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MAHESH INVERSE TENSION INDEX FOR GRAPHS

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ABSTRACT. A topological index of a chemical structure is a number that correlates the chemical structure with chemical reactivity or physical properties. Several topological indices have been defined on graphs using degrees of vertices/edges, for instance first and second Zagreb indices. In this paper, we introduce a new topological index of a graph called Mahesh inverse tension index using reciprocal of tension on edges. Further, we establish some inequalities and compute Mahesh inverse tension index for some standard graphs.

1. INTRODUCTION

For standard terminology and notion in graphs, we refer the reader to the textbook of Harary [1]. The non-standard will be given in this paper as and when required.

Throughout this paper, $G = (V, E)$ denotes a graph (finite, undirected and simple) and $V = V(G)$ and $E = E(G)$ denote vertex set and edge set of G, respectively. Two non-distinct edges in a graph are adjacent if they are incident on a common vertex. We consider that an edge in a graph is not adjacent to itself. The letters k, l, m, n, and r denote positive integers or zero.

The distance between two vertices u and v in G, denoted by $d(u, v)$ is the number of edges in a shortest path (also called a graph geodesic) connecting them. We say

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that a graph geodesic P is passing through an edge e in G if e is an edge in P. The number of geodesics in G is denoted by f.

The stress of a vertex is a node centrality index, which has been introduced by Shimbel in 1953. The stress of a vertex in a graph is the number of geodesics (shortest paths) passing through it [10]. By the motivation of stress of a vertex, Rajendra et al. [9] have introduced two topological indices of for graphs called first stress index and second stress index, using stresses of vertices. The concept of stress of a vertex was inspirational for introducing the notion of tension on edge in a graph, which has been studied recently by K. Bhargava, N.N. Dattatreya, and R. Rajendra in their paper [2]. Let G be a graph and e be an edge in G. The tension on e, denoted by $\tau_G(e)$ or simply $\tau(e)$, is defined as the number of geodesics in G passing through e. Tension on an edge is always ≥ 1. A graph G is said to be k-tension-regular if all its edges are of tension k. The total tension of G, denoted by $N_\tau(G)$, is defined as,

$$N_\tau(G) = \sum_{e \in E} \tau(e). \tag{1.1}$$

In molecular graph theory, the molecular graphs represent the chemical structures of a chemical compound and it is often found that there is a correlation between the molecular structure descriptor with different physico-chemical properties of the corresponding chemical compounds. These molecular structure descriptors are commonly known as topological indices which are some numeric parameter obtained from the molecular graphs and are necessarily invariant under automorphism. Thus topological indices are very important useful tool to discriminate isomers and also shown its applicability in quantitative structure-activity relationship, structure-property relationship and nanotechnology including discovery and design of new drugs.

The first Zagreb index $M_1(G)$ of a simple graph G is defined (see [3,4]) as

$$M_1(G) = \sum_{v \in V(G)} d(v)^2.$$

In [8], Rajendra et al. have introduced a topological index of a graph G called Tosha index (denoted by $\mathcal{T}(G)$) using tension on edges:

$$\mathcal{T}(G) = \sum_{e \in E(G)} \tau(e)^2. \tag{1.2}$$

The Zagreb indices have been defined using degrees of vertices in a graph to explain some properties of chemical compounds at molecular level [3, 4]. It is exciting to study concepts involving values on edges like tosha-degree, tension etc., (See [2, 5–7]). By the motivation of Zagreb indices, in this paper we introduce a new topological index on graphs called Mahesh inverse tension index for graphs using reciprocal of tensions on edges and we try to obtain some results.

2. Mahesh Inverse Tension Index

Definition 2.1. *The Mahesh inverse tension index $\mathcal{M}(G)$ of a graph G is defined by*

$$\mathcal{M}(G) = \sum_{e \in E(G)} \tau(e)^{-1}. \tag{2.1}$$

Observation: From (1.2) and (2.1), it is clear that, for any graph G, $\mathcal{M}(G) \leq \mathcal{T}(G)$ and $\mathcal{M}(G)$ need not be an integer whereas $\mathcal{T}(G)$ is always a non-negative integer.

Proposition 2.1. *For any graph G, we have the following inequalities:*

$$|E| f^{-1} \leq \mathcal{M}(G) \leq |E| \tag{2.2}$$

and

$$N_\tau(G)^{-1}|E| \leq \mathcal{M}(G).$$

Proof. Since $\tau(e) \geq 1$, $\tau(e)^{-1} \leq 1$, $\forall e \in E(G)$, and from the (2.1), we have

$$\mathcal{M}(G) \leq |E|. \tag{2.3}$$

Since $\tau(e) \leq f$, $f^{-1} \leq \tau(e)^{-1}$, $\forall e \in E(G)$, and from the (2.1), we have

$$|E| f^{-1} \leq \mathcal{M}(G). \tag{2.4}$$

Combining (2.3) and (2.4), we get (2.2).

