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IMPROVED ESTIMATION OF SURGERY DURATION USING EXPONENTIAL DISTRIBUTION FOR THE ELECTIVE SURGERY SCHEDULING PROBLEM

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ABSTRACT. Surgery scheduling problem is complicated since the surgery duration is uncertain and a good method to estimate the surgery duration accurately is needed. Failure to do so might affect the operating room (OR) utilization and patient's waiting time. This paper uses statistical distribution for the elective patient's surgery duration to produce a good estimation of length of the surgery duration. Our aim is to maximize the sum of the urgency values assigned to each surgery. The results show that Exponential Distribution (ED) is better than Uniform Distribution (UD) in scheduling the urgency value for scheduled surgeries. ED also reduces the urgency value for the unscheduled surgeries slightly.

1. INTRODUCTION

OR planning and scheduling are getting more attention in health care sector due to the high demand in developing an efficient OR system for the surgery scheduling problem. One of the challenges in creating an efficient surgery schedule is the accurate estimation of the surgery durations. The surgery duration is based on the real data from the hospital. Due to the many procedures that need to be taken, it will take a long time to obtain the real data. Therefore, in this paper we will generate the data for the surgery duration. Eventhough we do not have real data

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from the hospital, they provided us with the estimated duration for the Angiology and Vascular surgery.

The best fit among known distribution such as normal and log-normal distribution ([2, 6]) is usually used to model the variability in surgical procedures and estimated the durations. Another approaches to estimate the data is using the uniform distribution. This distribution is known as the simplest probability model and it is also applied to tests the randomness [9]. In [4] the authors proposed this distribution to estimate the surgery duration based on mean and median values obtained from the historical data.

In [1] and [10] was stated that reliability analysis is the study of various devices life times and the estimation of their possible failures. Exponential is one of the commonly used distributions to model the data reliability and variability. In scheduling problem, [3] proposed this distribution to generate the arrival times for the emergency patients. We refer to this distribution to generate the data for the surgery duration due to its accuracy and diversity.

This paper focuses on the problem of generating data for the surgery duration using two statistical distributions. By using the data information from the hospital and [5], we will generate data using the uniform and exponential distribution and compare the results. Section 2 will briefly explain the heuristic procedures to solve the elective surgery scheduling problem. Section 3 gives a detailed description of the generated data using the distribution. Results and discussion are presented in Section 4. Finally, the main conclusions of this paper can be found in Section 5.

2. Materials and Methods

In this section, we refer to the mathematical model and developed two heuristics procedure to solve the surgery scheduling problem efficiently. We developed a Simple Heuristic procedure (SH) to produce a good initial solution and then modified the Local Search (LS) heuristic to improve the schedule. Although integer linear programming (ILP) method is good to obtain an optimal solutions, sometimes it can be hard to solve for a large size problem. Therefore, we use heuristic method to quickly find good solutions. Further explanation of both heuristics can be found in our previous study [8].

2.1. **Integer Linear Programming Model.** In this section, an integer linear programming model is presented. The model is used for the elective surgery scheduling problem. All the notations, decision and binary variables used in the model are presented below.

Parameters	
i	Index for surgery
d	Index for days
r	Index for operating rooms
cir	Index for number of surgeons available for each surgery speciality, esp
Urg_i	Indicator of the urgency of surgery i.
Dur_i	Estimated duration of surgery i.
Gr_i	Number of operation room group where i is performed.
Day_{gr_t}	Number of days to program for operation room group t.
OR_{gr_t}	Number of rooms available in operation room group t.
S_{gr_t}	Number of surgeries to be programmed in operation room group t.
Dur_{max}	Maximum time available for each room in single day.
$S_{min\_r\_d}$	Minimum number of surgeries to be programmed in each room, r on each day, d.
$Beds_{d_t}$	Number of beds available for operated patients in group t on each day, d.
esp_{cir}	Surgery speciallity with a cir number of surgeons.
$n_{esp_{cir}}$	Number of surgeries programmed that belong to esp_{cir}.
$t_{esp_{cir}}(i)$	Set that indicates the surgeries i selected that pertain to surgery speciality esp_{cir}.

