Gamma(Z'_{2q}))) = q - \Delta'(\Gamma(Z'_{2q}))$. □

Theorem 2.4. *If q is any prime with $q > 3$, then $\gamma_e(L(\Gamma(Z'_{3q}))) = 2$.*

Proof. Let $L(\Gamma(Z'_3 q))$ is vertex set of $\{3, 6, 9, \ldots, 3(q-1), q, 2q\}$. Let a vertex $e \in V(L(\Gamma(Z_{3q})))$ with $deg(e) = \Delta$. Suppose f be another vertex with $deg(f) = \Delta$ in $L(\Gamma(Z_{3q}))$, then either $e = q,\ f = 2q$ or $u = 2q, v = q$. Then $ef = 2q \times q = 2q^2$ which does not divide by $3p$. Then e and f are non-adjacent vertices in $L(\Gamma(Z_{3q}))$.

Let w be any other vertex in $L(\Gamma(Z_{3q}))$ such that $we = few = 0$. In other words, the remaining vertices in $L(\Gamma(Z_{3q}))$ are adjacent to either e or f. Clearly, the efficient domination set $E = \{e, w\}$ or $E = \{f, w\}$ and hence, $\gamma_e(L(\Gamma(Z'_{3q}))) = 2$. □

Theorem 2.5. *If p is any prime, then $\gamma_e(L(\Gamma(Z_{q^2}))) = 1$.*

Proof. $L(\Gamma(Z_{q^2}))$ is a set of $\{q, 2q, 3q, \ldots, (q-1)\}$. Clearly, q is adjacent to every vertices in $V(L(\Gamma(Z_{q^2})))$. Also any two vertices in $L(\Gamma(Z_{q^2}))$ is adjacent. Hence, $\gamma_e(L(\Gamma(Z_{q^2}))) = 1$. □

3. Bounds of $L(\Gamma(Z_n))$

Theorem 3.1. *Let $L(\Gamma(Z_n))$ be the zero divisor graph of line graph. Then $\gamma_e(L(\Gamma(Z_n))) \leq \beta_1(L(\Gamma(Z_n))) - 1$, where $\beta_1(L(\Gamma(Z_n)))$ be independent domination number of line graph.*

Proof. Let R be an independent set of vertices in $\Gamma(Z_n^{'})$ such that $|R| = \beta_1(\Gamma(Z_n))$. Then $\Gamma(Z_n)$ contains no large independent set. This means that every vertex e in is adjacent to at least one vertex of R. Therefore R is an edge dominating set. Also $E = L(\Gamma(Z_n^{'})) - \beta_1(\Gamma(Z_n)) - 1$ is an efficient dominating set of $L(\Gamma(Z_n^{'}))$. That is $|E|$, $|E| \leq |R| - 1$. Therefore $\gamma_e(L(\Gamma(Z_n))) \leq \beta_1(L(\Gamma(Z_n))) - 1$. □

Theorem 3.2. *Let R be a finite commutative ring and let $L(\Gamma(Z_n^{'}))$ be the zero divisor graph of line graph. Then*

$$\frac{q}{1 + \Delta(L(\Gamma(Z_n)))} \leq \gamma_e(L(\Gamma(Z_n))).$$

Proof. Let $E = \{e_1, e_2, \ldots, e_k\}$ be a set of $L(\Gamma(Z_n^{'}))$ and each vertex dominates at most itself and $\Delta(L(\Gamma(Z_n)))$ other vertices, so the conclusion of the theorem follows. □

Theorem 3.3. *Let $L(\Gamma(Z_n^{'}))$ be the zero divisor graph of line graph. Then*

$$\gamma_e(L(\Gamma(Z_n))) \leq q - \delta_1(L(\Gamma(Z_n))) + 1.$$

Proof. Let $E = \{e_1, e_2, \ldots, e_k\}$ be an efficient dominating set in $L(\Gamma(Z_n^{'}))$. Thus, there exists at least one vertex e of minimum degree of edge $\delta_1(L(\Gamma(Z_n)))$.

