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BIOCONVECTION IN CASSON FLUID FLOW WITH GYROTACTIC MICROORGANISMS AND HEAT TRANSFER OVER A LINEAR STRETCHING SHEET IN PRESENCE OF MAGNETIC FIELD

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ABSTRACT. A study related to bioconvection of magnetohydrodynamic boundary layer flow, heat and mass exchange of Casson fluid containing gyrotactic microorganisms above a linearly stretching surface is considered. The Partial differential equations which govern the physical situation are transformed into a system of coupled nonlinear ordinary differential equations using similarity transformations. Taylor's series solution for momentum, energy, the diffusive concentration of nanofluid, and concentration of microorganism's equations are obtained using the differential transform method and compared with numerical solutions. The effects of a range of non-dimensional parameters on bioconvection fluid flow and heat transfer are analyzed through graphs.

1. INTRODUCTION

Flow and heat exchange of nanofluids with distinct properties in the boundary layer over a continuous flat stretching surface is the physical situations that come across in many industrial and engineering applications. Attention on the boundary layer of Newtonian and non-Newtonian fluid flows and heat exchange with various effects made great applications in industries, such as the polymer eviction from a dye and wire drawing, etc. Other Engineering executions of the

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stretching sheet are polymer sheet ejection, tinning, manufacturing glass fibers, paper and cooling of metallic plates, etc. Within the past few decades, we can find a lot of work done by eminent scholars on boundary layer flow and heat exchange above the linear stretching membrane. Crane [1] was the first to achieve an exact solution for the stream over a stretching plate problem. Rajgopal et al. [2] extended Crane's work with micropolar fluid. An analysis is made on the flow of an incompressible viscous fluid exceeding the stretching sheet through stagnation point by Chaim [3] and later many eminent scholars have made an effort to obtain analytical or numerical results of boundary layer flow of unlike fluids overstretching or shrinking sheet in presence of the magnetic field.

Bioconvection of nanofluid and microorganisms is worth study because of modern bio nanomaterial manufacturing purposes. The bioconvection flow occurs at the boundary of the fluid containing microorganisms or the bacteria and in such unstable incidents boundary layer containing microorganisms spreads as bioconvection cells and these can be categorized as gyrotactic, oxytocic, and gravitaxis microorganisms. Pedley et al. [4] initiated the use of the bioconvection term concerning microscopic convection due to motile microorganisms. A numerical solution of thermo-bioconvection suspended due to gyrotactic microorganisms was carried by Alloui et al. [5]. Khan et al. [6] investigated the combined effect on boundary layer flow with heat and mass exchange of water-based nanofluid mixed with gyrotactic microorganisms in presence of magnetic field and Navier slip. Numerical results are obtained using Oberbeck-Boussinesq approximation and similarity transformation. Mehmood et al. [7] revealed the effect of stagnation point flow over a stretching sheet containing gyrotactic microorganisms in presence of an induced magnetic field. Akbar et al. [8] analyzed numerically the combined effect of bioconvection, Brownian motion, and thermophoresis of gyrotactic microorganisms and nanoparticles over a stretching sheet surface in presence of a magnetic field. Recently, Raju et al. [9] brought numerical results on the effects of thermophoresis andBrownian motion on the radiative flow of Casson fluid over a moving wedge containing gyrotactic microorganisms in presence of a magnetic field. Khan et al. [10] applied the homotopy analysis method to investigate mixed convection of non-Newtonian fluid films such as Casson fluid and Williamson fluid flow containing both nanoparticles and gyrotactic microorganisms. Chakraborthy et al. [11] examined the combined impacts of magnetic and convective boundary state on bioconvection of nanofluid containing gyrotactic microorganisms with convective boundary conditions. Khan [12] studied bioconvection in steady second-grade nanofluids thin flow including nanoparticles and gyrotactic microorganisms using passively controlled nanofluids model boundary conditions and the governing equations are solved analytically using HAM.