The ILP model proposed by [7] involve the parameters in the above table are formulated as follows:

$$\max \sum_{d=1}^{Day_{gr_t}} \sum_{r=1}^{OR_{gr_t}} \sum_{i=1}^{S_{gr_t}} Urg_i \times x_{ird}. \tag{2.1}$$

Objective function (2.1) maximizes the sum product of the decision variable and the urgency value of each surgery scheduled.

$$s.t: \sum_{i=1}^{S_{gr_t}} Dur_i \times x_{ird} \leq Dur_{max} \qquad \forall r = 1, \ldots, OR_{gr_t}, \quad \forall d = 1, \ldots, Day_{gr_t}. \tag{2.2}$$

Constraint (2.2) is the time constraint where all surgeries need to be done within the time frame 8.00 AM to 4.00 PM which is 480 minutes per operation room, r in day, d.

$$\sum_{d=1}^{Day_{gr_t}} \sum_{r=1}^{OR_{gr_t}} x_{ird} \leq 1, \qquad \forall i = 1, \ldots, S_{gr_t}. \tag{2.3}$$

Constraint (2.3) ensures that each surgery is assigned only once at a given time per operating room (only 1 surgery can be done at a given time per operating room, represent a particular type of assignment restrictions since they allow the unique assignment or non-assignment of each one of the decision variables).

$$\sum_{i=1}^{S_{gr_t}} x_{ijk} \geq S_{min\_r\_d} \qquad \forall r = 1, \ldots, OR_{gr_t}, \quad \forall d = 1, \ldots, Day_{gr_t}. \tag{2.4}$$

Constraint (2.4) represent the minimum number of surgeries that can be performed in each operation room, r.

$$\sum_{r=1}^{OR_{gr_t}} \sum_{i=1}^{S_{gr_t}} x_{ird} \leq Beds_{d_t}, \qquad \forall d = 1, \ldots, Day_{gr_t}. \tag{2.5}$$

Constraint (2.5) is the capacity constraint where all patients will have recovery bed after the surgery.

$$\sum_{i=1}^{n_{esp_{cir}}} x_{t_{esp_{cir}}(i)rd} = 0, \qquad \forall d = 1, \ldots, Day_{gr_t}, \quad \forall r \neq 1, \quad \text{for} \quad t_{esp_{cir}} \leq 4. \tag{2.6}$$

Constraint (2.6) is the surgeon constraint where for every surgery scheduled, there must be four or less surgeons ($cir \leq 4$) and surgeon that performed only in room 1, giving value of 0 for any possible assignment in other rooms. For our research, we only considered to assign at least two surgeons for each surgery which is lead surgeon and assistant surgeon.

$$x_{ird} = \{0, 1\}, \qquad i = 1, \ldots, S_{gr_t}, \quad r = 1, \ldots, OR_{gr_t}, \quad d = 1, \ldots, Day_{gr_t}. \tag{2.7}$$

Lastly, constraint (2.7) is the decision variable used in the model where,

$$x_{ird} = \begin{cases} 1; & \text{if surgery, } i \text{ performed in operation room, } r \text{ on day, } d \\ 0; & \text{otherwise} \end{cases}.$$

2.2. **Simple Heuristic.** This heuristic is developed to make sure that higher urgency surgeries will be prioritize so that patients with critical condition will get their treatment as soon as possible. The Shortest Processing Time (SPT) rule is applied as we consider patients in the same urgency group with shorter surgery durations. This rule allows the heuristic to schedule more surgeries in the ORs while considering the urgency value assigned to each surgery. Now we have the initial sequence for the surgery list. The final step is to assign the surgeries to ORs according to the initial sequence to generate the initial solution.

2.3. **Local Search.** Local search heuristic will help to improve the initial solution from the simple heuristic procedure. This heurisitic works by randomly moving in the neighborhood to find better solution. If the new solution is better than current solution, it will replaces the current solution with the new solution. The iteration will continue until no better solution can be found. In our case, we have modified the heuristic to suit our problem. Our local search will search the unscheduled surgeries by each row until no further improvement can be founded. The current solution will be replaced by the new solution if the new total urgency value is better than the current one.

3. Experimental Design

In this section, we present the generated data using uniform and exponential distribution. Since we are not provided with the real data, we will used the information of the estimated surgery duration from the hospital to generate our data. Based on the previous study, we used a uniform distribution to generate data for the surgery duration. In this section, we use an exponential distribution to generate our data and we compare the data between both distribution. We also refer the information for the surgery duration of 265 patients from other paper and test using uniform and exponential distribution.

