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A NOTE ON VERTEX-EDGE DOMINATING COLORING

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ABSTRACT. A vertex u in a graph is said to *ve-dominate* an edge e = vw if $u \in \{v, w\}$ or $uv \in E(G)$ or $uw \in E(G)$. An edge coloring is said to be a ve-dominating if no two edges ve - dominated by a single vertex receive the same color. The minimum number of colors required for a ve - dominating coloring of a graph G is called *ve - chromatic number* of G and is denoted by $\chi_{ve}(G)$. In this paper we find ve - chromatic number for special type of graph, called necklace.

1. INTRODUCTION

Let $G = (V, E)$ be a non-trivial connected graph of finite order. A vertex v in a graph is said to *ve-dominate* an edge e = uw if either $v \in \{u, w\}$ or $vu \in E(G)$ or $vw \in E(G)$. A subset $D \subseteq V(G)$ is said to be a *ve - dominating set* of a graph G if every edge in the graph is dominated by a vertex in D. The minimum cardinality of a *ve - dominating set* of a graph is called *ve - domination number* of the graph and is denoted by $\gamma_{ve}(G)$. The study of ve - domination number has been initiated in [1]. An *edge coloring* of a graph is called ve-dominating coloring if the edges ve - dominated by a single vertex receive different colors. The minimum number of colors required for a ve - dominating coloring of a graph G is called *ve - chromatic number* of G and is denoted by $\chi_{ve}(G)$. For a vertex v of a graph G, the ve - degree of v is defined as the number of edges ve - dominated by the vertex v and is denoted by $deg_{ve}(v)$. The minimum and maximum ve - degrees of the graph are defined

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as $\delta_{ve}(G) = \min\{deg_{ve}(v)|v \in V(G)\}$ and $\Delta_{ve}(G) = \max\{deg_{ve}(v)|v \in V(G)\}$, respectively.

Observation 1.1. *[2] For any graph G, $\chi_{ve}(G) \geq \Delta_{ve}(G)$.*

2. χ_{ve} OF NECKLACE GRAPH

Definition 2.1. *A Halin graph G is a plane graph obtained from a planar embedding of a tree T of order at least 4, whose vertices are of degree one or atleast 3 by joining all the vertices of degree 1 in tree T as a cycle C', so that C' is the boundary of the unbounded face.*

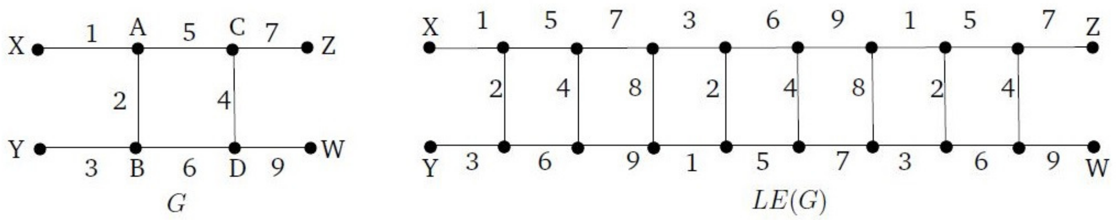
Definition 2.2. *The tree T and the cycle C' is called the characteristic tree and the adjoint cycle of G respectively.*

Definition 2.3. *A caterpillar is a tree such that the removal of the leaves becomes a path.*

Definition 2.4. *A Halin graph G is called a cubic Halin graph if $deg(v) = 3$ for all $v \in G$. A cubic Halin graph in which the characteristic tree is a caterpillar is called necklace.*

Lemma 2.1. *Let G be a graph with $\chi_{ve}(G) \geq 9$. Let A, B, C, D be four vertices of degree 3 on C_4. Let X,Y,Z,W be the other neighbors of A and B, C and D respectively. Then LE(G) is a graph obtained from G by replacing edge-induced subgraph $G[< A, B, C, D >]$ by a ladder of length 7. Then $\chi_{ve}(LE(G)) \leq \chi_{ve}(G)$*

Proof.



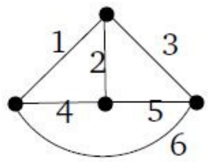
□

Notation 1. *Let $G_i = LE(G_{i-1})$ where $i \geq 2$ and $G_1 = LE(G)$.*

Theorem 2.1.

