A NEW RESEMBLANCE MEASURE ON INTUITIONISTIC FUZZY SETS AND ITS APPLICATION IN SERIAL CRIME DETECTION

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ABSTRACT. Serial Crimes is a major problem in our society and long term psychological impacts on the people of the society. Moreover, the investigation process for serial crimes are very troublesome sometime due to absence of evidences and sometimes the investigator finds difficult to solve the crimes due to a large number of similar criminal cases. In this paper, crime linkage, which is the process of studying and detecting serial crimes by an investigator is discussed to utilize a novel Resemblance measure of Intuitionistic fuzzy set along with an approach in Intuitionistic fuzzy multi criteria decision making. Further, a case study has been carried out on an existing data set to validate the proposed Resemblance measure.

1. INTRODUCTION

Crime linkage analysis can be defined as the process by which a criminal investigator investigates a finite set of crimes with the help of shreds of evidence like person's fingerprints, DNA, objects available at the crime spot and try to make decisions whether there are linked crimes among the set of crimes via same offenders or not. If the pieces of evidence are available or reliable, then the decision making process becomes confident and accurate, but sometimes the lack of forensic evidence makes the investigation unreliable and uncertain. The mental state of any human being varies from person to person. Any activity done by a human being

at any instant reflects the mental state of the person at that instant. So a criminal activity done by an offender will reflect his/her mental state, behavioral pattern, way of thinking, conscious and subconscious mind to some extent (Grubin [6]; Woodhams et al., [16]). So, by studying the similarities and by processing and coding the activities of offenders properly we can interpret them logically. But in reality, sometimes an absence of precise evidence makes the mathematical interpretation or coding uncertain and hence they are fuzzy in nature. In real crime scene, every crime may not be identical, although the same offender does it. In the practice of crime investigation, explanations and report of criminal cases are recorded in the written form via passages or registers by different investigators for different crimes. However, this information is found from various sources are uncertain in nature and found to be fluctuating between occurrence and non-occurrence of those events. This is the main reason why Intuitionistic fuzzy set has been used for expressing crimes in terms of evidences. In this paper, a novel resemblance measure is proposed to express similarity between two IFSs and the proposed Resemblance measure is utilized in a fuzzy MCDM approach for crime linkage analysis. The MCDM case study has been carried out on an existing data set (Goala and Dutta [4]) to validate the proposed Resemblance measure.

Fuzzy set and logic are widely applied in criminal investigation methodologies to deal with the uncertainty. Queck et al., [12] developed a fuzzy neural network to detect the match finger prints. Grubestic [5] used fuzzy clustering technique for detection of crime hotspot area in a city. Sheng et al., [13] developed a methodology to find the crime pattern and suggested police duty positioning utilizing fuzzy time series analysis. Nurul et al., [11] developed a method for detection of most potential area under crime execution using with the geographical information system (GIS) via using AHP. Stofel et al., [15] developed a fuzzy clustering method for detection of pattern of crime via forensic data. Shrivastav et al., [14] predicted crimes via used fuzzy time series analysis. Albertetti et al., [1] utilized a fuzzy Multi-attributive decision making (MADM) to analyze the serial crimes from large number of crimes. Adeyiga et al., [8] proposed a technique of fuzzy clustering to identify the attribute of criminals for criminal profiling. Gupta et al., [7] considered different types of geo-physical and demographic characteristics to map down to help in crime detection. Goala and Dutta [3] used distance measure on the hesitant fuzzy set to find the distinction between two crimes and gave the pair

wise comparisons in fuzzy MCDM for crime linkage. Goala and Dutta [4] introduced a novel resemblance function for IFSs and used the resemblance function for detecting serial crimes.

2. PRELIMINARIES

In this section, necessary terms of fuzzy set, Intuitionistic fuzzy sets and Intuitionistic fuzzy similarity measures are discussed.

Definition 2.1. *[17] Let X is a universal set of discourse, then a fuzzy set A is defined as*

$$\tilde{A} = \{(x, \mu_A(x)) : x \in X, \mu_A(x) \in [0, 1]\},$$

where μ_A is a membership between 0 and 1 which represents the belongingness objects within its universe of discourse.

