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H-E SUPER MAGIC HAMILTON DECOMPOSITION OF GRAPHS

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ABSTRACT. A Hamilton decomposition of G is a partition of its edge set into disjoint Hamilton cycles. A Hamilton cycle in a graph G is a cycle passing through every vertex of G.The graph G is said to be $H-E$ Super Magic Hamilton Decomposition if there exists a mapping $f : V(G) \cup E(G) \to \{1, 2, 3, \ldots, p + q\}$ such that for every copy H in the decomposition $\sum_{v\in V(H)} f(v) + \sum_{e\in E(H)} f(e)$ is constant with each subgraph is hamiltonian. Also $f(E(G)) = 1, 2, \ldots, q$. In this paper we prove that the anti prism graph A_n and Harary graph $H_{k,n}$ are $H-E$ Super Magic Hamilton decomposable graph.

1. INTRODUCTION

We consider regular graphs. The vertex and edge sets of a graph G are denoted by V(G) and E(G) respectively. A graph labeling is an assignment of integers to the edges or vertices or both, depend upon certain conditions.$E-$ super magic decomposition of even regular graph have been explained [11]. Also the graph G confesses an $(H_1, H_2, \ldots, H_n)$ covering if a covering of G is a family of subgraphs $H_1, H_2, \ldots, H_n$ such that every edge of $E(G)$ belongs to at least one of the subgraphs $H_i, 1 \leq i \leq n$. G confesses an $H-$covering,if every H_i is isomorphic to a given graph H, A family of subgraphs $H_1, H_2, \ldots, H_n$ of G is a $H-$ decomposition of G if all the subgraphs are isomorphic to a graph$H$, $E(H_i) \cap (H_j) = \oslash$ where $i \neq j$

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and $E(G) = \bigcup\limits_{i=1}^{n} E(H_i)$. In this case, we write $G = H_1 \oplus H_2 \oplus \cdots \oplus H_n$ and G is said to be $H-$decomposable.Various types of decomposition of graphs have been studied in literature by imposing suitabe conditions on the subgraphs H_i. Geometric decomposition of graphs for various standard graphs have been explained in[9].In this paper we introduce the concept $H - E$ Super Magic Hamilton Decomposition.The graph G is said to be $H-E$ Super Magic Hamilton Decomposition if there exists a mapping $f : V(G) \cup E(G) \to \{1, 2, 3, \ldots, p + q\}$ such that for every copy H in the decomposition $\sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e)$ is constant with each subgraph is hamiltonian. Also $f(E(G)) = 1, 2, \ldots, q$.If $f(V(G)) = 1, 2, \ldots, p$ then the decomposition is said to be $H - V$ Super Magic Hamilton Decomposition. In 1960 [1] W.S. Andrews explained about Magic Squares with magic number$[MN]$ $\frac{n(n^2+1)}{2}$ for square of n sides.Guterrez and Llado [2] in 2005 first described $H-$super magic labeling .For basic definition and notation we follow [3]. Hamilton decompostion of complete graph have been proved by Hilton[4]. Inayah et al. [5] studied various types of labeling along with path decomposition.In 2016 Jude and Swathi [6] studied Hamilton decomposition of Harary graph. In 2012 Marimuthu and Balakrishnan [7] proved that if a graph G of odd order can be decomposed into two Hamilton cycles, then G is an E-super vertex magic graph. Petersen [8] studied even regular graph has a $2k-$factor. In 2020 Remya and Sudha [10] described star decomposition of graphs with cordial labeling. In this paper we prove that certain regular graphs are decomposed into hamilton cycles with magic constant.

Definition 1.1. *The anti prism graph A_n for $n \geq 3$ is defined as a $4-$regular graph with $2n$ vertices and $4n$ edges.It consists of outer and inner C_n, while the two cycles connected by edges v_iu_i and $v_iu_{1+i(modn)}$ for $i = 1, 2, 3, \ldots, n$.*

Lemma 1.1. [11] *Let G be a $m-$ factor decomposable graph and let g be a bijection from $E(G)$ onto $\{1, 2, 3, \ldots, q\}$. Then g can be extended to an $m-$factor$-E-$super magic labeling of G if and only if $k_e = \sum_{e \in E(G)} g(e)$ is constant for every m-factor G' in the decomposition of G.*

2. Main results

Theorem 2.1. *The anti prism graph A_n is hamiltonian decomposable graph of length $2n$ with magic constant $4n^2 + n$, when $n \geq 10$ is a positive integer.*

Proof. Let $G=(V,E)$ be the graph A_n. The order and size of this graph are $2n$ and $4n$ respectively.(ie)$|V(G)|=2n$ and$|E(G)|=4n$.Let $\{v_1,v_2,\dots,v_n\}$ be the set of vertices of the outer cycle and$\{u_1,u_2,\dots,u_n\}$ be the set of vertices of the inner cycle.Let $\{v_iu_i\}\cup\left\{v_iu_{i+1(modn)}\right\}$ be the set of edges.