The differential transform method (DTM) is efficient and it is also known as Zhou's method, which is extensively used by many researchers. DTM is a simple method of solving the differential equation provided with initial or boundary conditions and Taylor's series solution can be obtained. This method is very flexible and very simple that can be easily computerized and here, it is not required to choose any auxiliary parameter as in HAM. Recently many researchers have succeeded in obtaining the convergence of series solutions for the coupled nonlinear differential equations. It is found that, DTM act as an alternative way which gives fast convergence series solutions for the system of non-linear ordinary and partial differential equations that cannot be solved analytically. Mirzaee [13] has applied DTM to solve linear and nonlinear systems of ordinary differential equations. Hatami et al. [14] used DTM for Newtonian and non-Newtonian nanofluids flow analysis and the results are compared well in agreement with numerical results. Sepasgozar et al. [15] applied DTM to get the series solution for momentum and heat exchange equations of non-Newtonian fluid flow in an axisymmetric channel with a porous wall.

The present work is on the bioconvection of Casson fluid flowing along the x-axis above the stretching sheet including gyrotactic microorganisms in presence of a magnetic field. Further, we have examined interesting aspects of Brownian motion and thermophoresis and heat exchange analysis of the considered fluid. In the literature survey, it is come to know that, no work is done on this particular physical situation with the Casson model and gyrotactic microorganisms and most of the similar vertical flow problems are solved either numerically or by using HAM. Currently, we have made an effort to solve the governing equations using the differential transform method and numerical method. Both the results are good in agreement and those are represented through graphs.

2. Mathematical Formulation

Consider incompressible water-based Casson nanofluid in which gyrotactic microorganisms are added. It is assumed that the mixed fluid is dilute so that the microorganisms are alive. In $xy-$plane, the stretching sheet is stretched along the $x-$axis and the fluid is allowed to flow above the sheet in presence of an applied induced magnetic field normal to the surface of the sheet. The effect of an induced magnetic field is neglected. The velocity of the stretching sheet surface is assumed to be linear. Let D_B be Brownian diffusion coefficient and D_T be thermophoresis diffusion coefficient effect on the heat conduction. Let T_w, C_w, and N_w are the temperature, volume fraction of nanoparticles, and diffusive concentration of microorganisms at the wall respectively and T_∞ and C_∞ denotes temperature, the concentration of nanofluid, and concentration of microorganism diffusive coefficient at infinite distance from the wall which is as shown in the Figure 1.

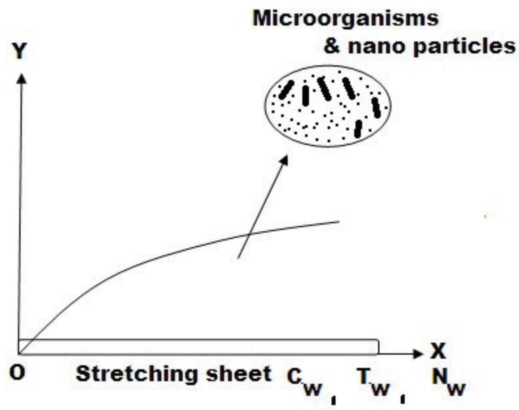

Figure 1. Geometrical sketch of the model

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\begin{cases} u\dfrac{\partial u}{\partial x}+v\dfrac{\partial v}{\partial y}=\nu\left(1+\dfrac{1}{\beta}\right)\dfrac{\partial^2 u}{\partial y^2}-\dfrac{\sigma B_0^2}{\rho}u-\left(\dfrac{1-C_\infty}{\rho_f}\right)\rho_\infty g\alpha(T-T_\infty) \\ -\left(\dfrac{\rho_p-\rho_\infty}{\rho_f}\right)g(C-C_\infty)-\left(\dfrac{\rho_m-\rho_f}{\rho_f}\right)g\gamma(N-N_\infty) \end{cases}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left[\frac{\partial T}{\partial y}\right]^2\right)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}+\frac{bW_c}{C_w-C_\infty}\left(\frac{\partial}{\partial y}\left[N\frac{\partial C}{\partial y}\right]\right)=D_m\frac{\partial^2 N}{\partial y^2}$$

Here, u and v are the components of velocity along x and y axis respectively and T is the fluid temperature, α is volumetric expansion coefficient of the fluid, ρ_p is the density of nanoparticles, ρ_m is the microorganism density, ρ is the Casson fluid density, γ is an average volume of microorganisms, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient, $\tau=\frac{(\rho C)_p}{(\rho C)_f}$ is the ratio of effective heat capacity of the fluid with ρ_f and ρ_p, β is the Casson fluid parameter.