Definition 2.2. *[9] An Intuitionistic fuzzy set (IFS in short) on a universe of discourse X is defined as*

$$A = \{\langle x, (\mu_A(x), \nu_A(x)) \rangle : x \in X, \mu_A(x), \nu_A(x) \in [0, 1]\},$$

where $\mu_A(x)$, $\nu_A(x)$ is called the degree of membership and non-membership of x in A satisfying the condition $0 \leq \mu_A(x) + \mu_A(x) \leq 1$.

In addition, the hesitancy degree of IFS is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ which can considered as the degree of lack of uncertainty associated with the degree of membership or non-membership in A.

Definition 2.3. *[5] Let $A, B \in IFS(X)$, then we have*
  a) $A \subseteq B$ iff $\forall x \in X$, $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;
  b) $A = B$ iff $\forall x \in X$, $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$;
  c) $A^C = \{(x, \nu_A(x), \mu_A(x)) : x \in X\}$;
  d) $\bigcap A_i = \{(x, \wedge\mu_{A_i}(x), \vee\nu_{A_i}(x)) : x \in X\}$, where A_i is class of IFSs;
  e) $\bigcup A_i = \{(x, \vee\mu_{A_i}(x), \wedge\nu_{A_i}(x)) : x \in X\}$, where A_i is class of IFSs.

2.1. **Resemblance Measure.** Goala and Dutta [4] had noticed that similarity measures are unable to find the similarity degree among more than two IFSs. It is a common practice in real life situation to make comparisons among the things for checking their equivalency or similarity. For this drawback of similarity measures, Goala and Dutta [4] introduced the concept of resemblance measure for IFSs.

Definition 2.4. *[4] Consider*

$$A = \{\langle x, (\mu_{A_1}(x), \nu_{A_1}(x))\rangle, \langle x, (\mu_{A_2}(x), \nu_{A_2}(x))\rangle, \cdots, \langle x, (\mu_{A_n}(x), \nu_{A_n}(x))\rangle\}$$

be a collection of IFSs. Resemblance measure is a function from A to $[0, 1]$ defined as:

$$R(A) = \frac{1}{n}\sum_{i=1}^{n}(1 - |\bar{\mu}_{A_i} - \mu_{A_i}| - |\bar{\nu}_{A_i} - \nu_{A_i}|),$$

where $\bar{\mu}_{A_i} = \frac{1}{n}\sum_{1=1}^{n}\mu_{A_i}$ and $\bar{\nu}_{A_i} = \frac{1}{n}\sum_{i=1}^{n}\nu_{A_i}$.

Example 1. *Let $A_1 = \{\langle x, (0.1, 0.8)\rangle\}$, $A_2 = \{\langle x, (0.9, 0.1)\rangle\}$ and $A_3 = \{\langle x, (0.5, 0.5)\rangle\}$ be three IFNs. Then, Resemblance measure of these IFSs are given by*

$$R(\{A_1, A_2, A_3\}) = \frac{1}{3}\sum_{i=1}^{3}(1 - |\bar{\mu}_{A_i} - \mu_{A_i}| - |\bar{\nu}_{A_i} - \nu_{A_i}|).$$

In general, for any IFSs $\tilde{A}_j = \{\langle x_j, (\mu_{A_{ji}}(x_j), \nu_{A_{ji}}(x_j))\rangle : x_j \in X\}$. The Generalized Resemblance measure of $\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n\}$ can be defined as:

$$R\left(\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n\}\right) = \begin{cases} \sum_{j=1}^{n} w_j \frac{1}{l_i}\sum_{i=1}^{l_i}(1 - |\bar{\mu}_{A_{ji}} - \mu_{A_{ji}}| - |\bar{\nu}_{A_{ji}} - \nu_{A_{ji}}|) ; n \neq 1 \\ 1, n = 1 \end{cases}$$

where l_i is the number of IFSs in $\tilde{A}_j$ and w_j is the weight of $\tilde{A}_j$ with $w_1 + w_2 + \cdots + w_n = 1$, $R\left(\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n\}\right) = 1$ for $n = 1$, since an IFSs can be considered as similar or resemblance to itself.

