ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **10** (2021), no.1, 121–128
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.1.12

CONNECTED ANTI-FUZZY EQUITABLE DOMINATING SET IN ANTI-FUZZY GRAPHS

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ABSTRACT. In this paper, the concept of connected anti-fuzzy equitable dominating set is introduced. The connected anti-fuzzy equitable dominating number is obtained and also studied the relationship between the connected anti-fuzzy equitable domination number and anti-fuzzy equitable domination number. Some bounds and interesting results for connected anti-fuzzy equitable dominating set are obtained.

1. INTRODUCTION

The notion of fuzzy set introduced by L. A. Zadeh [8] to represent vagueness mathematically and tried to solve the problems by assigning a particular membership values to every element of a given set. Graph is one of the most suitable ways of representing the relationship between objects. Sometimes there exist uncertainties in the description of the objects or in its relationships or in both which has to be designed as fuzzy graph model. In fuzzy graph, the relation attains only the minimum membership value among the objects. Sometimes, if complexity occurs among the relation, then it may be lead to obtain maximum membership values. For such cases, we can use anti-fuzzy graph models to solve the problems. In 2012, Akram [1] studied the concept of anti-fuzzy structures in fuzzy graphs. R.Seethalakshmi and R. B. Gnanajothi [7] introduced the

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2020 *Mathematics Subject Classification.* 05C62, 05E99, 05C07.
Key words and phrases. anti-fuzzy graph, equitable dominating set, connected anti-fuzzy equitable dominating set.

operations on anti-fuzzy graph. A. Somasundram and S.Somasundaram [6] presented several types domination parameters such as independent domination, total domination, connected domination and domination in Cartesian product and composition of fuzzy graphs. R.Muthuraj and A. Sasireka [5] introduced domination in anti-fuzzy graphs. Some works in complementary nil domination in fuzzy graphs can be found in [3, 4]. In 2020, Firthous Fatima and K. Janofer [2] introduced the concept of anti-fuzzy equitable domination set, anti-fuzzy equitable independent set and anti-fuzzy equitable independent dominating set of an anti-fuzzy graph. In this paper, the concept of connected anti-fuzzy equitable dominating set is introduced. Also, the connected anti-fuzzy equitable domination number is obtained. Some theorems and results on this parameter are stated and proved.

2. Preliminaries

Definition 2.1. *[1] A fuzzy graph $G = (\sigma, \mu)$ is said to be an anti-fuzzy graph with a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$, for all $u, v \in V$, we have $\mu(u, v) \geq \sigma(u) \vee \sigma(v)$ and it is denoted by $G_{AF}(\sigma, \mu)$.*

Definition 2.2. *[1] The order p and size q of an anti-fuzzy graph $G = (\sigma, \mu)$ are defined to be $p = \sum_{u\in V} \sigma(u)$ and $q = \sum_{uv\in E} \mu(uv)$. It is denoted by $O(G)$ and $S(G)$.*

Definition 2.3. *[5] Let G be an anti-fuzzy graph and let $u, v \in V$. If $\mu(u, v) = \sigma(u) \vee \sigma(v)$ then u dominates v (or v dominates u) in G. A subset D of V. A set $D \subseteq V$ is said to be a dominating set of an anti-fuzzy graph G if for every vertex $v \in V - D$ there exists $u \in D$ such that u dominates v.*

Definition 2.4. *[5] A dominating set D of an anti-fuzzy graph G is called a minimal dominating set if there is no dominating set D' such that $D' \subseteq D$.*

Definition 2.5. *[5] The maximum scalar cardinality taken over all minimal dominating set is called domination number of an anti-fuzzy graph G and is denoted by γ_{AFG}.*

3. Connected Anti-Fuzzy Equitable Dominating Set

Definition 3.1. *An anti-fuzzy equitable dominating set X of an anti-fuzzy graph G is called a connected anti-fuzzy equitable dominating set (CAFEDS) if induced anti-fuzzy sub-graph $< X >$ is connected.*

Definition 3.2. *The maximum scalar cardinality taken over all minimal connected anti-fuzzy equitable dominating set of anti-fuzzy graph G is called the connected anti-fuzzy equitable domination number and is denoted by γ^{ced}_{AFG}.*

Example 1. *Consider a connected anti-fuzzy graph G, Here, $d(u_1) = 0.5$, $d(u_2) =$*

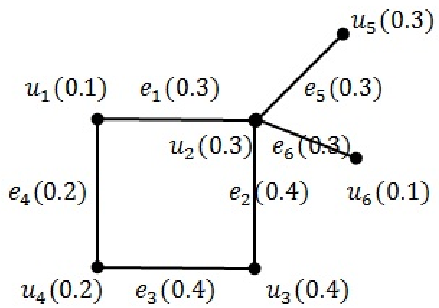

FIGURE 1. Fuzzy Graph

$1.3, d(u_3) = 0.8, d(u_4) = 0.6, d(u_5) = 0.3, d(u_6) = 0.3$ *The sets* $\{u_1, u_2, u_6\}$ *and* $\{u_2, u_3, u_6\}$ *are the minimal connected anti-fuzzy equitable dominating sets. Since they satisfy the definition of connected anti-fuzzy equitable dominating set and also, no proper subsets of these sets are not a connected anti-fuzzy equitable dominating sets. The maximum scalar cardinality of these minimal dominating sets is* $\gamma^{ced}_{AFG} = 0.8$, *corresponding to the connected anti-fuzzy equitable dominating set* $\{u_2, u_3, u_6\}$.