The boundary conditions for the flow and heat transfer with Casson model are:

$$\left.\begin{array}{l}\nu=0,\quad u=ax,\quad T=T_w,\quad C=C_w,\quad N=N_w,\quad \text{as}\quad y\to 0,\\ u\to 0,\ \ T\to T_\infty,\quad C\to C_\infty,\quad N\to N_\infty,\quad \text{as}\quad y\to\infty,\end{array}\right\}$$

where $a>0$, is stretching rate. Using the similarity transformation into the governing equations:

$$\left.\begin{array}{l}\eta=\frac{y}{x}Ra_x^{\frac{1}{4}}f(\eta),\quad \psi=mRa_x^{\frac{1}{4}}f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\\ \chi(\eta)=\dfrac{N-N_\infty}{N_w-N_\infty},\ \ Ra_x=\dfrac{(1-C_\infty)\alpha g\Delta T_f}{m\nu}x^3\end{array}\right\}$$

We form the following coupled nonlinear ordinary differential equations:

$$\left(1+\frac{1}{\beta}\right)f_{\eta^3}-\left(\frac{1}{2P_r}\right)f_\eta^2+\left(\frac{3P_r}{4}\right)ff_{\eta^2}-Mf_\eta+\theta-N_r\phi-R_b\chi=0, \tag{2.1}$$

$$\theta_{\eta^2}+\left(\frac{3}{4}\right)f\theta_\eta+N_b\theta_\eta\phi_\eta+N_t\theta_\eta^2=0, \tag{2.2}$$

$$\phi_{\eta^2}+\left(\frac{3}{4}\right)L_ef\phi_\eta+\left(\frac{N_t}{N_b}\right)\theta_{\eta^2}=0, \tag{2.3}$$

$$\chi_{\eta^2} + \left(\frac{3}{4}\right) S_c f \chi_\eta - P_e \left[\phi_\eta \chi_\eta + \phi_{\eta^2}(\chi + \sigma)\right] = 0. \tag{2.4}$$

The associated dimensionless boundary conditions are

$$\left.\begin{array}{l} f(0) = 0,\ \ f_\eta(0) = \lambda,\ \ \theta(0) = 1,\ \ \phi(0) = 1,\ \ \chi(0) = 1,\ \ as \quad \eta \to 0, \\ f^{'}(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0, \quad \chi(\infty) = 0, \quad\quad as \quad\quad \eta \to \infty. \end{array}\right\} \tag{2.5}$$

The dimensionless parameters used in equations ((2.2) to (2.5)) are given by

$$\left.\begin{array}{l} P_r = \dfrac{\nu}{m}, M = \dfrac{\sigma B_0^2 x^2}{\rho \nu R a_x^{\frac{1}{2}}}, N_r = \dfrac{(\sigma_p - \sigma_\infty)\Delta C_w}{\sigma_f (1 - C_\infty)\alpha \Delta T_f}, N_t = \dfrac{\tau D_T (T_w - T_\infty)}{m T_\infty} \\ R_b = \dfrac{\gamma \Delta N_w \Delta \rho}{\sigma_f \alpha (1 - C_\infty)\Delta T_w}, N_b = \dfrac{\tau D_B (C_w - C_\infty)}{m}, S_c = \dfrac{m}{D_m}, \\ L_e = \dfrac{m}{D_B}, P_e = \dfrac{b W_c}{(C_w - C_\infty)}, \sigma = \dfrac{N_\infty}{(N_w - N_\infty)}, \lambda = \dfrac{a x^2}{m R_x^{\frac{1}{2}}}{}' \end{array}\right\} \tag{2.6}$$

where L_e is the traditional Lewis number, P_e is the bioconvection Peclet number, N_r is the buoyancy ratio parameter, R_b is the bioconvection Rayleigh number, N_b is the Brownian motion parameter and N_t is the thermophoresis parameter, S_c is the Schmidt number, σ is the (dimensionless) bioconvection constant, M is the modified magnetic parameter and λ is the slip parameter."