Example 2. *Let $\tilde{A}_1$, $\tilde{A}_2$ and $\tilde{A}_3$ be the three IFSs such that*

$$\tilde{A}_1 = \{\langle A_{11}, (0.9, 0.1)\rangle, \langle A_{12}, (0.1, 0.8)\rangle, \langle A_{13}, (0.5, 0.5)\rangle, \langle A_{14}, (0.9, 0.1)b\rangle,$$
$$\langle A_{15}, (0.7, 0.2)\rangle, \langle A_{16}, (0.3, 0.6)\rangle\}$$

$$\tilde{A}_2 = \{\langle A_{21}, (0.7, 0.2)\rangle, \langle A_{22}, (0.9, 0.1)\rangle, \langle A_{23}, (0.7, 0.2)\rangle, \langle A_{24}, (0.1, 0.8)\rangle,$$
$$\langle A_{25}, (0.3, 0.6)\rangle, \langle A_{26}, (0.7, 0.2)\rangle\}$$

$$\tilde{A}_3 = \{\langle A_{31}, (0.7, 0.2)\rangle, \langle A_{32}, (0.5, 0.5)\rangle, \langle A_{33}, (0.7, 0.2)\rangle, \langle A_{34}, (0.9, 0.1)\rangle,$$
$$\langle A_{35}, (0.9, 0.1)\rangle, \langle A_{36}, (0.5, 0.5)\rangle\}$$

Suppose $w_j = \frac{1}{6}$, $j = 1, 2, \cdots, 6$ for reflecting the equal importance of each criteria. Therefore, resemblance value among $\tilde{A}_1$, $\tilde{A}_2$ and $\tilde{A}_3$ can be given by

$$R\left(\left\{\tilde{A}_1, \tilde{A}_2, \tilde{A}_3\right\}\right) = \sum_{j=1}^{6} w_j \frac{1}{3} \sum_{i=1}^{3} \left(1 - \left|\bar{\mu}_{A_{ji}} - \mu_{A_{ji}}\right| - \left|\bar{\nu}_{A_{ji}} - \nu_{A_{ji}}\right|\right)$$

$$= \sum_{j=1}^{6} \frac{1}{18} \sum_{i=1}^{3} \left(1 - \left|\bar{\mu}_{B_i} - \mu_{B_i}\right| - \left|\bar{\nu}_{B_i} - \nu_{B_i}\right|\right)$$

$$= 0.625926$$

This value of resemblance will give an overall degree of similarity among $\tilde{A}_1$, $\tilde{A}_2$ and $\tilde{A}_3$.

3. A NOVEL RESEMBLANCE MEASURE

In this section, a novel Resemblance measure has been introduced for Intuitionistic fuzzy sets. Consider

$$A = \left\{\langle x, (\mu_{A_1}(x), \nu_{A_1}(x))\rangle, \langle x, (\mu_{A_2}(x), \nu_{A_2}(x))\rangle, \cdots, \langle x, (\mu_{A_n}(x), \nu_{A_n}(x))\rangle\right\}$$

be a collection of IFSs. The Resemblances measure from A to $[0, 1]$ is defined as:

$$R(A) = \frac{1}{2}\left\{\cos\left(\frac{\pi}{3}\sum_{i=1}^{n}\left|\bar{\mu}_{A_i} - \mu_{A_i}\right|\right) + \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}\left|\bar{\nu}_{A_i} - \nu_{A_i}\right|\right)\right\},$$

where $\bar{\mu}_{A_i} = \frac{1}{n}\sum_{1=1}^{n}\mu_{A_i}$ and $\bar{\nu}_{A_i} = \frac{1}{n}\sum_{1=1}^{n}\nu_{A_i}$.

Example 3. *x Let $A_1 = \{\langle x, (0.9, 0.1)\rangle\}, A_2 = \{\langle x, (0.7, 0.2)\rangle\}$ and $A_3 = \{\langle x, (0.7, 0.2)\rangle\}$ be any three IFNs. Then resemblance measure is given by*

$$R(\{A_1, A_2, A_3\}) = \frac{1}{2}\left\{\cos\left(\frac{\pi}{3}\sum_{i=1}^{3}\left|\bar{\mu}_{A_i} - \mu_{A_i}\right|\right) + \cos\left(\frac{\pi}{3}\sum_{i=1}^{3}\left|\bar{\nu}_{A_i} - \nu_{A_i}\right|\right)\right\}$$

$$= 0.975765.$$

In general, for any IFS $\tilde{A}_j = \left\{\langle x_j, (\mu_{A_{ji}}(x_j), \nu_{A_{ji}}(x_j))\rangle : x_j \in X\right\}$, where $j = 1, 2, \ldots, n$, the Generalized Resemblance function on $\left\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n\right\}$ is defined as:

$$R\left(\left\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n\right\}\right)$$

