

## STRUCTURE OF T–SEMIRING

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**ABSTRACT.** In this paper, we study the conditions under which the class of T-semirings are additively and/or multiplicative idempotent. We also study the structures of T-semiring. In a totally ordered T-semiring, we prove that the additive and multiplicative structures are maximum addition and maximum multiplication respectively.

### 1. INTRODUCTION

The word idempotent signifies the study of semirings in which the addition operation is idempotent  $u + u = u$ . The best-known example for idempotent semiring is the max-plus semiring. Interest has been shown in such structures arose in late 1950s through the observation that certain problems of discrete optimization could be linearized over suitable idempotent semirings. Recently the subject has established connections with discrete event systems automata theory, non-expansive mappings, optimization theory. Idempotent semiring is a fundamental structure that has many applications in Computer Science. Idempotent semiring is a ring with additive idempotent. Recently modal operators of idempotent semirings are introduced in order to model the properties of programs and transition systems more suitably and to link algebraic and relational formalisms with dynamic and temporal logics. Some other applications of semiring areas are

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cryptography, optimization theory, graph theory, dynamical systems and automata theory.

The paper is organized as follows: Section 1 contains introduction. In section 2 some definitions are given, Section 3 presents structure of T - semiring. In section 4 we study the structure of totally ordered T - semiring and the last section is conclusion. Additional information can be found in [1-21].

## 2. PRELIMINARIES

We used the following definitions in this paper.

An algebraic structure  $(S, +, \cdot)$  is termed as semiring if the additive reduct is a semigroup; multiplicative reduct is a semigroup and  $u(x + y) = ux + uy$  and  $(x + y)u = xu + yu$  for every  $u, x, y$  in  $S$ . An additive s.g is additively idempotent if  $u + u = u$  for all  $u$  in  $S$ . A multiplicative s.g is multiplicatively idempotent if  $u^2 = u \forall u$  in  $S$ . A semiring is said to be a mono semiring if  $u + x = ux \forall u, x$  in  $S$ . A multiplicative s.g is assumed to be left (right) singular if  $ux = u(xu = x)$  for all  $u, x$  in  $S$ . An additive s.g is said to be left (right) singular if  $u + x = u(u + x = x)$  for all  $u, x$  in  $S$ . An element  $u$  is periodic if  $u^m = u^n$ , where  $m$  and  $n$  are positive integers. A multiplicative s.g is said to be periodic if every one of its elements is periodic. An additive s.g is rectangular band if  $u = u + x + u \forall u, x$  in  $S$ . A multiplicative s.g is rectangular band if  $u = uxu \quad \forall u, x$  in  $S$ . In a semiring  $S$ , the s.g  $(S, \cdot)$  is zeroid if for all  $u$  in  $S$  such that  $ux = x$  or  $xu = x$  for some  $x$  in  $S$ . In a semiring  $S$ , the additive s.g is zeroid if for all  $u$  in  $S$  such that  $u + x = x$  or  $x + u = x$  for some  $x$  in  $S$ . An additive s.g is commutative if  $u + x = x + u$  for all  $u, x$  in  $S$ . A multiplicative s.g is commutative if  $ux = xu \forall u, x$  in  $S$ . A component  $u$  in a multiplicative s.g is known as left and right cancellable, if  $ux = uy(xu = yu)$  for any  $x, y$  in  $S$  implies  $x$  equals to  $y$ . An element  $u$  in an additive s.g is known as left and right cancellable, if  $u + x = u + y$  and  $x + u = y + u$  for any  $x, y$  in  $S$  implies  $x$  equals to  $y$ . A semiring  $S$  is almost idempotent if  $u + u^2 = u^2 \forall u, x$  in  $S$ . A semiring  $S$  is said to satisfy the Integral Multiple Property (IMP) if  $u^2 = nu \forall u$  in  $S$ , where the positive integer  $n$  depends on the element  $u$ .

### Note.

- (1) In this paper  $S$  is said to be a T - semiring satisfying the identity  $u^2 + uxu = u$  for all  $u, x$  in  $S$ .
- (2) For undefined concepts refer (iv).

## 3. STRUCTURE OF T - SEMIRING

**Lemma 3.1.** *Let  $S$  be a  $T$  - semiring. Then  $S$  is an idempotent semiring in the following cases.*

- (a)  $S$  contains the multiplicative identity.
- (b)  $(S, +)$  is right cancellative.
- (c)  $S$  is multiplicatively subidempotent semiring.
- (d)  $(S, +)$  is left singular.