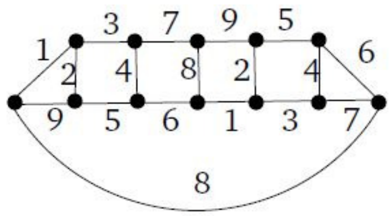
$$\chi_{ve}(Ne_h) = \begin{cases} 6, & \textit{if } h = 1 \\ 9, & \textit{if } h \equiv 2(mod\ 3) \\ 10, & \textit{if } h \equiv 0 \textit{ or } 1(mod\ 3) \textit{ and } h \geq 15, h = 9, 12 \\ 11, & \textit{if } h = 6, 10, 13 \\ 12, & \textit{if } h = 3, 7 \\ 14, & \textit{if } h = 4 \end{cases}$$

Proof. **Case 1:** When h = 1, $|E(Ne_1)| = 6$. A single vertex dominates all the six edges. Therefore, $\chi_{ve}(Ne_1) = 6$.



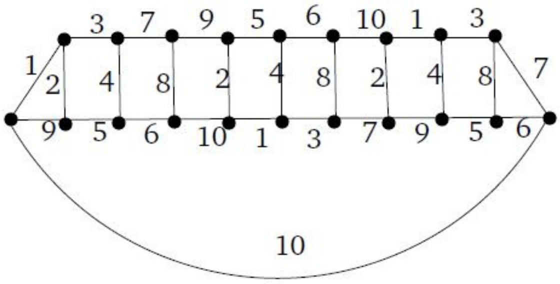
Ne_1

Case 2: If $h \equiv 2(mod\ 3)$, then $Ne_h = G_i$ for some i. By Lemma 2.1, $\chi_{ve}(Ne_h) \leq \chi_{ve}(Ne_5) \leq 9$. Since $\chi_{ve}(Ne_h) \geq \Delta_{ve} = 9$, $\chi_{ve}(Ne_h) = 9$.



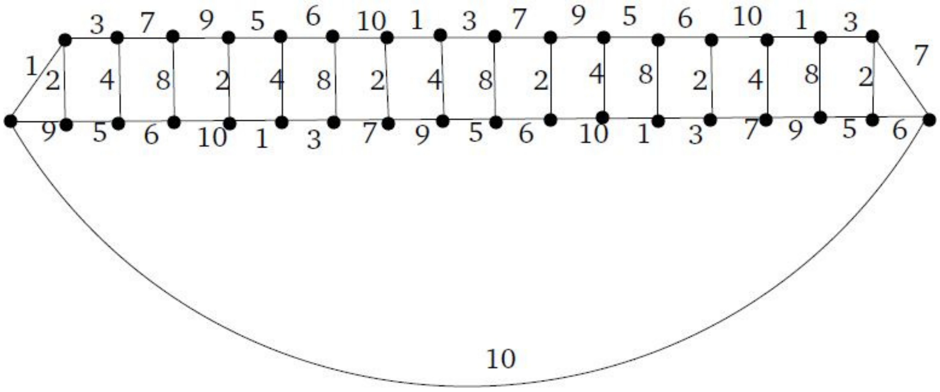
Ne_5

Case 3: If $h \equiv 0(mod\ 3)$. Then $|E(Ne_h)| = 3(3k) + 3 = 9k + 3$. Using 9 colors to color its edges, each color class contains atmost k edges. Therefore, the remaining 3 edges cannot be colored with 9 colors. Hence, $\chi_{ve}(Ne_h) \geq 10$. By Lemma 2.1 $10 \leq \chi_{ve}(Ne_h) \leq \chi_{ve}(Ne_9) \leq 10$. Therefore, $\chi_{ve}(Ne_h) = 10$.



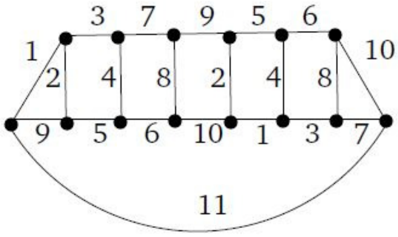
Ne_9

Case 4: If $h \equiv 1(mod\, 3)$. Then $|E(Ne_h)| = 3(3k+1)+3 = 9k+6$. Using 9 colors to color its edges, each color class contains atmost k edges. Therefore, the remaining 6 edges cannot be colored with 9 colors. Hence, $\chi_{ve}(Ne_h) \geq 10$. By Lemma 2.1, $10 \leq \chi_{ve}(Ne_h) \leq \chi_{ve}(Ne_{16}) \leq 10$. Therefore, $\chi_{ve}(Ne_h) = 10$.



Ne_{16}

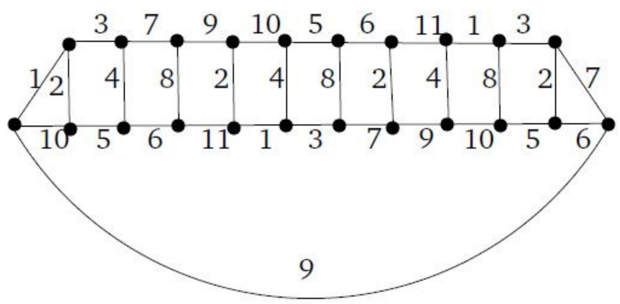
Case 5: When h = 6, 10, 13.
When h = 6.