$$= \begin{cases} \sum_{j=1}^{n} \frac{w_j}{2}\left\{\cos\left(\frac{\pi}{3}\sum_{i=1}^{l_i}\left|\bar{\mu}_{A_{ji}} - \mu_{A_{ji}}\right|\right) + \cos\left(\frac{\pi}{3}\sum_{i=1}^{l_i}\left|\bar{\nu}_{A_{ji}} - \nu_{A_{ji}}\right|\right)\right\}; n \neq 1 \\ 1, n = 1 \end{cases},$$

where l_i is the number of IFSs in $\tilde{A}_j$ and w_j is the weight of $\tilde{A}_j$ with $w_1+w_2+\cdots+w_n = 1$. In Particular, $R(\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n\}) = 1$ for $n = 1$, since an IFS can be considered as resemble to itself.

$$\tilde{A}_1 = \{\langle x_1, (0.9, 0.1)\rangle, \langle x_2, (0.1, 0.8)\rangle, \langle x_3, (0.5, 0.5)\rangle, \langle x_4, (0.9, 0.1)\rangle,$$
$$\langle x_5, (0.7, 0.2)\rangle, \langle x_6, (0.3, 0.6)\rangle\}$$
$$\tilde{A}_2 = \{\langle x_1, (0.7, 0.2)\rangle, \langle x_2, (0.9, 0.1)\rangle, \langle x_3, (0.7, 0.2)\rangle, \langle x_4, (0.1, 0.8)\rangle,$$
$$\langle x_5, (0.3, 0.6)\rangle, \langle x_6, (0.7, 0.2)\rangle\}$$
$$\tilde{A}_3 = \{\langle x_1, (0.7, 0.2)\rangle, \langle x_2, (0.5, 0.5)\rangle, \langle x_3, (0.7, 0.2)\rangle, \langle x_4, (0.9, 0.1)\rangle,$$
$$\langle x_5, (0.9, 0.1)\rangle, \langle x_6, (0.5, 0.5)\rangle\}$$

taking $w_i = \frac{1}{3}$; $i = 1, 2, 3$.

$$R\left(\left\{\tilde{A}_1, \tilde{A}_2, \tilde{A}_n\right\}\right)$$
$$= \sum_{j=1}^{3} \frac{1}{2} \cdot \frac{1}{3}\left\{\cos\left(\frac{\pi}{3}\sum_{i=1}^{6}\left|\bar{\mu}_{A_{ji}} - \mu_{A_{ji}}\right|\right) + \cos\left(\frac{\pi}{3}\sum_{i=1}^{6}\left|\bar{\nu}_{A_{ji}} - \nu_{A_{ji}}\right|\right)\right\} = 0.79866.$$

The proposed Resemblance measure satisfies the following three theorems (Goala and Dutta [4]):

Theorem 3.1. $0 \leq R \leq 1$.

Theorem 3.2. *If $R(\{A_1, A_2, \cdots, A_n\}) = 1$ for any finite n, $A_1 = A_2 = \cdots = A_n$.*

Theorem 3.3. *If $\{A_1, A_2, \cdots, A_n\} = \{B_1, B_2, \cdots, B_n\}$, then $R(\{A_1, A_2, \cdots, A_n\}) = R(\{B_1, B_2, \cdots, B_n\})$. But the converse may not be true.*

Now, the proofs of the theorems are given one by one:

Proof of Theorem 3.1. Given $R(\{A_1, A_2, \cdots, A_n\}) = 1$ we have the following chain of implications:

$$\Rightarrow \quad \frac{1}{2}\left\{\cos\left(\frac{\pi}{3}\sum_{i=1}^{n}\left|\bar{\mu}_A - \mu_{A_i}\right|\right) + \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}\left|\bar{\nu}_A - \nu_{A_i}\right|\right)\right\} = 1$$
$$\Rightarrow \quad \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}\left|\bar{\mu}_A - \mu_{A_i}\right|\right) + \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}\left|\bar{\nu}_A - \nu_{A_i}\right|\right) = 2$$
$$\Rightarrow \quad \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}\left|\bar{\mu}_A - \mu_{A_i}\right|\right) = 1, \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}\left|\bar{\nu}_A - \nu_{A_i}\right|\right) = 14$$
$$\Rightarrow \quad \sum_{i=1}^{n}\left|\bar{\mu}_A - \mu_{A_i}\right| = 0, \sum_{i=1}^{n}\left|\bar{\nu}_A - \nu_{A_i}\right| = 0$$