*Proof.*

- (a) Given that  $S$  is a  $T$  - semiring then  $u^2 + u^3 = u$  for all  $u$  in  $S$  since  $S$  contains multiplicative identity implies  $1 + 1 = 1$ , and

$$(3.1) \quad \implies u + u = u \longrightarrow (1).$$

Therefore  $(S, +)$  is idempotent since  $S$  is a  $T$  semiring  $u^2 + uxu = u$  for all  $u, x$  in  $S$

$$\implies u^2 + u \cdot 1 \cdot u = u \implies u^2 + u^2 = u \implies u(u + u) = u.$$

Using equation (3.1) in above we get

$$(3.2) \quad u^2 = u.$$

Therefore  $(S, \cdot)$  is idempotent Hence from equations (3.1) and (3.2),  $S$  is an idempotent semiring.

- (b) By hypothesis

$$(3.3) \quad u^2 + uxu = u \text{ for all } u, x \text{ in } S$$

and

$$(3.4) \quad u^2 + u^3 = u \text{ for all } u \text{ in } S.$$

From (3.3)

$$(3.5) \quad u^2 + u(u + u)u = u \implies u^2 + u^3 + u^3 = u \implies u + u^3 = u.$$

From (3.4) and (3.5):  $u^2 + u^3 = u + u^3$ . Since  $(S, +)$  is right cancellative implies

$$(3.6) \quad u^2 = u.$$

Therefore  $(S, \cdot)$  is idempotent.

Now substituting equation (3.6) in equation (3.4) we obtain  $u + u = u$ .  
Therefore  $(S, +)$  is idempotent. Hence  $S$  is an idempotent semiring.

(c) Consider

$$(3.7) \quad u^2 + u^3 = u \text{ for all } u \text{ in } S,$$

since  $S$  is multiplicative subidempotent

$$(3.8) \quad u + u^2 = u \text{ for all } u \text{ in } S$$

$$(3.9) \quad u^2 + u^3 = u^2$$

From (3.7) and (3.9),

$$(3.10) \quad u = u^2.$$

Therefore  $(S, \cdot)$  is idempotent substituting equation (3.10) in equation (3.8) we get  $u + u = u$ . Therefore  $(S, +)$  is idempotent. Hence  $S$  is an idempotent semiring.

(d) By hypothesis

$$(3.11) \quad u^2 + u^3 = u \text{ for all } u \text{ in } S$$

since  $(S, +)$  is left singular then equation (3.11) becomes  $u^2 = u$ . Again consider  $u^2 + u^3 = u \implies u + u \cdot u = u \implies u + u = u$ . Therefore  $(S, +)$  is idempotent. Hence  $S$  is an idempotent semiring.

□

**Example 3.2.** We have framed an example for the above lemma in multiplicative idempotent semiring, idempotent and  $T$  - semiring with  $S = \{u, x, y\}$ .

+	u	x	y
u	u	x	y
x	u	x	y
y	u	x	y

·	u	x	y
u	u	x	y
x	u	x	y
y	u	x	y

**Theorem 3.3.** If  $S$  is a  $T$  - semiring and  $(S, \cdot)$  is left singular or right singular semigroup, then  $(S, +)$  is idempotent.

*Proof.* Given  $u^2 + uxu = u$  for all  $u, x$  in  $S$  since  $(S, \cdot)$  is left singular then above equation becomes  $u + u \cdot u = u$  and  $u + u = u$ . Therefore  $(S, +)$  is idempotent. □

**Remark 3.4.** In a  $T$ -semiring if  $(S, \cdot)$  is idempotent, then  $(S, +)$  is idempotent.

**Theorem 3.5.** Let  $S$  be a  $T$ -semiring. Then  $(S, \cdot)$  is periodic under the following cases.

- (a)  $S$  is an almost idempotent semiring.
- (b)  $S$  is a mono semiring.

*Proof.* (a) We have

$$(3.12) \quad u^2 + u^3 = u \text{ for all } u \text{ in } S$$

since  $S$  is almost idempotent semiring

$$(3.13) \quad u + u^2 = u^2 \text{ for all } u \text{ in } S.$$

From (3.12)  $u(u + u^2) = u$  using equation (3.13) we obtain  $u(u^2) \implies u^3 = u$  Therefore  $(S, \cdot)$  is periodic.