3. Differential Transform Method (DTM)

Using DTM [14] the equations((2.1) - (2.6)) can be transformed in the following Differential forms:

$$\left.\begin{array}{l} \left(1 + \frac{1}{\beta}\right)(r+1)(r+2)(r+3)F[r+3] = \\ M(r+1)F[r+1] - \theta[r] + N_r \phi[r] + R_b \chi[r] + \\ \frac{1}{2} P_r \sum\limits_{m=0}^{r} (r-m+1)F[r-m+1](m+1)F[m+1] - \\ \frac{3}{4} P_r \sum\limits_{m=0}^{r} F[r-m](m+1)(m+2)F[m+2] \end{array}\right\} \tag{3.1}$$

where $F[0] = 0$, $F[1] = 0$, $F[2] = a_1$;

$$\left.\begin{aligned}&(r+1)(r+2)\theta[r+2] = -\tfrac{3}{4}\sum_{m=0}^{r} F[r-m](m+1)\theta[m+1]-\\&N_b\sum_{m=0}^{r}(r-m+1)\theta[r-m+1](m+1)\theta[m+1]-\\&N_t\sum_{m=0}^{r}(r-m+1)\theta[r-m+1](m+1)\theta[m+1]\end{aligned}\right\} \tag{3.2}$$

where $\theta[0] = 1$, $\theta[1] = a_2$;

$$\begin{aligned}(r+1)(r+2)\phi[r+2] &= -\frac{3}{4}L_e\sum_{m=0}^{r} F[r-m](m+1)\phi(m+1)\\&\quad-\frac{N_t}{N_b}\sum_{m=0}^{r}(r+1)(r+2)\theta[r+2]\end{aligned} \tag{3.3}$$

where $\phi[0] = 1$, $\phi[1] = a_3$;

$$\left.\begin{aligned}&(r+1)(r+2)\chi[r+2] =\\&P_e\sum_{m=0}^{r}(r-m+1)\phi[r-m+1](r+1)\chi[r+1]+\\&P_e\sum_{m=0}^{r}\chi[r-m](m+1)(m+2)\phi[m+2]+\\&P_e\sigma(m+1)(m+2)\phi[m+2]-\\&\tfrac{3}{4}S_c\sum_{m=0}^{r}F[r-m](m+1)\chi[m+1]\end{aligned}\right\} \tag{3.4}$$

where $\chi[0] = 1$, $\chi[1] = a_4$.

Further, $F[r]$, $\theta[r]$, $\phi[r]$ and $\chi[r]$ are the differential transform of $f(\eta)$, $\theta(\eta)$, $\phi(\eta)$ and $\chi(\eta)$ respectively. a_1, a_2, a_3 and a_4 are the constants and these can be determined with the aid of equations ((3.1) - (3.4)) and the boundary conditions. For $s = 0, 1, 2, 3\dots$ we get

$$\left.\begin{aligned}&F[3] = \tfrac{N_r+R_b-1}{6\left(1+\frac{1}{\beta}\right)}\\&\theta[2] = -\left(\tfrac{1}{2}a_2a_3N_b + \tfrac{a_2^2N_t}{2}\right)\\&\phi[2] = \tfrac{N_t\left(\frac{1}{2}a_2a_3N_b+\frac{1}{2}a_2^2N_t\right)}{Nb}\\&\chi[2] = \tfrac{1}{2}\left[a_3a_4P_e + \tfrac{N_tP_e\left(a_2a_3N_b+a_2^2N_t\right)}{N_b}(1+\sigma)\right]\end{aligned}\right\}$$