$$\Rightarrow \quad |\bar{\mu}_A - \mu_{A_i}| = 0, |\bar{\nu}_A - \nu_{A_i}| = 0 \text{ for all } i.$$

$$\Rightarrow \quad \bar{\mu}_A = \mu_{A_i}, \bar{\nu}_A = \nu_{A_i}$$

$$\Rightarrow \quad \mu_{A_j} = \mu_{A_i}, \nu_{A_j} = \nu_{A_i}$$

$$\Rightarrow \quad A_j\left(\mu_{A_j}, \nu_{A_j}\right) = A_i\left(\mu_{A_i}, \nu_{A_i}\right)$$

$$\Rightarrow \quad A_1 = A_2 = \cdots = A_n$$

If $(\{A_1, A_2, \cdots, A_n\}) = 1$ for any finite, then $A_1 = A_2 = \cdots = A_n$. $\qquad\square$

Proof of Theorem 3.2. Given $R(\{A_1, A_2, \cdots, A_n\}) = 1$ we have the following chain of implications:

$$\Rightarrow \quad \frac{1}{2}\left\{\cos\left(\frac{\pi}{3}\sum_{i=1}^{n}|\bar{\mu}_A - \mu_{A_i}|\right) + \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}|\bar{\nu}_A - \nu_{A_i}|\right)\right\} = 1$$

$$\Rightarrow \quad \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}|\bar{\mu}_A - \mu_{A_i}|\right) + \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}|\bar{\nu}_A - \nu_{A_i}|\right) = 2$$

$$\Rightarrow \quad \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}|\bar{\mu}_A - \mu_{A_i}|\right) = 1, \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}|\bar{\nu}_A - \nu_{A_i}|\right) = 1$$

$$\Rightarrow \quad \sum_{i=1}^{n}|\bar{\mu}_A - \mu_{A_i}| = 0, \sum_{i=1}^{n}|\bar{\nu}_A - \nu_{A_i}| = 0$$

$$\Rightarrow \quad |\bar{\mu}_A - \mu_{A_i}| = 0, |\bar{\nu}_A - \nu_{A_i}| = 0 \text{ for all } i$$

$$\Rightarrow \quad \bar{\mu}_A = \mu_{A_i}, \bar{\nu}_A = \nu_{A_i} \text{ for all } i$$

$$\Rightarrow \quad \mu_{A_j} = \mu_{A_i}, \nu_{A_j} = \nu_{A_i} \text{ for all } i \text{ and } j$$

$$\Rightarrow \quad A_j\left(\mu_{A_j}, \nu_{A_j}\right) = A_i\left(\mu_{A_i}, \nu_{A_i}\right) \text{ for all } i \text{ and } j$$

$$\Rightarrow \quad A_1 = A_2 = \cdots = A_n$$

i.e. if $R(\{A_1, A_2, \cdots, A_n\}) = 1$ for any finite n, then $A_1 = A_2 = \cdots = A_n$. $\qquad\square$

Proof of Theorem 3.3. If $\{A_1, A_2, \cdots, A_n\} = \{B_1, B_2, \cdots, B_n\}$, then we have

$$R(\{A_1, A_2, \cdots, A_n\}) = \frac{1}{2}\left\{\cos\left(\frac{\pi}{3}\sum_{i=1}^{n}|\bar{\mu}_A - \mu_{A_i}|\right) + \cos\left(\frac{\pi}{3}\sum_{i=1}^{n}|\bar{\nu}_A - \nu_{A_i}|\right)\right\}$$

$$= \frac{1}{2}\left\{\cos\left(\frac{\pi}{3}\sum_{j=1}^{n}|\bar{\mu}_B - \mu_{B_j}|\right) + \cos\left(\frac{\pi}{3}\sum_{j=1}^{n}|\bar{\nu}_B - \nu_{B_j}|\right)\right\}$$

because $A_i = B_j$ for some i and j.

The converse part may not be true.