From (1.1), we have

$$N_\tau(G)^{-1} = \left[\sum_{e \in E} \tau(e)\right]^{-1} \leq \tau(\alpha)^{-1}, \tag{2.5}$$

where α is an edge in G. Inequality (2.5) gives

$$\sum_{\alpha \in E} N_\tau(G)^{-1} \leq \sum_{\alpha \in E} \tau(\alpha)^{-1} \quad \Longrightarrow \quad N_\tau(G)^{-1}|E| \leq \mathcal{M}(G).$$

□

Proposition 2.2. *If G is a subgraph of a tree T, then*

$$\mathcal{M}(G) \geq \mathcal{M}(T) - \sum_{e \in E(T)-E(G)} \tau_T(e)^{-1}.$$

Proof. Let G be a subgraph of a tree T. Since, in a tree, between any two vertices there is one and only one path, $\tau_G(e) \leq \tau_T(e)$ for any edge e in G. So, $\tau_G(e)^{-1} \geq \tau_T(e)^{-1}$, $\forall e \in E(G)$. Therefore, from (2.1),

$$\begin{aligned}
\mathcal{M}(G) &= \sum_{e \in E(G)} \tau_G(e)^{-1} \\
&\geq \sum_{e \in E(G)} \tau_T(e)^{-1} \\
&\geq \sum_{e \in E(T)} \tau_T(e)^{-1} - \sum_{e \in E(T)-E(G)} \tau_T(e)^{-1} \\
&= \mathcal{M}(T) - \sum_{e \in E(T)-E(G)} \tau_T(e)^{-1}.
\end{aligned}$$

□

Proposition 2.3. *If G is k-tension regular, then*

$$\mathcal{M}(G) = \frac{|E|}{k}.$$

Proof. If G is k-tension regular, then $\tau(e) = k$, $\forall e \in E(G)$ and so from (2.1), we have

$$\mathcal{M}(G) = \sum_{e \in E(G)} k^{-1} = \frac{|E|}{k}.$$

□

Corollary 2.1.

(1) *For the complete graph K_n on n vertices, $\mathcal{M}(K_n) = \binom{n}{2}$.*
(2) *For the complete bipartite graph $K_{m,n}$, $\mathcal{M}(K_{m,n}) = \dfrac{mn}{(m+n-1)}$.*
(3) *For the cycle C_n on n vertices,*

$$\mathcal{M}(C_n) = \begin{cases} \dfrac{64n}{(n-1)(n+1)}, & \text{if } n \text{ is odd;} \\ \dfrac{64}{(n+2)}, & \text{if } n \text{ is even.} \end{cases}$$

Proof.

(1) In the complete graph K_n, for any edge e, we have $\tau(e) = 1$. Therefore K_n is 1-tension regular graph. Hence by Proposition 2.3, we have

$$\mathcal{M}(K_n) = 1^{-1}|E(K_n)| = \binom{n}{2}.$$

(2) In the complete bipartite graph $K_{m,n}$, for any edge e, we have $\tau(e) = m + n - 1$. Therefore $K_{m,n}$ is $(m+n-1)$-tension regular graph. Hence by Proposition 2.3, we have

$$\mathcal{M}(K_{m,n}) = (m+n-1)^{-1}|E(K_{m,n})| = \frac{mn}{(m+n-1)}.$$

(3) Let e be any edge in the cycle graph C_n on n vertices. Then

$$\tau(e) = \begin{cases} \dfrac{(n-1)(n+1)}{8}, & \text{if } n \text{ is odd;} \\ \dfrac{n(n+2)}{8}, & \text{if } n \text{ is even.} \end{cases}$$

Therefore C_n is

$$\begin{cases} \dfrac{(n-1)(n+1)}{8}\text{-tension regular}, & \text{if } n \text{ is odd;} \\ \dfrac{n(n+2)}{8}\text{-tension regular}, & \text{if } n \text{ is even.} \end{cases}$$

Hence by Proposition 2.3, we have

$$\mathcal{M}(C_n) = \begin{cases} \left[\dfrac{(n-1)(n+1)}{8}\right]^{-1} |E(C_n)|, & \text{if } n \text{ is odd;} \\ \left[\dfrac{n(n+2)}{8}\right]^{-1} |E(C_n)|, & \text{if } n \text{ is even.} \end{cases}$$

$$= \begin{cases} \dfrac{64n}{(n-1)(n+1)}, & \text{if } n \text{ is odd;} \\ \dfrac{64}{(n+2)}, & \text{if } n \text{ is even.} \end{cases}$$

□

Proposition 2.4. *If T is a tree with m edges $e_1, e_2, ..., e_m$, then*

$$\mathcal{M}(T) = \sum_{i=1}^{m} \frac{1}{|V(C_{i1})||V(C_{i2})|},$$

where C_{i1} and C_{i2} are the components of $T - e_i$, $1 \leq i \leq m$. Further, For a path P_n with n vertices,

$$\mathcal{M}(P_n) = \sum_{i=1}^{n-1} \frac{1}{i(n-i)}.$$

Proof. For the edge e_i in T, let C_{i1} and C_{i2} be the components of $T - e_i$. Then

$$\tau(e_i) = |V(C_{i1})||V(C_{i2})|,\ 1 \leq i \leq m.$$

Hence from (2.1), we have $\mathcal{M}(T) = \sum_{i=1}^{m} \frac{1}{|V(C_{i1})||V(C_{i2})|}$.