Similarly, we can find $F[4]$, $\theta[4]$ $\phi[4]$, $\chi[2]$, $\chi[3]$ and taking $P_r = 6.2$, $\beta = 1$, $M = 5$, $N_r = 0.5$, $R_b = 0.1$, $N_t = 0.1$, $N_b = 0.1$, $L_e = 10$, $S_c = 0.1$, $P_e = 1$, $\sigma = 0.2$, $R_a = 0.5$, $D_a = 0.5$ and solving for all the five transformed equations in five unknowns using the boundary condition and Pade approximation for $f^{'}(\eta) = 0$ as $\eta \to \infty$ we obtain $a_1 = 0.144761$, $a_2 = -0.159598$, $a_3 = -0.4891398$, $a_4 = -0.619069$ and the better approximation for the functions $f(\eta)$, $\theta(\eta)$, $\phi(\eta)$, $\chi(\eta)$ as Taylor's series solutions are obtained as follows

$$\left.\begin{array}{l}
f(\eta) = 0.0723803575\eta^2 - 0.033333333\eta^3 + 0.0120192699\eta^4 \\
-0.0038352688\eta^5 + 0.0009907633\eta^6 - 0.0002295507\eta^7 \\
+0.0000587013\eta^8 - 0.0000160486\eta^9 + 0.0000042669\eta^{10} \\
-0.0000011624\eta^{11} + 0.0000003229\eta^{12} - \cdots \\
\theta(\eta) = 1 - 0.159598\eta - 0.00517687\eta^2 - 0.000111948\eta^3 \\
+0.0120192699\eta^4 - 0.0038352688\eta^5 + 0.000024909\eta^6 \\
-0.00000958335\eta^7 + 0.00000722131\eta^8 + 0.000000464485\eta^9 \\
-0.00000018687\eta^{10} + 0.000000157907\eta^{11} - 0.0000000919743\eta^{12} - \cdots \\
\phi[\eta] = 1 - 0.48914\eta^2 + 0.000111948\eta^3 + 0.0214074\eta^4 \\
-0.00630385\eta^5 + 0.00152499\eta^6 - 0.0014524\eta^7 + 0.000757427\eta^8 \\
-0.000298733\eta^9 + 0.000148427\eta^{10} - 0.000075264\eta^{11} \\
+0.0000345801\eta^{12} - \cdots \\
\chi[\eta] = 1 - 0.619069\eta + 0.157618\eta^2 - 0.0277013\eta^3 + 0.0297124\eta^4 \\
-0.0212832\eta^5 + 0.00917625\eta^6 - 0.00430325\eta^7 + 0.00259966\eta^8 \\
-0.00144392\eta^9 + 0.000719194\eta^{10} - 0.000368404\eta^{11} + 0.000196706\eta^{12} - \cdots
\end{array}\right\}$$

4. Results and discussion

In this study, DTM and the numerical solutions are obtained for all the governing equations and it is intended to analyze an influence of wide range of parameters on the velocity of the fluid, temperature profile, concentration of nano particles and motile gyrotactic microorganisms profile through graphs.

Analysis of velocity profile: Figures 2-5 are the velocity profile of the Casson fluid for values of a range of parameters. In the Figure 2, velocity of the Casson nanofluid stream increases due to applied magnetic field normal to the surface for growing values of β. Figure 3 displays, decrease in the velocity of the fluid for the increase in the bioconvection Rayleigh number R_b. This is due to effect of buoyancy, the velocity of the Casson fluid decrease near the boundary of the stretching sheet. Figure 4 shows the decreasing behavior of velocity of the fluid for different values of bioconvection ratio parameter (N_r), This effect is due to the fact that the increases in the concentration of the nanoparticles near the sheet. In Figure 5, it is found that the velocity of Casson fluid decreases with growing values of Prandtl number. The effect of growing values of the Prandtl number leads to increase in the concentration of nanoparticles hence the viscosity coefficient increase, therefore the velocity of the fluid decreases.