Counter example: Let $A_1 = \{\langle x, (1,0)\rangle\}, A_2 = \{\langle x, (0,0)\rangle\}$ be two IFSs. Then $R(\{A_1, A_2\}) = 0.75$. Again, let $B_1 = \{\langle x, (0,1)\rangle\}, B_2 = \{\langle x, (0,0)\rangle\}$ two IFSs. Then $R(\{B_1, B_2\}) = 0.75$. Thus, though $R(\{A_1, A_2\}) = R(\{B_1, B_2\})$ but $\{A_1, A_2\} \neq \{B_1, B_2\}$, i.e. if $\{A_1, A_2, \cdots, A_n\} = \{B_1, B_2, \cdots, B_n\}$. Then

$$R(\{A_1, A_2, \cdots, A_n\}) = R(\{B_1, B_2, \cdots, B_n\}).$$

□

4. METHODOLOGY

Consider $\{C_1, C_2, \cdots, C_n\}$ be the set of crimes taken under investigation, i.e. from $\{C_1, C_2, \cdots, C_n\}$ we have to determine the sub-collection of crimes which are connected common offenders. Consider $\{X_1, X_2, \cdots, X_m\}$ be the activities of the offenders collected from the crime scene analysis. Now, the situation is represented by the following matrix (Klir [10]):

$$
\begin{array}{c}
\begin{array}{cccc}
C_1 & C_2 & \cdots & C_n
\end{array} \\
\begin{array}{c}
X_1 \\
X_2 \\
\vdots \\
X_m
\end{array}
\left[
\begin{array}{cccc}
A_{11} & A_{12} & \cdots & A_{1n} \\
A_{21} & A_{22} & \cdots & A_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
A_{m1} & A_{m2} & \cdots & A_{mn}
\end{array}
\right]
\end{array}.
$$

Here, A_{ij} are the IFSs that denote the degree of relationship between the activity X_i and the crime C_j. Therefore, the crime C_j are signified as IFSs in terms of activities of offenders during crime as

$$C_j = \{< X_i, A_{ij} > | X_i \in X\},$$

where $i = 1, 2, \cdots m$ and $j = 1, 2, \cdots n$. Then, Resemblance measure is applied to each sub-collection of the set of crime $\{C_1, C_2, \cdots, C_n\}$, to check which sub-collection of crimes has higher resemblance value, i.e., to find out which sub-collection of crimes are related by same offenders (Goala and Dutta [4]).

Most obviously, the higher resemblance reflects the higher the possibility of relationship among the corresponding subset of crimes.

Now, a threshold value (Goala and Dutta [3]; Goala and Dutta [4]) for the resemblance measure has been fixed a real number between $[0, 1]$ above or equal to which the decision maker may consider that the corresponding subset of crimes are related by common offender or offenders. By checking resemblance measures of each collection of crimes, whether they exceed the threshold value or not one can get the set of related crimes easily.

5. A Case Example

Beasley [2] carried out a case study on seven criminals by interviewing them. Goala and Dutta [4] selected five crimes for their studies C_1, C_2, C_3, C_4 and C_5 and expressed them via IFSs. For the case study, following six activities of offenders are taken as criteria:

  i. Money or jewelry taken from the victim: X_1;
 ii. Proper planning: X_2;
iii. Cruelty: X_3;
 iv. Use down weapon: X_4;
  v. Chance of being quickly overpowered and killed: X_5;
 vi. Forensically aware: X_6.

The relationship between crimes and activities are represented by following matrix: (Goala and Dutta [4]):

$$
\begin{array}{c c c c c c}
 & C_1 & C_2 & C_3 & C_4 & C_5 \\
\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{array} &
\left[\begin{array}{c}
(0.9, 0.1) \\ (0.1, 0.8) \\ (0.5, 0.5) \\ (0.9, 0.1) \\ (0.7, 0.2) \\ (0.3, 0.6)
\end{array}\right. &
\begin{array}{c}
(0.7, 0.2) \\ (0.9, 0.1) \\ (0.7, 0.2) \\ (0.1, 0.8) \\ (0.3, 0.6) \\ (0.7, 0.2)
\end{array} &
\begin{array}{c}
(0.7, 0.2) \\ (0.5, 0.5) \\ (0.7, 0.2) \\ (0.9, 0.1) \\ (0.9, 0.1) \\ (0.5, 0.5)
\end{array} &
\begin{array}{c}
(0.7, 0.2) \\ (0.9, 0.1) \\ (0.9, 0.1) \\ (0.3, 0.6) \\ (0.3, 0.6) \\ (0.7, 0.2)
\end{array} &
\left.\begin{array}{c}
(0.9, 0.1) \\ (0.1, 0.8) \\ (0.5, 0.5) \\ (0.9, 0.1) \\ (0.7, 0.2) \\ (0.5, 0.5)
\end{array}\right]
\end{array}
$$