(b) Again consider equation  $u^2 + u^3 = u$  for all  $u$  in  $S$  since  $S$  is mono semiring then above equation implies  $u^2 \cdot u^3 = u \implies u^5 = u$  Therefore  $(S, \cdot)$  is periodic.  $\square$

**Proposition 3.6.** Let  $S$  be a  $T$ -semiring. If  $(S, +)$  is idempotent, then  $u^n + u = u$  for all  $u$  in  $S$ .

*Proof.* Given

$$(3.14) \quad u^2 + uxu = u \text{ for all } u, x \text{ in } S$$

Further:

$$\begin{aligned} &\implies u^2 + u^2 + uxu = u^2 + u \\ &\implies u^2 + uxu = u^2 + u, \text{ since } (S, +) \text{ is idempotent} \\ &\implies u = u^2 + u \\ &\implies u^3 + u^2 = u^2 \\ &\implies u^3 + u^2 + uxu = u^2 + uxu \\ &\implies u^3 + u = u \text{ by (3.14).} \end{aligned}$$

Continuing like this we get  $u^n + u = u$  for all  $u$  in  $S$ .  $\square$

**Proposition 3.7.** Let  $S$  be a  $T$ -semiring and  $S$  satisfies IMP. Then  $(S, +)$  is periodic.

*Proof.* By hypothesis  $u^2 + u^3 = u$  for all  $u$  in  $S$ . Then  $u^2 + u^2 \cdot u = u$  since  $S$  satisfies IMP then above equation becomes  $nu + (nu)u = u$ , and further

$$\implies nu + nu^2 = u \implies nu + n(nu) = u \implies nu + n^2u = u \implies n(1 + n)u = u.$$

Therefore  $(S, +)$  is periodic.  $\square$

**Theorem 3.8.** *Let  $S$  be a  $T$  - semiring. If  $(S, \cdot)$  is zeroid, then  $(u + x)u = u$  or  $u(u + x) = u$  for all  $u, x$  in  $S$*

*Proof.* Consider

$$(3.15) \quad u^2 + uxu = u \text{ for all } u, x \text{ in } S.$$

By hypothesis  $(S, \cdot)$  is zeroid,  $ux = x$  or  $xu = x$ . From (3.15),  $u^2 + uxu = u$  and further,  $u^2 + xu = u$ . Since  $ux = x$ , we have  $(u + x)u = u$ .

Now consider  $xu = x$ . From (3.15),  $u^2 + uxu = u$ , and further  $u^2 + ux = u$ , i.e.,  $u(u + x) = u$ . Therefore  $(u + x)u = u$  or  $u(u + x) = u$   $\square$

**Theorem 3.9.** *Let  $S$  be a  $T$  - semiring and zerosum semiring. Then*

(a)  $u^2 + u = u + u^2$  for all  $u$  in  $S$ .

(b)  $u^2xu = u$ .

*Proof.* (a) We have  $u^2 + uxu = u$  for all  $u, x$  in  $S$ . Then,  $u^2 + u^2 + uxu = u^2 + u$  and since  $S$  is zerosum semiring  $0 + uxu = u^2 + u$  we have

$$(3.16) \quad u^2 + u = uxu.$$

Again, we take  $u^2 + uxu = u$  and receive  $u + u^2 + uxu = u + u$  since  $S$  is zerosum semiring  $u + u^2 + uxu = 0$ ,  $u + u^2 + uxu + uxu = 0 + uxu$  and  $u + u^2 + 0 = 0 + uxu$ . Since  $S$  is zerosum semiring we have

$$(3.17) \quad u + u^2 = uxu$$

From equation (3.16) and (3.17) we get  $u^2 + u = u + u^2$ .

(b) Since  $u^2 + u^3 = u$  for all  $u$  in  $S$  we have  $u(u + u^2) = u$ , and further  $u(uxu) = u$ . Therefore  $u^2xu = u$ .  $\square$

**Theorem 3.10.** *If  $S$  is a  $T$  - semiring and  $(S, \cdot)$  is rectangular band, then*

(a)  $S$  is an almost idempotent semiring.

(b)  $(u^2 + u)(u + u^2) = u$  for all  $u$  in  $S$ .

*Proof.* (a) From

$$(3.18) \quad u^2 + uxu = u \text{ for all } u, x \text{ in } S$$

since  $(S, \cdot)$  is rectangular band

$$(3.19) \quad u^3 = u$$

and

$$(3.20) \quad u^2 + u = u.$$

Also, from (3.18)  $u^2 + u^3 = u$  for all  $u$  in  $S \implies u^2 + u = u$  by (3.19),

$$u(u^2 + u) = u \cdot u \implies u^3 + u^2 = u^2 \implies u^2 + u^3 + u^2 = u^2 + u^2.$$

Since  $(S, \cdot)$  is rectangular band then  $(S, +)$  is idempotent then above equation implies

$$(3.21) \quad u + u^2 = u^2 \text{ for all } u \text{ in } S.$$

Therefore  $S$  is an almost idempotent semiring.