3.1. **Generated data for surgery list.** **Table 1** present the details of the surgery lists with the size of 30 and 265 patients. Data of 30 patients is from our previous study while data of 265 patients is referred from other paper. They provided the information of surgery duration for the Angiology and Vascular surgery which is the surgery type that we considered in our study. In this paper, we focus on two datas (small data and big data) to test our heuristic procedure. The number of ORs available for scheduling, N_{Room} and number of days to schedule the surgeries, N$_{Day}$ are set randomly to test our proposed heuristic in different demand settings. Urgency value for each surgery i, Urg_i and surgery type, $Type_i$ are randomly generated using the uniform distribution. The priority of each surgery is determined by the value of the Urg_i, where value 1 is the least urgent surgery while value 4 is the most urgent surgery and must be scheduled first. Since it is hard to obtain information on all types of surgeries, we only consider two types of surgery. The surgery type correspond to the Angiology and Vascular surgery. As for the surgery duration for each surgery i, Dur_i, we will use a uniform and exponential distribution to generate our data. Further explanation is presented in Section 3.2.

TABLE 1. Details of the surgery lists.

Size	N$_{Room}$	N$_{Day}$	Urg_i	$Type_i$	Dur_i(min)
30	2	2	[1,4]	[1,2]	[30,120]
265	5	5	[1,4]	[1,2]	[45,120]

3.2. **Surgery duration, Dur_i using uniform and exponential distribution.** The details of generated data for the surgery duration using uniform and exponential distribution is presented in **Table 2**. We consider two data sets with the size of 30 and 265 patients.

We generate the surgery duration by using a Uniform Distribution (UD) and Exponential Distribution (ED). Both distributions is given by the notation $X \sim U[a,b]$ and $X \sim Exp(\lambda)$. The Exponential Distribution (ED) is ranged from 0 to $+\infty$, where the maximum relative length is infinite. However, in our case there is a limit on the maximum length of a surgery in the OR. Therefore, we limit the surgery duration in the range $[a,b]$ with new value of μ.

TABLE 2. Details for Uniform Distribution (UD) and Exponential Distribution (ED).

Size	Distribution	Notation	Range	Parameter
30	Uniform	$X \sim U[30,120]$	[30,120]	$\mu = 75$, $\sigma = 7.5$
	Exponential	$X \sim Exp(0.0141)$	[0,+∞)	$\mu = \sigma = 71.0667$
		$X \sim Exp(0.0144)$	[30,120]	$\mu = \sigma = 69.4$
265	Uniform	$X \sim U[45,120]$	[45,120]	$\mu = 82.5$, $\sigma = 6.25$
	Exponential	$X \sim Exp(0.0215)$	[0,+∞)	$\mu = \sigma = 46.5$
		$X \sim Exp(0.0136)$	[45,120]	$\mu = \sigma = 73.2755$

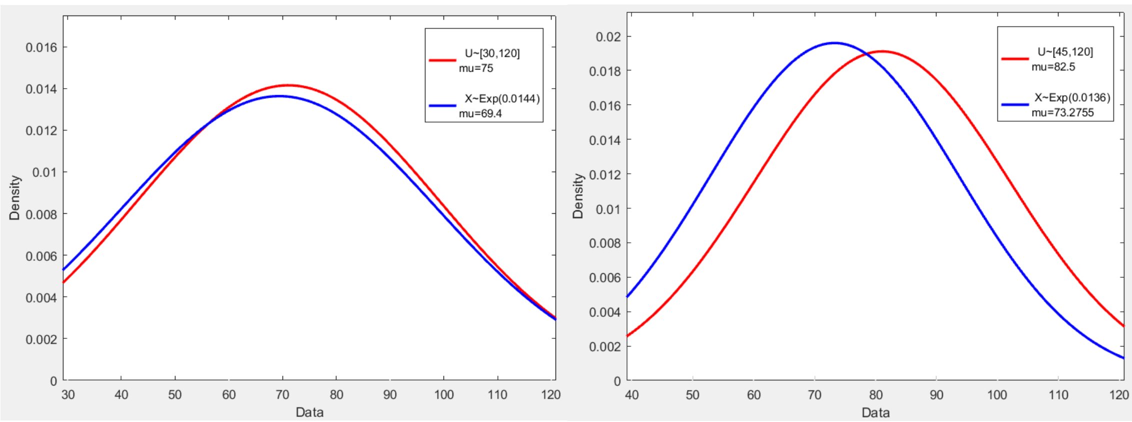
For data set with the size of 30 patients, we generate the surgery duration based on the estimated duration from the hospital which is between 30 to 120 minutes. The duration is for the Angiology and Vascular surgery in Hospital Tengku Ampuan Afzan (HTAA), Kuantan. As for the the data set with the size of 265 patients, we refer to the data information for the same surgery type from other paper.

Figure 1(A) and **Figure 1(B)** shows the comparison of the surgery duration between the uniform distribution (UD) and exponential distribution (ED) for the size of 30 and 265 patients. The **red** line present the UD data while **blue** line present the ED data.