Construct a mapping $f:E(G)\to\{1,2,\dots,4n\}$ by

$$\begin{aligned}
f(v_iu_i) =& i; 2\le i\le n-1,\\
f(v_iu_{i+1}) =& 4n-i; 1\le i\le n-1,\\
f(v_iv_{i+1}) =& 2n+i; 1\le i\le n-2,\\
f(u_iu_{i+1}) =& n+i; 1\le i\le n-1,\\
f(v_1v_n) =& 4n,\\
f(v_nv_{n-1}) =& 3n,\\
f(u_1u_n) =& 1,\\
f(u_1v_1) =& 2n,\\
f(u_nv_n) =& 3n-1,\\
f\ (u_1v_n) =& n.
\end{aligned}$$

The Hamilton decomposition of antiprism graph of length $2n$ are $\{v_1u_2,u_2v_2,$ $v_2u_3,u_3v_3,v_3u_4,u_4v_4,v_4u_5,\dots,v_{n-1}u_n,u_nu_1,u_1v_n,v_nv_1\}$, $\{v_1v_2,v_2v_3,v_3v_4,v_4v_5,$ $v_5v_6,\dots,v_{n-1}v_n,v_nu_n,u_nu_{n-1},\dots,u_2u_1,u_1v_1\}$ and

$$\begin{aligned}
\sum_{e\in E(H_1)} f(e) =& [1+2+3+\cdots+n]+[(4n-1)+(4n-2)+(4n-3)+\cdots\\
&+(4n-(n-1))]+4n=4n^2+n.
\end{aligned}$$

Also $\sum_{e\in E(H_2)} f(e)=4n^2+n.$ □

Theorem 2.2. *If k,n are positive integer such that $n>k$, then Harary graph $H_{k,n}$ have a hamiltonian decomposition graph of length n with magic constant $\frac{kn^2+2n}{4}$.*

Proof. Let $G=(V,E)$ be the graph $H_{k,n}$. The order and size of this graph are n and $\frac{kn}{2}$, respectively. G can be decomposed into Hamiltonian cycles say $H_1\oplus H_2\oplus H_3\oplus\ldots\oplus H_{\frac{k}{2}}$ each cycles of length n. Put $k=r$. Procedure for $\frac{k}{2}$ Hamilton Decomposition of $H_{k,n}$ is done by [6].

Case (i) When r and n are positive even integers. The edge of the graph can be labeled as shown in Table 1.

TABLE 1. The edge label of $H_{k,n}$ if r and n are positive even integers.

H_1	H_2	H_3	$\dots$	$H_{\frac{k}{2}-1}$	$H_{\frac{k}{2}}$
1	2	3	$\dots$	$\frac{k}{2}-1$	$\frac{k}{2}$
k	$k-1$	$k-2$	$\dots$	$\frac{k}{2}+2$	$\frac{k}{2}+1$
$k+1$	$k+2$	$k+3$	$\dots$	$\frac{3k}{2}-1$	$\frac{3k}{2}$
$2k$	$2k-1$	$2k-2$	$\dots$	$\frac{3k}{2}+2$	$\frac{3k}{2}+1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\frac{k}{2}\left(n-2\right)$	$\frac{k}{2}\left(n-2\right)-1$	$\frac{k}{2}\left(n-2\right)-2$	$\dots$	$\frac{k}{2}\left(n-3\right)+2$	$\frac{k}{2}\left(n-3\right)+1$
$\frac{k}{2}\left(n-2\right)+1$	$\frac{k}{2}\left(n-2\right)+2$	$\frac{k}{2}\left(n-2\right)+3$	$\dots$	$\frac{k}{2}\left(n-1\right)-1$	$\frac{k}{2}\left(n-1\right)$
$\frac{kn}{2}$	$\frac{kn}{2}-1$	$\frac{kn}{2}-2$	$\dots$	$\frac{k}{2}\left(n-1\right)+2$	$\frac{k}{2}\left(n-1\right)+1$

The sum of the edge labels for the Hamiltonian cycles are

$$\sum_{e\in E(H_1)} f(e) = 1+k+k+1+2k+\cdots+\frac{k}{2}(n-2)+\frac{k}{2}(n-2)+1+\frac{kn}{2}$$
$$= \frac{kn^2+2n}{4}.$$

In similar way ,we calculate the magic constant for each decomposition $H_2, H_3, H_4, \dots, H_{\frac{k}{2}}$ as $\frac{kn^2+2n}{4}$.

Case (ii) When r is even and n is an odd integer with $r \not\equiv 0(mod4)$.The edge of the graph can be labeled as shown in Table 2.

TABLE 2. The edge label of $H_{k,n}$ if r is even and n is an odd integer with $r \not\equiv 0(mod4)$.