Let $\delta_1(L(\Gamma(Z_n))) \in V(L(\Gamma(Z_N)))$. Since at least one vertex of D is adjacent to a vertex of minimum degree. Then $|E| \leq |V| - \delta_1 L(\Gamma(Z_n)) + 1$. Therefore $\gamma_e(L(\Gamma(Z_n))) \leq q - \delta_1(L(\Gamma(Z_n))) + 1$. □

Theorem 3.4. *Let $L(\Gamma(Z_n^{'}))$ be the zero divisor graph of line graph. Then*

$$\gamma_e(L(\Gamma(Z_n))) + diam(L(\Gamma(Z_n))) \leq q - 1.$$

Proof. Let $R = \{e_1, e_2, \ldots, e_k\}$ subset of $V(L(\Gamma(Z_n)))$ be the set of edges which constitutes the longest path between any two distinct vertices of $L(\Gamma(Z_n^{'}))$. Such that $|R| = diam L(\Gamma(Z_n))$.

Let $E = \{f_1, f_2, \ldots, f_k\}$ be any minimal dominating set of $L(\Gamma(Z_n^{'}))$. But $|E| \cup R \leq q - 1$. Therefore $\gamma_e(L(\Gamma(Z_n)) + diam(L(\Gamma(Z_n))) \leq q - 1$. □

Theorem 3.5. *For any graph* $L(\Gamma(Z_2^{'} \times Z_q^{'}))$ *where* $q > 2$ *is any prime number, holds* $\gamma_e(L(\Gamma(Z_2 \times Z_q))) = 1$.

Proof. Let $L(\Gamma(Z_2^{'} \times Z_q^{'}))$ is defined as $V = \{(0,1),(0,2),(0,3,\ldots,(0,q-1),(1,0)\} = \{e_1, e_2, \ldots, e_q\}$. The total number of vertices in $L(\Gamma(Z_2^{'} \times Z_q^{'}))$ is q and $L(\Gamma(Z_2^{'} \times Z_q^{'}))$ is a complete graph.

It is clear that, e_q is adjacent to every other vertices in $V(L(\Gamma(Z_n)))$. Thus, $\gamma_e(L(\Gamma(Z_2 \times Z_q))) = 1$ is one.$\{eq\}$ is dominating in exactly one edge $V(L(\Gamma(Z_n)))$. Therefore, $\gamma_e(L(\Gamma(Z_2 \times Z_q))) = 1$. □

Theorem 3.6. *For any connected* $p.q$ *be a graph,* $\gamma_e(L(\Gamma(Z_{2q}^{'}))) + \gamma_i(L(\Gamma(Z_{2q}^{'}))) \leq q$ *Equality holds if* G *isomorpshim of* $K_{1,n-1}$.

Proof. Let $E = \{e_1, e_2, \ldots, e_k\}$ be a subset of $V(L(\Gamma(Z_n)))$ be a set of edges which covers all the edges in line graph. Then E is a minimal edge dominating set of $L(G)$. Further, if the sub graph $< D >$ contains the set of edges e_i, $1 \leq i \leq k$, such that $dege_i = 0$. Then E itself is an independent dominating set of line graph.

Otherwise, $R = E^{'} \cup I$,where $E \subseteq E^{'}$ and $I \subseteq V(L(\Gamma(Z_n^{'}))) - E$ form a minimal edge independent dominating set of line graph. Since $V(L(\Gamma(Z_n^{'}))) = q = E(G)$. It follows that $|R| \cup |E| \leq q$.

Therefore $\gamma_e(L(\Gamma(Z_{2q}^{'}))) + \gamma_i(L(\Gamma(Z"_{2q}))) \leq q$. □

4. Conclusion

In this paper, we studied the efficient dominating set of line graph of zero divisor over commutative ring $Z_n^{'}$. We have studied the efficient domination number of line graph of Z_n, where n is product of primes. Also, we discussed the edge dominating number of line graph over Z_{pq}. We found some relationship between domination and efficient domination number of zero divisor graphs.

If we gray to cut the signal of only one building block in college, we can pull off the sub net Router in that building block, thus that other building blocks will collect their connection properly.Zero Divisor Graph and its edge Domination, we can get proper and good range of the net connection in the whole college campus. Based on star zero divisor of line graph Z_{10}.

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