ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.12, 11133–11140
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.12.93

APPLICATION OF HEURISTIC METHOD IN DUAL-HESITANT FUZZY TRANSPORTATION PROBLEM

G. JOTHILAKSHMI[1], S. KRISHNA PRABHA, AND P. THIRUMURUGAN

ABSTRACT. Different methodologies and theories were originated in long term research challenge to pact with ambiguity in factual world issues. To handle the ambiguity in various categories of problems, a broad assortment of tools are developed by fuzzy sets further with their expansions, corresponding as, interval-valued fuzzy sets, Atanassov's intuitionistic fuzzy sets and type-2 fuzzy sets etc are introduced. In order to covenant with the hesitant circumstances that are scarcely considered by the prior contrivances a novel extent of fuzzy set namely hesitant fuzzy sets has been established. Dual-hesitant fuzzy set is applied for handling imprecise, hesitant or imperfect facts and expertise circumstances in factual-existence effective investigate predicaments. So far many researchers have applied different methods like North West corner method, least cost method, Vogel's approximation method, allocation table method to solve various fuzzy transportation problems so as to find the optimum elucidation. In this work an innovative technique named as Heuristic Method for unraveling dual-hesitant fuzzy transportation problem is established. An arithmetical exemplar is illustrated with the new technique and the result obtained through this method is compared with the existing methods. This proposed method gives an optimum solution.

[1]*corresponding author*

2020 *Mathematics Subject Classification.* 90B06, 03E72.
Key words and phrases. score function, dual-hesitant fuzzy transportation problem, Heuristic Method, dual-hesitant fuzzy numbers.

1. Introduction

Transportation is a robust accord to gather together the attempt of how to afford the goods to the clients in extra skillful techniques. They guarantee the expert progress and perceptive receptivity of stocks and refined commodities. The transportation predicament was developed by Hitchock [8] in 1941, followed by Charnes et al [1] who proposed the stepping stone technique in 1953. Dantzig [2] introduced the primal simplex transportation method with constraints in 1963 to solve the transportation problem.

Due to overwhelming aspects in present significance's all the constraints of the transportation problems may not be renowned openly. This kind of unclear information cannot be characterized clearly by picking a random variable from a probability distribution. Zadeh [7] in 1965 introduced the fuzzy numbers to handle these situations.

Hajjari and Abbasbandy in 2011 [6] found a method of magnitude for defuzzification of fuzzy numbers. Gurupada Maity et al in 2019 [5] developed an algorithm to get the best elucidation for the dual-hesitant fuzzy transportation problem with some constraints. Allocation table method for solving dual-hesitant fuzzy transportation problem is introduced by Krishna Prabha et al in 2020 [10].

Researchers like Deepak (2016) [4] have contributed many results in this field. By using the same example projected by Gurupada Maity et al [9] for untangle a dual-hesitant fuzzy transportation problem which was solved by using VAM, we have suggested heuristic method to solve the same example and compared the results[3].

The work is structured as follows, introduction to the concepts was given in section 1.A preface to fuzzy sets and dual-hesitant fuzzy sets are sported in section 2. A procedure for deciphering the problem is put forward in section 3. A numerical example is illustrated in section 4, and we culminate the work in section 5.

2. Preliminaries

Definition 2.1. *A HFS is declared precisely in the subsequent method: $H=\{(x_i, h(x_i)): x_i \in X\}$where $h(x_i)$ is a set of numerous values in the interval[0, 1] for each $x_i \in X$, which indicates the feasible membership degree of the element $x_i \in X$ in the* *set H.In the normal sense, each element of h(x_i) is called a Hesitant Fuzzy Element (HFE), represented by h_i.*

Definition 2.2. *Let X be a fixed set; then a DHFS D on X is defined as follows: D= {(x, h(x), g(x) : x∈ X} where h(x) and g(x) are mappings that take set-values in [0, 1]; they are denoted as the possible membership degree and non-membership degree of any element x ∈ X, to the set D, respectively, with the conditions 0≤ h_D, g_D ≤ 1, 0 ≤ h_D+g_D ≤ 1, for any h_D∈ h(x); g_D ∈ g(x). Dual-Hesitant fuzzy element (DHFE) is understood as the pair d(x) = ((h(x), g(x)), and it is denoted in the functional form as d = (h, g).*