Figure 1(A) present the surgery duration data of 30 patients for the UD and ED. The graph of data for the distributions are very close since the differences of the mean value, μ, is only 5.6 minutes. As the data size is small, the data has less variability which is why the graph is as shown in **Figure 1(A)**.

Figure 1(B) present the graph of the surgery duration for the UD and ED for 265 patients. By referring to the figure, the graph is differ as the UD skewed to right while the ED skewed to the left. As the data size increases, the data has more diversity. Not only that, the mean value also differ for 10 minutes.

This differences will affect the performances of our proposed heuristics and quality of the solutions. The results obtained using different data distribution is presented in **Section 4**.



(A) 30 patients

(B) 265 patients

FIGURE 1. Uniform Distribution (UD) VS Exponential Distribution (ED)

4. Results and Discussion

Based on the Local Search heuristic from the previous study, we test the proposed heuristic with the generated data from the uniform and exponential distribution. The performances of the heuristic with the generated data is measured by the total computational time for uniform distribution, Time$_{UD}$, total computational time for exponential distribution, Time$_{ED}$, total urgency value for the **scheduled** surgeries, Urg_S, and total urgency value for the **unscheduled** surgeries, Urg_{US} for both distributions.

Table 3 shows the total computational time for the UD and ED using the local search heuristic to solve the surgery scheduling problem. It can be observed that there is no significant difference between the total computational time for both distributions. Although the size of the data set is increase to 265 patients, the Time$_{UD}$ and Time$_{ED}$ is very fast as it only takes less than two seconds to solve the problem.

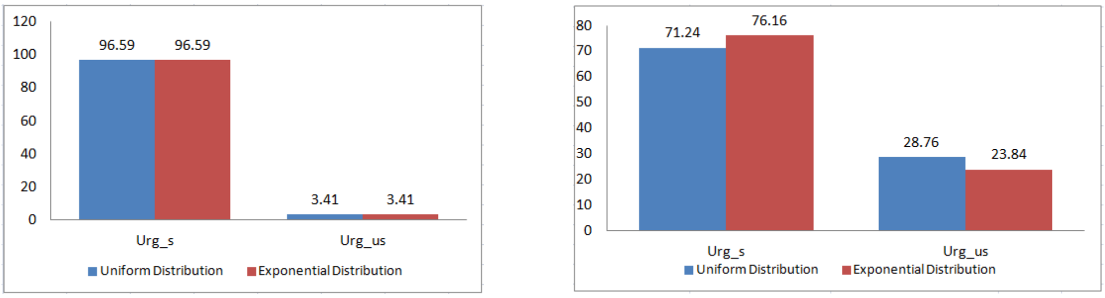
Table 3. Total computational time for UD and ED

Size	Time$_{UD}$(s)	Time$_{ED}$(s)
30	0.067007	0.075795
265	1.868582	1.603512

Figure 2(A) shows the result of data set with the size of 30 patients. Both distributions produce the same result for the total of urgency value for scheduled, Urg_S and unscheduled surgeries, Urg_{US}. Since the data size is small, there is less variability in the data which lead to the same result. The result is good as the Urg_S is 96.59% and the Urg_{US} is only 3.41%.

Based on **Figure 2(B)**, we can see the differences between the total of urgency value for scheduled surgeries, Urg_S and unscheduled surgeries, Urg_{US} for UD and ED. The ED improves the solution by increasing the value of Urg_S by 6.9% and decreases the value of Urg_{US} by 17.11%. This concludes that ED is better than UD.

This is because by using a large size of data, we also increase the variability of the data. In general, the surgery duration generated by the exponential distribution is better than the uniform distribution as it improve the surgery scheduling problem.



(A) 30 patients (B) 265 patients

FIGURE 2. Uniform Distribution (UD) VS Exponential Distribution (ED)

5. CONCLUSION

This paper is mainly focused on generating the estimated surgery duration for the surgery scheduling problem. Uniform and exponential distribution is used to generate our data. Both data is tested using the local search heuristic and the results show that using exponential distribution to generate the surgery duration is better than the uniform distribution. By using data from the exponential distribution, we have increased the total number of urgency value for the scheduled surgeries as well as reducing the total number of urgency value for the unscheduled surgeries. The total computational time for both distributions is not significant since it is very fast even when the size of data set is increased. To schedule surgical procedure, we must have a statistical distribution that accounts for the variability inherent in the surgical times. Not only that, the reliability factor is also important which determines the data accuracy. Thus, the exponential distribution is better than uniform distribution in generating data for the surgery duration. In the future, other statistical method can be used to improve the estimation of the surgery duration.

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