Temperature profile: Figures 6-9 are the temperature profiles for different values of non dimensional parameters and we found that raise of warm for the growing of values buoyancy Rayleigh number, Brownian motion parameter, thermophoresis parameter and decrease of temperature for the raising values of slipping parameter. In Figure 6 boost of slipping parameter λ leads to decrease in the temperature profile due to the reason is that growing stretching parameter values for the Casson fluid leads to decrease in the pressure and temperature. Figure 7 shows the enhancement of temperature profile for the boost of Brownian motion parameter because the increase in Brownian motion parameter increases the collision frequency of the particles in the fluid and hence heat flow grows. The influence of thermophoresis parameter N_t is shown in Figure 8. Temperature profile of the fluid increases with increase value of N_t.The reason is that, nano particles exhibits different values of N_t. The influence of buoyancy Rayleigh number on the temperature is displayed in the Figure 9. Temerature profile raise with the raise of buoyancy Rayleigh parameter R_b reason behind this is the velocity of the fluid grows because microorganisms drag the fluid so that the concentration of the particle in the fluid decreases, therefore the probability of random motion increases and leads to temperature enhancement.

Concentration of nano particles and microorganisms: Concentration profiles of nano particles and microorganisms are described in the Figures 10 to 13. In Figure 10, it is clear that the concentration of nanoparticles decreases for the increasing values of stretching parameter λ. This is due to the movement of nano particles away from sheet and hence the Brownian motion coefficient decreases. Figure 11 displays the decrease of the nanoparticles concentration for the growing values of thermophoresis non dimensional parameter. The reason is that, the increase in the thermophoresis parameter directly effects on the movement of nanoparticles towards the cold reason hence the concentration of the nanoparticles in the fluid decreases. Figure 12 explains the relation between the concentration of nanoparticles and the buoyancy ratio parameter. Here we can observe that, the concentration of nanoparticles increases for the increasing values of buoyancy ratio parameter N_r. Due to increasing values of the buoyancy parameter, there will be upward force exerted by the fluid that opposes weight of object in a fluid pressure increase with depth as a result of the weight of the overlaying fluid, thus the pressure at the bottom of a fluid is greater than at the top, hence the pressure difference result in a net upward force and the nanoparticles sink to the bottom. Figure 13 describes the increase of the concentration of the nanoparticles for the enhanced values of the magnetic field. When the magnetic field is applied to the nanoparticles leads to the formation of magnetic dipoles along the direction of the magnetic field, therefore the particle created chain link clusters along the direction of the applied magnetic field so that the concentration of the particles increases. Figure 14 explains about the relation between the concentration of microorganisms and the Schmidt number. The density of the microorganisms decreases with the raising values of the Schmidt number. As Schmidt number increases dynamic viscosity of the fluid increases but the density and mass diffusivity of the nanoparticles decreases hence concentration of the microorganisms decreases. Figure 15 represents the behavior of the thermophoresis parameter on the density of the microorganisms. Concentration of microorganisms decreases for the increasing values of thermophoresis parameter. Thermophoresis parameter is most commonly used to analyze temperature effect on the mobile particles, since the moving particles have different capacity of temperature gradient, therefore when temperature of moving particles increases the concentration of microorganisms falls down. Figure 16 relates the concentration of microorganisms and the bioconvection constant. We can observe the decrease of the concentration of microorganisms for the increasing values of the bioconvection constant, it is because of the fact that, the bioconvection occurs when the mobile organisms falls down, since the density of the microorganisms are denser than the fluid therefore the accumulation of the microorganisms will not occur. Figure 16 reveals the relation of motile microorganisms and the non dimensional Peclet number. Increase in the Peclet number reduces the concentration of the microorganisms. The Peclet number helps to reduce the thickness of the boundary layer.