2.1. **Ranking of dual hesitant fuzzy sets [5].** Let D= {(x, $h(x)$, $g(x)$: x∈ X} be a DHFS, where X ={x_1,x_2,x_3,...x_n} and d = (h, g) be a DHFE. We define a score function S_d on the DHFS, represented as follows:

$$s_d = \left| \frac{1}{k} \sum_{i=1}^{k} h_d(x_i) - \frac{1}{k} \sum_{i=1}^{k} g_d(x_i) \right|.$$

Let d_1 and d_2 be any two DHFSs. With regard to a given score function, Zhu et al (2012) defined order relations as follows:
Case 1: If $s_{d_1} > s_{d_2}$, then d1 is called superior to d2, denoted by d1 > d2.
Case 2: If $s_{d_1} < s_{d_2}$, then h1 is called inferior to h2, denoted by d1< d2.
Case 3: If $s_{d_1} = s_{d_2}$, then h1 is called indifferent from h2, denoted by $d_1 \sim d_2$

3. Algorithm for heuristic method

Step-1: Construct a Dual-Hesitant Fuzzy Transportation Table (DHFTT).
Step-2: Verify if the TP is balanced or not, if not, make it balanced.
Step-3: Find the Penalty for every row and column by evaluating the difference between the two minimum cost cells for each row and column. These values are called as penalties, P, respectively
Step-4: Find the summations of row and Column cost. These values are called as T
Step-5: Find the product PT by multiplying the penalty 'P' and the total cost 'T
Step-6: Spot out the row/column having minimum 'PT'.
Step-7: Choose the cell having minimum cost in row/column recognized in Step-6.

Step-8: Make maximum feasible allocation to the cell chosen in Step-7, if the cost of this cell is also minimum in its column/row. Otherwise allocations are avoided and go to Step-9.
Step-9: Identify the row/column having next to lowest 'PT'.
Step-10: Choose the cell having minimum cost in row/ column recognized in step 9.
Step-11: Make maximum feasible allocation to the cell chosen in Step-10.
Step-12: Cross out the satisfied row/column.
Step-13: Repeat the procedure until all the requirements are satisfied.
Step-14: Now shift this allocation to the original DHFTT.
Step-15: To conclude, compute the total profit of the DHFTT. This calculation is the sum of the product of cost and resultant allocated value of the DHFTT.

4. NUMERICAL EXAMPLE

Consider Dual-Hesitant Fuzzy Transportation problem with three sources S_1, S_2, S_3 and three destinations D_1, D_2, $D_{3.}$ By using the score function we convert the given dual-Hesitant Fuzzy Transportation problem into crisp values, we get the following table.

$$\mathbf{s_d} = \left| \frac{\mathbf{1}}{\mathbf{k}} \sum_{\mathbf{i=1}}^{\mathbf{k}} \mathbf{h_d(x_i\ )} - \frac{\mathbf{1}}{\mathbf{k}} \sum_{\mathbf{i=1}}^{\mathbf{k}} \mathbf{g_d(x_i\ )} \right|$$

$$s_{11} = \left| \frac{1}{3} \sum_{i=1}^{3} h_d(x_i\ ) - \frac{1}{3} \sum_{i=1}^{3} g_d(x_i\ ) \right|$$

Table 1: Formulating dual-hesitant fuzzy cost

	D1	**D2**	**D3**	**Supply**
S1	{ {0:5;0:4;0:1}, {0:4;0:5; 0:9}} (20, 25, 30)	{{0:7;0:6;0:5;0:2}, {0:1;0:3;0:4;0:5}} (14, 16, 20, 35)	{{*0:6; 0:4;0:3* }, {*0:2;0:6; 0:7*}} (22, 25, 36)	20