(b) From (3.20) and (3.21)  $(u^2 + u)(u + u^2) = u \cdot u^2$  we receive  $(u^2 + u)(u + u^2) = u^3$ , i.e.,  $(u^2 + u)(u + u^2) = u$ .  $\square$

**Theorem 3.11.** *Let  $S$  be a  $T$  - semiring. If  $(S, +)$  is a rectangular band and  $(S, +)$  is commutative, then  $u + u = u^2 + u^2$  for all  $u$  in  $S$ .*

*Proof.* Consider

$$(3.22) \quad u^2 + uxu = u \text{ for all } u, x \text{ in } S$$

So,  $u^2 + u^3 = u$  for all  $u$  in  $S$ , since  $(S, +)$  is rectangular band  $u + x + u = u$  for all  $u, x$  in  $S$ . Thus,  $u(u + x + u) = u \cdot u$  and  $u^2 + ux + u^2 + uxu = u^2 + uxu$ . Using (3.15),  $u^2 + ux + u = u$  we receive  $u^3 + uxu + u^2 = u^2$  and  $u^3 + u^2 + uxu = u^2$ , since  $(S, +)$  is commutative which implies  $u^3 + u = u^2$ ,  $u^2 + u^3 + u = u^2 + u^2$  and  $u + u = u^2 + u^2$ ,

Therefore  $u + u = u^2 + u^2$  for all  $u$  in  $S$ .  $\square$

**Theorem 3.12.** *If  $S$  is a  $T$  - semiring and  $(S, \cdot)$  is left cancellative, then  $u + u^2 = x + x^2$  for all  $u, x$  in  $S$*

*Proof.* Consider  $u^2 + u^3 = u$  for all  $u$  in  $S$ . Then

$$(3.23) \quad u^2x + u^3x = ux.$$

Also  $x^2 + x^3 = x$  for all  $x$  in  $S$  implies

$$(3.24) \quad ux^2 + ux^3 = ux.$$

From (3.23) and (3.24),  $u^2x + u^3x = ux^2 + ux^3$ , and further  $ux(u + u^2) = ux(x + x^2)$  since  $(S, \cdot)$  is left cancellative.

Therefore  $u + u^2 = x + x^2$  for all  $u, x$  in  $S$ .  $\square$

## 4. STRUCTURE OF TOTALLY ORDERED T - SEMIRING

**Note.**

- (1) Throughout this paper positively totally ordered is denoted by p.t.o. and negatively totally ordered is denoted by n.t.o.
- (2) For undefined concepts refer (XII).

**Definition 4.1.** In a totally ordered semiring  $(S, +, \cdot, \leq)$

- (a)  $(S, +)$  and  $(S, \cdot)$  are p.t.o, if  $u + x \geq u, x(ux \geq u, x) \forall u, x$  in  $S$ ;
- (b)  $(S, +)$  and  $(S, \cdot)$  are n.t.o, if  $u + x \leq u, x(ux \leq u, x) \forall u, x$  in  $S$ .

**Definition 4.2.** An element  $u$  in a partially ordered semigroup  $(S, +, \leq)$  is non-negative (non-positive) if  $u + u \geq u(u + u \leq u)$ . A partially ordered semigroup  $(S, \cdot, \leq)$  is non-negative (non-positive) if  $u^2 \geq u(u^2 \leq u)$ .

**Definition 4.3.** An element  $u$  in a totally ordered semiring is said to be a minimal/maximal if  $u \leq x(u \geq x)$  for every  $x \in S$ .

**Theorem 4.4.** If  $S$  is a totally ordered  $T$  - semiring and  $(S, +)$  is p.t.o, then  $(S, \cdot)$  is non-positively ordered.

*Proof.* We have  $u^2 + uxu = u$  for all  $u, x$  in  $S$ . Since  $(S, +)$  is p.t.o  $u = u^2 + uxu \geq u^2$ , then  $u \geq u^2$  for all  $u$  in  $S$ . Therefore  $(S, \cdot)$  is non-positively ordered.  $\square$

**Proposition 4.5.** Let  $S$  be a totally ordered  $T$  - semiring. If  $S$  has multiplicative identity 1 and  $(S, +)$  is p.t.o., then

- (a)  $(S, \cdot)$  is n.t.o.;
- (b) 1 is the maximum element.