Now, the crimes are represented via IFSs as follows:

$$C_1 = \{\langle X_1, (0.9, 0.1)\rangle, \langle X_2, (0.1, 0.8)\rangle, \langle X_3, (0.5, 0.5)\rangle, \langle X_4, (0.9, 0.1)\rangle,$$
$$\langle X_5, (0.7, 0.2)\rangle, \langle X_6, (0.3, 0.6)\rangle\}$$
$$C_2 = \{\langle X_1, (0.7, 0.2)\rangle, \langle X_2, (0.9, 0.1)\rangle, \langle X_3, (0.7, 0.2)\rangle, \langle X_4, (0.1, 0.8)\rangle,$$
$$\langle X_5, (0.3, 0.6)\rangle, \langle X_6, (0.7, 0.2)\rangle\}$$
$$C_3 = \{\langle X_1, (0.7, 0.2)\rangle, \langle X_2, (0.5, 0.5)\rangle, \langle X_3, (0.7, 0.2)\rangle, \langle X_4, (0.9, 0.1)\rangle,$$
$$\langle X_5, (0.9, 0.1)\rangle, \langle X_6, (0.5, 0.5)\rangle\}$$
$$C_4 = \{\langle X_1, (0.7, 0.2)\rangle, \langle X_2, (0.9, 0.1)\rangle, \langle X_3, (0.9, 0.1)\rangle, \langle X_4, (0.3, 0.6)\rangle,$$
$$\langle X_5, (0.3, 0.6)\rangle, \langle X_6, (0.7, 0.2)\rangle\}$$
$$C_5 = \{\langle X_1, (0.9, 0.1)\rangle, \langle X_2, (0.1, 0.8)\rangle, \langle X_3, (0.5, 0.5)\rangle, \langle X_4, (0.9, 0.1)\rangle,$$
$$\langle X_5, (0.7, 0.2)\rangle, \langle X_6, (0.5, 0.5)\rangle\}$$

Now, the new resemblance measure has been applied on each collection of crimes and the corresponding resemblance values are given below in tables (Table 1-Table 4):

TABLE 1. Pairwise Resemblance measure

$R(\{C_1, C_2\})$	0.865051
$R(\{C_1, C_3\})$	0.975985
$R(\{C_1, C_4\})$	0.878438
$R(\{C_1, C_5\})$	0.997722
$R(\{C_2, C_3\})$	0.903634
$R(\{C_2, C_4\})$	0.99408
$R(\{C_2, C_5\})$	0.87356
$R(\{C_3, C_4\})$	0.923254
$R(\{C_3, C_5\})$	0.978262
$R(\{C_4, C_5\})$	0.886948

TABLE 2. Resemblance measure among three crimes

$R(\{C_1, C_2, C_3\})$	0.79866
$R(\{C_1, C_2, C_4\})$	**0.780509**
$R(\{C_1, C_2, C_5\})$	0.776171
$R(\{C_1, C_3, C_4\})$	0.828602
$R(\{C_1, C_3, C_5\})$	0.957583
$R(\{C_1, C_4, C_5\})$	0.797746
$R(\{C_2, C_3, C_4\})$	0.848828
$R(\{C_2, C_3, C_5\})$	0.805187
$R(\{C_2, C_4, C_5\})$	0.795401
$R(\{C_3, C_4, C_5\})$	0.835128

TABLE 3. Resemblance measure among four crimes

$R(\{C_1, C_2, C_3, C_4\})$	0.623963
$R(\{C_1, C_2, C_3, C_5\})$	0.707178
$R(\{C_1, C_2, C_4, C_5\})$	0.57974
$R(\{C_1, C_3, C_4, C_5\})$	0.737302
$R(\{C_2, C_3, C_4, C_5\})$	0.638164

TABLE 4. Resemblance measure among five crimes

$R(\{C_1, C_2, C_3, C_4, C_5\})$	0.50157

Now, set a threshold value for the resemblance values as $\delta = 0.95$. From the table it is clear that

$$R(\{C_1, C_3\}), R(\{C_1, C_5\}), R(\{C_2, C_4\}), R(\{C_3, C_5\}) \geq \delta,$$

$R(\{C_1, C_3, C_5\}) \geq \delta.$

Therefore, it is quite obvious that the crimes $\{C_1, C_3\}, \{C_1, C_5\}$, $\{C_2, C_4\}$ and $\{C_3, C_5\}$ can be considered as related by common offenders. Similarly, the set of crimes $\{C_1, C_3, C_5\}$ can be considered as related by common offenders. Thus it can be concluded that $\{C_1, C_3, C_5\}$ and $\{C_2, C_4\}$ can be considered as related by common offenders. This gives same result that put forward in case study of Goala and Dutta [4].