Ne_6

For Ne_6, $|E(Ne_6)| = 21$. If we use 9 colors, then each ve-color class contains atmost 2 edges and atleast 3 edges are uncolored. If we use 10 colors, then each ve-color class contains atmost 2 edges and atleast one edge is uncolored. Therefore, $\chi_{ve}(Ne_6) \geq 11$.

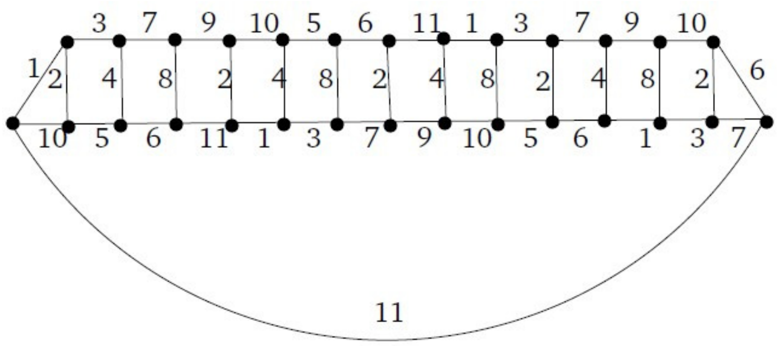
When h = 10.



Ne_{10}

For Ne_{10}, $|E(Ne_{10})| = 33$. If we use 9 colors, then each ve-color class contains atmost 3 edges and atleast 6 edges are uncolored. If we use 10 colors, then each ve-color class contains atmost 3 edges and atleast 3 edges are uncolored. Therefore, $\chi_{ve}(Ne_{10}) \geq 11$.

When h = 13.

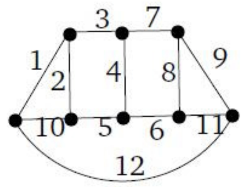


Ne_{13}

For Ne_{13}, $|E(Ne_{13})| = 42$. If we use 9 colors, then each ve-color class contains atmost 4 edges and atleast 6 edges are uncolored. If we use 10 colors, then each ve-color class contains atmost 4 edges and atleast 2 edges are uncolored. Therefore, $\chi_{ve}(Ne_{13}) \geq 11$.

Case 6: When h = 3, 7.

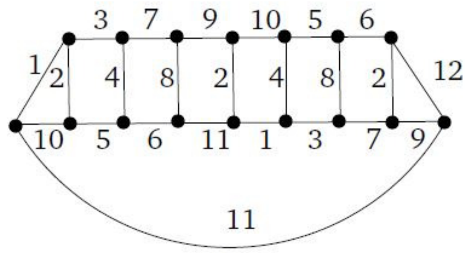
When h = 3.



Ne_3

For Ne_3, $|E(Ne_3)| = 12$. Any two edges are ve-dominated by a vertex. Therefore, $\chi_{ve}(Ne_3) = |E(Ne_3)| = 12$.

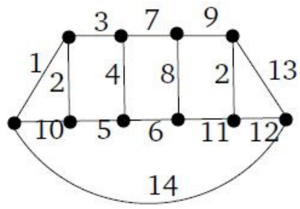
When h = 7.



Ne_7

For Ne_7, $|E(Ne_7)| = 24$. If we use 9 colors, then each ve-color class contains atmost 2 edges and atleast 6 edges are uncolored. If we use 10 colors, then each ve-color class contains atmost 2 edges and atleast 4 edges are uncolored. If we use 11 colors, then each ve-color class contains atmost 2 edges and atleast 2 edges are uncolored. Therefore, $\chi_{ve}(Ne_7) \geq 12$.

Case 7: When h = 4



Ne_4

For Ne_4, $|E(Ne_4)| = 15$. If we use 9 colors, then each ve-color class contains atmost 1 edge and atleast 6 edges are uncolored. If we use 10 colors, then each ve-color class contains atmost 1 edge and atleast 5 edges are uncolored. If we use 11 colors, then each ve-color class contains atmost 1 edge and atleast 4 edges are uncolored. If we use 12 colors, then each ve-color class contains atmost 1 edge and atleast 3 edges are uncolored. If we use 13 colors, then each ve-color class contains atmost 1 edge and atleast 2 edges are uncolored. Therefore, $\chi_{ve}(Ne_4) \geq 14$. □

REFERENCES

[1] J. R. LEWIS, S. T. HEDETNIEMI, T. W. HAYNES, G. H. FRICKE: *Vertex-edge domination*, Util. Math., **81**(2010), 193 - 213.

[2] R. MADHIYALAGAN, A. WILSON BASKAR: *Vertex-Edge dominating coloring of graphs*, Communicated.

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