S2	{ {0:4;0:2;}, {0:3;0:5}} (12, 15)	{{0:7;0:6;0:3}, {0:1; 0:3; 0:6}} (30, 35, 40)	{{*0:6; 0:5; 0:3* }, {*0:2; 0:3; 0:5*}} (22, 27, 30)	24
S3	{ {0:3;0:2;0:1}, {0:2;0:6; 0:7}} (30, 40, 45)	{{0:2;0:1}, {0:5; 0:9}} (25,32)	{{*0:6;0:5;0:3;0:2* }, {*0:2;0:3;0:5;0:7*}} (32, 35, 40, 50)	35
Demand	35	24	20	

$$s_{11} = \left| \frac{1}{3}\ (0:5+0:4+0:1) - \frac{1}{3}\ (0:4+0:5+\ 0:9)\,, \frac{1}{3}\ (20+25+30) \right|$$

$$s_{11} = \left| \frac{1}{3}\ (1) - \frac{1}{3}\ (1.8)\,, \frac{1}{3}\ (75) \right|$$

$$= |0.333 - 0.6, 25| = |0.333 - 0.6, 25| = |-0.27, 25| = \mathbf{0}.27, 2$$

TABLE 2. Defuzzification values

	D1	**D2**	**D3**	**Supply**
S1	0.27 / 25	0.175/21.25	0.07 / 27.67	20
S1	0.1 / 13.5	0.2 / 35	0.14 / 26.33	24
S1	0.3 /38.33	0.55 / 28.5	0.025 /39.25	35
Demand	35	24	20	

Table 3: Penalty and Total Cost

	D1	**D2**	**D3**	**Supply**	**Row penalty (P)**	**Total (T)**	**PxT**
S1	0.27	0.18	0.07	20	0.1	0.52	0.05
S2	0.1	0.2	0.14	24	0.04	0.44	0.02
S3	0.3	0.555	0.02	35	0.27	0.88	0.24
Demand	35	24	20				

Column penalty	0.17	0.03	0.05				
Total (T)	0.67	0.92	0.24				
PXT	0.11	0.02	0.01				

Table 4: Optimum Allocation

	D1	**D2**	**D3**	**Supply**	**Row penalty (P)**	**Total (T)**	**PxT**
S1	0.27	0.18 (20)	0.07	20	0.1 | 0.1 | – | – | – |	0.52	0.05 | 0.05 | – | – | – |
S2	0.1 (24)	0.2	0.14	24	0.04 | 0.1 | 0.1 | – | – |	0.44	0.02 | 0.04 | 0.04 | – | – |
S3	0.3 (11)	0.55 (4)	0.02 (20)	35	0.27 | 0.25 | 0.25 | 0.25 | 0.55 |	0.88	0.24 | 0.22 | 0.22 | 0.22 | 0.48 |
Demand	35	24	20				
Column penalty	0.17 0.17 0.2 0.3 - -	0.03 0.03 0.35 0.55 0.55	0.05 - - - - - - - -				
Total (T)	0.67	0.92	0.24				
PXT	0.11 0.11 0.13 0.2 - -	0.02 0.02 0.32 0.51 0.51	0.01 - - - - - - - -				

TABLE 5. Optimum allocation in original transportation problem

	D1	**D2**	**D3**	**Supply**
S1	0.245,	0.15,20	0.045	20
S2	0.075,24	0.2	0.14	24
S3	0.275, 11	0.525, 4	0.025,20	35
Demand	35	24	20	

TABLE 6. Comparison of different methods

North-West Corner method	**Vogel's Approximation method**	**Russell's method**	**Allocation method**	**Proposed method**
2230	2194	2194	2186	2069