Theorem 3.1. *Let G be an anti-fuzzy graph. Then a connected anti-fuzzy equitable dominating set exists if and only if G is connected anti-fuzzy graph.*

Proof. Proof of this theorem is straightforward from the definition of connected anti-fuzzy equitable dominating set. □

Theorem 3.2. *Every connected anti-fuzzy equitable dominating set of an anti-fuzzy graph G is an anti-fuzzy equitable dominating set.*

Proof. Let $G = (\sigma, \mu)$ be an anti-fuzzy graph. Let X be a connected anti-fuzzy equitable dominating set of G. Then for every $y \in V - X$ there exist a vertex $x \in X$ such that $xy \in E(G)$ and $|d(x) - d(y)| \leq 1$ and induced sub-graph $< X >$ is connected. Thus X is an anti-fuzzy equitable dominating set of an anti-fuzzy graph. □

Remark 3.1. *The converse of the above theorem is not true. It can be shown by the following example.*

Example 2. *Consider an anti-fuzzy graph G, Here, $X = \{u_1, u_3\}$ is an anti-fuzzy*

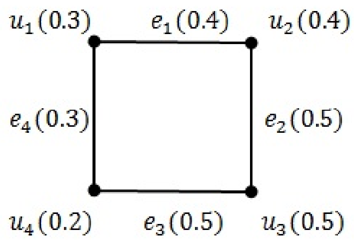

FIGURE 2. Fuzzy Graph

equitable dominating set, but it is not connected anti-fuzzy equitable dominating set. Since the induced anti-fuzzy sub-graph $< X >$ is not connected.

Theorem 3.3. *A connected anti-fuzzy equitable dominating set X of an anti-fuzzy graph G is a minimal connected anti-fuzzy equitable dominating set if and only if for each $x \in X$, one of the following condition holds.*

(1) *For some vertex $\alpha \in X, \sigma(x) < \sigma(\alpha)$*
(2) *There is a vertex $y \in V - X$ such that $N(y) \cap X = \{x\}$.*

Proof. Let X be a minimal connected anti-fuzzy equitable dominating set. Then for every vertex $x \in X, X' = X - \{x\}$ is not a connected anti-fuzzy equitable dominating set. This means that, there exists $x' \in V - X'$ such that x' is not dominated by any element of X.If $x' = x$, then x is adjacent to any vertex $\alpha \in X$ such that $\sigma(x) < \sigma(\alpha)$. Hence we get (i) and if $x' \neq x$, then $x' \in V - X$, then x is not adjacent to any vertex in $X - \{x\}$, but it is adjacent to some vertex $\alpha \in X$ then α is adjacent to only vertex $x \in V - X$. Thus $N(x) \cap X = \{\alpha\}$. Hence we get (ii). The converse is obvious. Conversely, suppose X is a connected anti-fuzzy equitable dominating set, and for each vertex $x \in X$, one of the two stated conditions holds. Now, we have to prove that X is minimal connected anti-fuzzy equitable dominating set. Suppose X is not a minimal connected anti-fuzzy equitable dominating set. Then there exists a vertex $x \in X$ such that $X - \{x\}$ is a connected anti-fuzzy equitable dominating set. Thus x is adjacent to atleast one vertex in $X - \{x\}$. Therefore condition (i) does not hold. Also, if $X - \{x\}$ is a dominating set, then every vertex in $V - X$ is adjacent to at least one vertex in $X - \{x\}$. Therefore, condition (ii) does not hold. Hence neither condition (i) nor condition (ii) holds, which is contradiction. Hence X is a minimal connected anti-fuzzy equitable dominating set. □

Theorem 3.4. *If G is a connected anti-fuzzy graph and $X \subset V$ is a connected anti-fuzzy equitable dominating set of G, then $V - X$ need not be connected anti-fuzzy equitable dominating set.*

Proof. The proof of this theorem can be proved by the following example. □

Example 3. *Consider an anti-fuzzy graph given in Fig 3. Here, $X = \{v_1, v_4\}$ is*

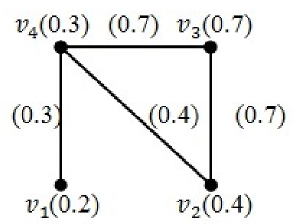

FIGURE 3. Fuzzy Graph

a connected anti-fuzzy equitable dominating set, but $V - X = \{v_2, v_3\}$ is a not a connected equitable dominating set.