H_1	H_2	H_3	$\dots$	$H_{k/2-1}$	$H_{k/2}$
		$\frac{k}{2}\times\frac{k}{2}$			
$\frac{k^2}{4}+1$	$\frac{k^2}{4}+2$	$\frac{k^2}{4}+3$	$\dots$	$\frac{k^2}{4}+\frac{k}{2}-1$	$\frac{k^2}{4}+\frac{k}{2}$
$\frac{k^2}{4}+k$	$\frac{k^2}{4}+k-1$	$\frac{k^2}{4}+k-2$	$\dots$	$\frac{k^2}{4}+\frac{k}{2}+2$	$\frac{k^2}{4}+\frac{k}{2}+1$
$\frac{k^2}{4}+k+1$	$\frac{k^2}{4}+k+2$	$\frac{k^2}{4}+k+3$	$\dots$	$\frac{k^2}{4}+\frac{3k}{2}-1$	$\frac{k^2}{4}+\frac{3k}{2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\frac{k^2}{4}+(n-\frac{k}{2}-2)\frac{k}{2}+1$	$\frac{k^2}{4}+(n-\frac{k}{2}-2)\frac{k}{2}+2$	$\frac{k^2}{4}+(n-\frac{k}{2}-2)\frac{k}{2}+3$	$\dots$	$\frac{k^2}{4}+(n-\frac{k}{2}-1)\frac{k}{2}-1$	$\frac{k^2}{4}+(n-\frac{k}{2}-1)\frac{k}{2}$
$\frac{k^2}{4}+\left(n-\frac{k}{2}\right)\frac{k}{2}$	$\frac{k^2}{4}+\left(n-\frac{k}{2}\right)\frac{k}{2}-1$	$\frac{k^2}{4}+\left(n-\frac{k}{2}\right)\frac{k}{2}-2$	$\dots$	$\frac{k^2}{4}+\left(n-\frac{k}{2}-1\right)\frac{k}{2}+2$	$\frac{k^2}{4}+\left(n-\frac{k}{2}-1\right)\frac{k}{2}+1$

The sum of the edge labels for the Hamiltonian cycles are

$$\begin{aligned}\sum_{e\in E(H_1)} f(e) &= MN+\frac{k^2}{4}+1+\frac{k^2}{4}+k+\frac{k^2}{4}+k+1+\ldots+\frac{k^2}{4}\\ &\quad+(n-\frac{k}{2}-2)\frac{k}{2}+1+\frac{k^2}{4}+(n-\frac{k}{2})\frac{k}{2}\\ &=\frac{k^3}{16}+\frac{k}{4}+\frac{k^2n}{4}-\frac{k^3}{8}+2k\left[\frac{\left(\frac{2n-k-4}{4}\right)\left(\frac{2n-k-4+4}{4}\right)}{2}\right]\\ &\quad+\frac{2n-k}{4}+\frac{nk}{2}-\frac{k^2}{4}=\frac{kn^2+2n}{4}\end{aligned}$$

In similar way, we calculate the magic constant for each decomposition $H_2, H_3, H_4,$ $\ldots, H_{\frac{k}{2}}$ as $\frac{kn^2+2n}{4}$. □

REFERENCES

[1] W. S. ANDREWS: *Magic Squares and Cubes*, Dover, 1960.

[2] A. GUTERREZ, A. LLADO: *Magic coverings*, J. Combin. Math. Combin. Comput., **55** (2005), 43-56.

[3] F. HARARY: *Graph Theory*, Addison-Wesley, Reading, Mass., 1969.

[4] A. J. W. HILTON: *Hamilton Decompositions of Complete Graphs*, Journal of Combinatorial Theory, **36** (1984), 125-134.

[5] N. INAYAH, A. LLADO, J. MORAGAS: *Magic and antimagic H-decompositions*, Discrete Math., **312** (2012), 1367-1371.

[6] J. A. CYNTHIA, N. R. SWATHI: *Hamilton Decomposition of Harary Graphs*, International Journal of Mathematics Trends and Technology, **34**(2) (2016), 59-63.

[7] G. MARIMUTHU, M. BALAKRISHNAN: *E-Super vertex magic labelings of graphs*, Discrete Appl. Math., **160** (2012), 1766-1774.

[8] J. PETERSEN: *Die Theorie der regularen Graphen*, Acta Math., **15** (1891), 193-220.

[9] J. REMYA, M.SUDHA: *Geometric decomposition of graph with various common ratio*, Advances in Mathematics: Scientific Journal, **9**(7) (2020), 5225-5229.

[10] J. REMYA, M.SUDHA: *Note on $H-$ totally magic cordial decomposition of graphs*, Journal of Xi′an University of Architecture and Technology. **12**(5) (2020), 3138-3141.

[11] S. P. SUBBIAH, J. PANDIMADEVI: *H-E-Super magic decomposition of graphs*,Electronic Journal of Graph Theory and Applications, **2**(2) (2014), 115-128.

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