6. DISCUSSION AND CONCLUSION

In this paper, a novel Resemblance measure on IFSs has been proposed. The resemblance measure has been used to determine the degree of relationships among the crimes. Also, the new resemblance measure has been used in a case study to an existing dataset and it is found that the new resemblance measure produces same result which shows the reliability of the proposed resemblance measure. As an extension of this study, attempt can be made to extend the existing resemblance measures to Intuitionistic Hesitant fuzzy sets or to Interval valued Intuitionistic fuzzy sets.

REFERENCES

[1] F. ALBERTETTI, P. COTOFREI, L. GROSSRIEDER, O. RIBAUX, K. STOFFEL: *The CriLiM Methodology: Crime Linkage with a Fuzzy MCDM Approach*, European Intelligence and Security Informatics Conference, 2013.

[2] J. O. BEASLEY: *Serial Murder in America: Case Studies of Seven Offenders*, Behavioral Sciences and the Law, 22, 395–414, 2004.

[3] S. GOALA, P. DUTTA: *A Fuzzy Multicriteria Decision-Making Approach to Crime Linkage*, International Journal of Information Technologies and Systems Approach, **11** (2018), 31-50.

[4] S. GOALA, P. DUTTA: *Intuitionistic Fuzzy Multi Criteria Decision Making Approach to Crime Linkage Using Resemblance Function*, International Journal of Applied and Computational Mathematics, **5**(4) (2019), art.np.112.

[5] T. H. GRUBESTIC: *On the application of fuzzy clustering from crime hot-spot detection*, Journal of Quantitative Criminology, 22, 77-105, 2006.

[6] D. GRUBIN, P. KELLY, C. BRUNSDON: *Linking serious sexual assaults through behaviour*, Home Offence Research Study, Home Offence: Research, Development and Statistics Directorate, **215**, 2001.

[7] S. GUPTA, S. KUMAR: *Crime Detection and Prevention using Social Network Analysis*, International Journal of Computer Applications, **126** (2015), 14-19.

[8] J. A. ADEYIGA, A. BELLO: *A Review of Different Clustering Techniques in Criminal Profiling*, International Journal of Advanced Research in Computer Science and Software Engineering, **6** (2017), 659-666.

[9] K. ATANASSOV: *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1986), 87–96.

[10] J. KLIR, B. YUAN: *Fuzzy Sets and Fuzzy Logic*, Pearson Education, 2009.

[11] H. NURUL, S. MOHD, S. HAFIZ: *Identification of Potential Crime Area Using Analytical Hierarchy Process (AHP) and Geographical Information System (GIS)*, International Journal of Innovative Computing, 2012.

[12] C. QUEK, K. B. TAN, V. K. SAGAR: *Pseudo-outer product based fuzzy neural network fingerprint verification system*, Neural Networks, **14** (2001), 305-323.

[13] T. L. SHENG, C. K. SHU, C. T. FU:*An intelligent decision support model using FSOM and rule extraction for crime prevention*, Expert System with Application, 2010.

[14] A. K. SHRIVASTAV, D. EKATA:,*Applicability of soft Computing Technique for Crime Forecasting: A Preliminary Investigation*, International Journal of Computer Science and Engineering Technology, **3** (2012), 415-421.

[15] K. STOFEL, P. COTOFREI, D. HAN: *Fuzzy Clustering based Methodology for Multidimensional Data Analysis in Computational Forensic Domain*, International Journal of Computer Information Systems and Industrial Management Applications, **4** (2012), 400-410.

[16] J. WOODHAMS, R. BULL, C. R. HOLLIN, R. N. KOCSIS: *Case linkage: Identifying crimes committed by the same offender in Criminal profiling*. Humana Press Inc., 2007.

[17] L. A. ZADEH: *Fuzzy Sets*, Information and Control, **8** (1965), 338-353.

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