A path P_n with n vertices, has $n-1$ edges $e_1, e_2, ..., e_{n-1}$ (shown in the following figure).

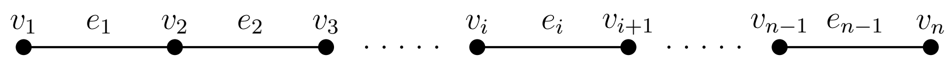

FIGURE 1. The path P_n on n vertices.

Clearly, $|V(C_{i1})| = i$ and $|V(C_{i1})| = n - i$ in $T - e_i$. Hence

$$\mathcal{M}(P_n) = \sum_{i=1}^{n-1} \frac{1}{|V(C_{i1})||V(C_{i2})|} = \sum_{i=1}^{n-1} \frac{1}{i(n-i)}. \qquad \square$$

Proposition 2.5. *Let W_n denote the wheel graph on $n \geq 5$ vertices. Then*

$$\mathcal{M}(W_n) = \frac{n(n-1)}{3(n-3)}.$$

Proof. For the wheel graph W_n with $n \geq 5$, there are $(n-1)$ radial edges and $(n-1)$ peripheral edges. If e_p is a peripheral edge in W_n, then $\tau(e_p) = 3$ and if e_r is a radial edge in W_n, then $\tau(e_r) = n - 3$. Hence from (2.1), we have

$$\begin{aligned}\mathcal{M}(W_n) &= \sum_{\text{peripheral edges}} \tau(e_p)^{-1} + \sum_{\text{radial edges}} \tau(e_p)^{-1} \\ &= (n-1) \cdot 3^{-1} + (n-1) \cdot (n-3)^{-1} \\ &= \frac{n(n-1)}{3(n-3)}. \qquad \square\end{aligned}$$

Proposition 2.6. *Let F_n denote the friendship graph on $2n+1$ vertices. Then*

$$\mathcal{M}(F_n) = \frac{n(2n+3)}{2n+1}.$$

Proof. In F_n, there are $3n$ edges, out of them $2n$ radial edges and n peripheral edges. If e_p is a peripheral edge in F_n, then $\tau(e) = 1$ and if e_r is a radial edge in F_n, then $\tau(e_r) = 2n + 1$. Hence from (2.1), we have

$$\begin{aligned}\mathcal{M}(F_n) &= \sum_{\text{peripheral edges}} \tau(e_p)^{-1} + \sum_{\text{radial edges}} \tau(e_p)^{-1} \\ &= n \cdot 1^{-1} + 2n \cdot (2n+1)^{-1} \\ &= \frac{n(2n+3)}{2n+1}.\end{aligned}$$

□

3. Conclusion

All graphs considered in this manuscript are simple. We have introduced a new topological index of a graph called Mahesh inverse tension index for graphs using reciprocal of tensions on edges and we obtained some results. The fact that modelling a molecule by a graph gives us many required information on the physico-chemical properties of the molecule at the end of some mathematical calculations made on the graph has been used in the last seven decades. In future, we would like to develop the theory of Mahesh inverse tension index, by finding methods of computation and its relations with chemical properties of molecules.

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References

[1] F. Harary: *Graph theory*, Addison Wesley, Reading, Mass, 1972.

[2] K. Bhargava, N. N. Dattatreya, R. Rajendra: *Tension on an edge in a graph*, Communicated for publication.

[3] I. Gutman, N. Trinajstić: *Graph theory and molecular orbitals. Total n-electron energy of alternant hydrocarbons*, Chem. Phys. Lett., **17** (1972), 535-538.

[4] I. Gutman, B. Ruščić, N. Trinajstić, C. F. Wilcox: *Graph theory and molecular orbitals*, XII. Acyclic polyenes, J. Chem. Phys., **62** (1975), 3399-3405.

[5] R. Rajendra, P. Siva Kota Reddy: *Tosha-degree of an edge in a graph*, Southeast Asian Bull. Math., **45** (2021), to appear.

[6] R. Rajendra, P. Siva Kota Reddy: *On Tosha-degree of an edge in a graph*, Eur. J. Pure Appl. Math., **15** (2020), to appear.

[7] R. Rajendra, P. Siva Kota Reddy: *Tosha-degree equivalence signed graphs*, Vladikavkaz Math. J., **22**(2) (2020), 48-52.

[8] R. Rajendra, P. Siva Kota Reddy, C. N. Harshavardhana: *Tosha index for graphs*, Proc. Jangjeon Math. Soc., **24** (2021), to appear.

[9] R. Rajendra, P. Siva Kota Reddy, I. N. Cangul: *Stress Indices of Graphs*, Adv. Stud. Contemp. Math., Kyungshang, **31** (2021), to appear.

[10] A. Shimbel: *Structural Parameters of Communication Networks*, Bull. Math. Biol., **15** (1953), 501-507.

[11] B. Zhou: *Zagreb indices*, MATCH Commun. Math. Comput. Chem., **52** (2004), 113–118.

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