Theorem 3.5. *Let G be an anti-fuzzy graph. Then $S(G) - O(G) \leq \gamma_{AFG}^{ced} \leq O(G) - \delta(G)$ where $O(G), S(G)$ and $\delta(G)$ are the order, size and minimum degree of an anti-fuzzy graph G.*

Proof. Let G be an anti-fuzzy graph and let X be a γ_{AFG}^{ced}-set. Let $O(G), S(G)$ and $\delta(G)$ are the order, size and minimum degree of an anti-fuzzy graph G. Then, there exists atmost $\frac{(O(G))}{2}$ edges incident from $V - X$ to X.

$$S(G) - \gamma_{AFG}^{ced} \leq O(G)$$
$$S(G) - O(G) \leq \gamma_{AFG}^{ced} \tag{3.1}$$

Let $d(\alpha) = \delta(G)$, for some $\alpha \in V$. Then α must be adjacent to some vertices in G. Therefore, $V - \{\alpha\}$ is a connected anti-fuzzy equitable dominating set. $\gamma_{AFG}^{ced} \leq O(G) - \delta(G)$. Hence $S(G) - O(G) \leq \gamma_{AFG}^{ced} \leq O(G) - \delta(G)$. □

Example 4. *Consider an anti-fuzzy graph G given in Fig.1. Here,* $O(G) = 1.4, S(G) = 1.9, \delta(G) = 0.3$ *and* $\gamma_{AFG}^{ced} = 0.8$*. Therefore,* $S(G) - O(G) \leq \gamma_{AFG}^{ced} \leq O(G) - \delta(G)$

$$1.9 - 1.4 \leq 0.8 \leq 1.4 - 0.3,$$
$$0.5 \leq 0.8 \leq 1.1.$$

Lemma 3.1. *Let* $G = (\sigma, \mu)$ *be anti-fuzzy graph whose vertex set* σ *is constant function, then* $S(G) - O(G) \leq \gamma_{AFG}^{ced} \leq O(G) - \Delta(G)$*.*

Theorem 3.6. *Let G be a connected anti-fuzzy graph, then* $\gamma_{AFG}(G) \leq \gamma_{ADF}^{ed}(G) \leq \gamma_{AFG}^{ced}(G)$*.*

Proof. Let G be a connected anti-fuzzy graph. By definition of the connected anti-fuzzy equitable dominating set of an anti-fuzzy graph G. It is clear that for any anti-fuzzy graph G, any connected anti-fuzzy equitable dominating set X is also an anti-fuzzy equitable dominating set and every anti-fuzzy equitable dominating set is also an anti-fuzzy dominating set. Hence $\gamma_{AFG}(G) \leq \gamma_{ADF}^{ed}(G) \leq \gamma_{AFG}^{ced}(G)$. □

Theorem 3.7. *Let G be a connected anti-fuzzy graph, then* $\gamma_{AFG}^{cd}(G) \leq \gamma_{AFG}^{ced}(G)$*.*

Proof. Let G be a connected anti-fuzzy graph. Since, every connected anti-fuzzy equitable dominating set for any connected anti-fuzzy graph G is a connected dominating set. Hence $\gamma_{AFG}^{cd}(G) \leq \gamma_{AFG}^{ced}(G)$. □

Theorem 3.8. *For any anti-fuzzy graph G with* $\delta(G) \geq 1$*,*

$$\lceil \frac{O(G)}{1 + \Delta(G)} \rceil \leq \gamma_{AFG}(G) \leq 2S(G) - O(G) + 1.$$

Proof. Let G be a connected anti-fuzzy graph. Clearly from the definition of the anti-fuzzy connected equitable dominating set, we have

$$\begin{aligned} \gamma_{AFG}^{ced} &\leq p-1 \\ &\leq 2S(G) - O(G) + 1. \end{aligned} \tag{3.2}$$

Then by theorem,

$$\lceil \frac{O(G)}{1+\Delta(G)} \rceil \leq \gamma_{AFG}(G).$$

Hence,

$$\lceil \frac{O(G)}{1+\Delta(G)} \rceil \leq \gamma_{AFG}(G) \leq 2S(G) - O(G) + 1.$$

□

Theorem 3.9. *Let G be a connected anti-fuzzy fuzzy graph with no non-anti fuzzy equitable edge and H is spanning anti-fuzzy sub-graph of G. Then* $\gamma_{AFG}^{ed}(G) \leq \gamma_{AFG}^{ed}(H)$.

4. CONCLUSION

Fuzzy graphs and anti-fuzzy graphs are very useful tools to study the uncertainty in varies real-life problems. In this work, we studied connected anti-fuzzy equitable dominating set and its number. We got some relationship between connected anti-fuzzy equitable domination number and related parameters.

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