*Proof.* (a) We have  $1^2 + 1 \cdot x \cdot 1 = 1$  for all  $x$  in  $S$ . Then

$$1 + x = 1 \implies u(1 + x) = u \cdot 1 \implies u + ux = u.$$

Since  $(S, +)$  is p.t.o.

$$(4.1) \quad u = u + ux \geq ux.$$

Again by T-semiring we have  $1^2 + 1 \cdot u \cdot 1 = 1$  for all  $u$  in  $S$  and

$$\implies 1 + u = 1 \implies (1 + u)x = x \cdot 1 \implies x + ux = x.$$

Since  $(S, +)$  is p.t.o.

$$(4.2) \quad x = x + ux \geq ux.$$



From (4.1) and (4.2),  $(S, \cdot)$  is n.t.o..

(b) We have  $1 + 1 \cdot x \cdot 1 = 1$  for all  $1, x$  is  $S$ . So,  $1 + x = 1$ . Since  $(S, +)$  is p.t.o.,  $1 = 1 + x \geq x$ . Therefore 1 is the maximum element.  $\square$

**Theorem 4.6.** *If  $S$  is a totally ordered  $T$  - semiring and  $(S, \cdot)$  is non-negatively ordered, then  $(S, +)$  is non-positively ordered.*

*Proof.* Consider  $u^2 + u^3 = u$  for all  $u$  in  $S$ . Since  $(S, \cdot)$  is non-negatively ordered  $u^2 \geq u$ . Therefore  $u = u^2 + u^3 \geq u + u \cdot u \geq u + u$ , i.e.,  $u \geq u + u$ . Therefore  $(S, +)$  is non-positively ordered.  $\square$

**Proposition 4.7.** *Let  $S$  be a totally ordered  $T$  - semiring. If  $S$  contains multiplicative identity.*

(a) *If  $(S, +)$  is p.t.o., then  $u + x = x + u = \max(u, x)$  for all  $u, x$  is  $S$ .*

(b) *If  $(S, \cdot)$  is p.t.o., then  $ux = xu = \max(u, x)$  for all  $u, x$  is  $S$ .*

*Proof.* (a) Let  $u, x \in S$ . Suppose  $u < x$ . Then  $u + u \leq u + x \leq u + u$ . Since  $(S, +)$  is idempotent,

$$(4.3) \quad u \leq u + x \leq u.$$

From (4.3),

$$(4.4) \quad u + x \leq x.$$

Since  $(S, +)$  is p.t.o.,

$$(4.5) \quad u + x \geq x.$$

From (4.4) and (4.5),  $u + x = x = \max(u, x)$ . Also  $u < x$  implies

$$(4.6) \quad u + u \leq x + u \leq x + x, \quad x + u \leq x.$$

From (4.3)

$$(4.7) \quad x + u \leq x.$$

Since  $(S, +)$  is p.t.o.,

$$(4.8) \quad x + u \geq x.$$

From (4.7) and (4.8)  $x + u = x = \max(u, x)$ .

Similarly, we can prove that  $u + x = x + u = \max(u, x)$  if  $x < u$ .

(b) Again Let  $u, x \in S$ . Suppose  $u < x$ . Then  $u^2 \leq ux \leq x^2$ . Since  $(S, \cdot)$  is idempotent,

$$(4.9) \quad u \leq ux \leq u.$$

From here

$$(4.10) \quad ux \leq x.$$

Since  $(S, \cdot)$  is p.t.o,

$$(4.11) \quad ux \geq x.$$

From (4.10) and (4.11),  $ux = x = \max(u, x)$ . Also  $u < x$  implies  $u^2 \leq xu \leq x^2$ . Since  $(S, \cdot)$  is idempotent,

$$(4.12) \quad u \leq xu \leq x.$$

From (4.9),

$$(4.13) \quad xu \leq x.$$

Since  $(S, \cdot)$  is p.t.o,

$$(4.14) \quad xu \geq x.$$

From (4.13) and (4.14),  $xu = x = \max(u, x)$ .

Similarly, we can prove that  $ux = xu = \max(u, x)$  if  $x < u$ . □

## 5. CONCLUSION

In this paper we have described and compared several structures of T - semiring. We gave the equational bases of them and also the varieties generated by them. Our future work can be continued in different directions on T - semiring.

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