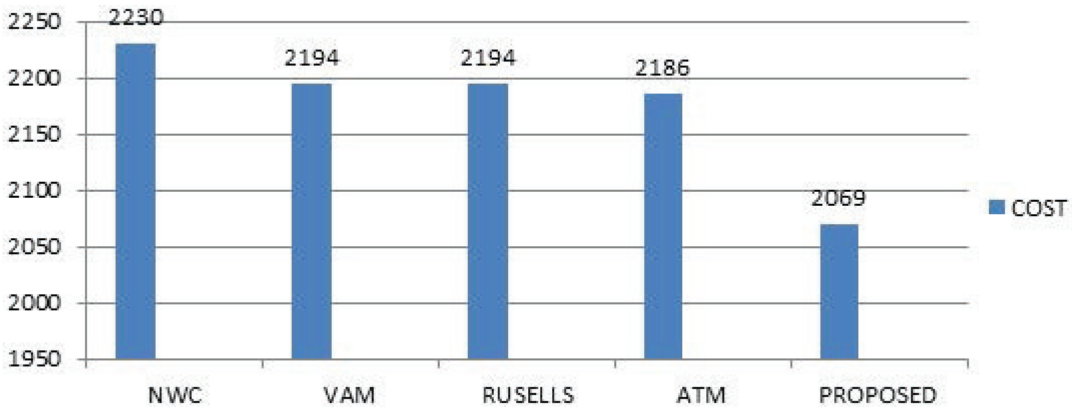

FIGURE 1

5. CONCLUSION

In order to decrease the transportation cost and to maximize the profit we have applied heuristic method to solve a Dual-Hesitant Fuzzy Transportation problem. By comparing our results to the exiting results we have verified that our approach yields a better optimum solution. The proposed algorithm will be very effective for real-life problem. This technique can be used to solve all types of Dual-Hesitant Fuzzy Transportation problems. Consequently this scheme can be utilized to solve the problems in supply chain management, assignment, cloud computing, travelling salesman etc.Further this can be extended to various types of Single-value, cubic, Bipolar, Interval bipolar fuzzy sets etc.

REFERENCES

[1] A. CHARNES, W. COOPER, A. HENDERSON: *An introduction to Linear Programming*, Wiley, New Work, 1953.

[2] G. B. DANTZIG: *Linear Programming and Extensions*, Princeton Univ Press, NJ., 1963.

[3] D. HUNWISAI, P. KUMAM: *A method for solving a fuzzy transportation problem via Robust ranking technique and ATM*, Cogent Mathematics, **4**, 2017.

[4] S. DEEPAK, J. JACOB: *Rough Approximations on Hesitant Fuzzy Sets.* IGI Global, 2016.

[5] G. GURUPADA: *A new approach for solving dual-hesitant fuzzy transportation problem with restrictions*, Sadhana, **44** (2019), art.no.75.

[6] T. HAJJARI, S. A. ABBASBANDY: *Promoter Operator for Defuzzification Methods*, Australian Journal of Basic and Applied Sciences, **5**(10) (2011), 1096-1105.

[7] A. HARI GANESH, M. SURESH, G. SIVAKUMAR: *On solving fuzzy transportation problem based on distance based defuzzification method of various fuzzy quantities using centroid*, Malaya Journal of Matematik, **S**(1) (2020), 410-426.

[8] F.L. HITCHCOCK: *The distribution of a product from several sources to numerous localities*, Journal of Mathematical Physics, **20**(1-4) (1941), 224-230.

[9] S. K. PRABHA, S. VIMALA: *ATM For Solving Fuzzy Transportation Problem Using Method Of Magnitude*, Iaetsd Journal for Advanced Research in Applied Sciences, **5**(3) (2018), 406-412.

[10] S. K. PRABHA, K. JEYALAKSHMI, M. THANGARAJ, K. SUBRAMANI: *Dual-Hesitant Fuzzy Transportation Problem With ATM*, Journal Of Critical Reviews, **7**(17) (2020), 1309-1315.

DEPARTMENT OF MATHEMATICS
THIAGARAJAR COLLEGE OF ENGINEERING
MADURAI
Email address: gjlmat@tce.edu

DEPARTMENT OF MATHEMATICS
PSNA COLLEGE OF ENGINEERING AND TECHNOLOGY
DINDIGUL, TAMILNADU
Email address: jvprbh1@gmail.com

DEPARTMENT OF ECE
PSNA COLLEGE OF ENGINEERING AND TECHNOLOGY
DINDIGUL, TAMILNADU
Email address: thiruece@psnacet.edu.in