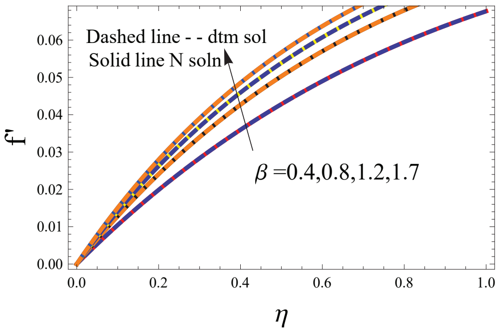

FIGURE 2. Velocity Profile of the fluids for β

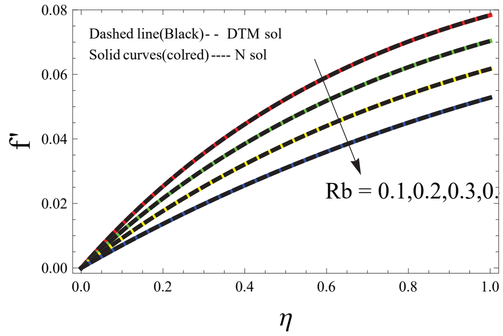

FIGURE 3. Velocity profile of the fluids for different values of the parameter R_b

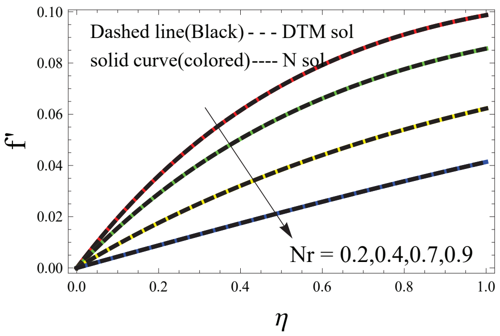

FIGURE 4. Velocity profile of the fluids for different values of N_r

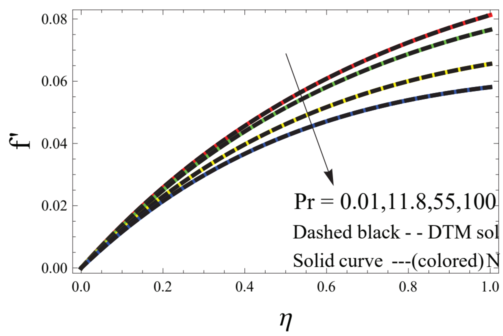

FIGURE 5. Velocity profile of the fluids for different values of P_r

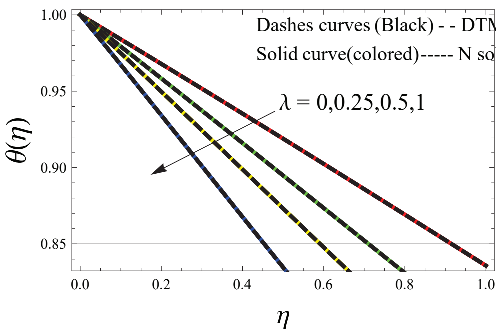

FIGURE 6. Temperature profile for different values of the slip parameter λ

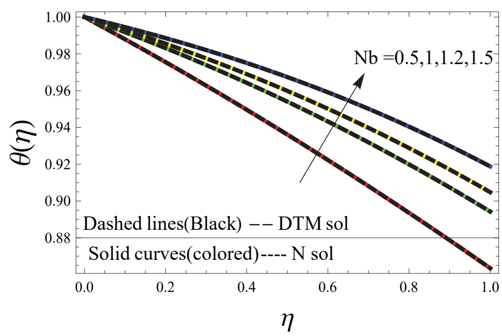

FIGURE 7. Temperature profile for different values of thermophoresis parameter N_b

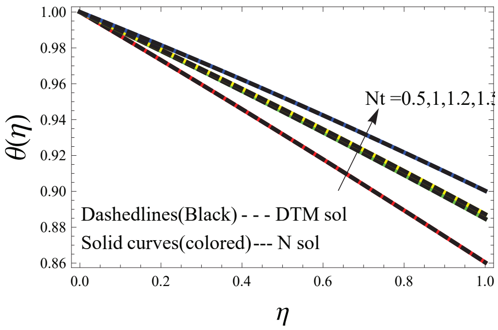

FIGURE 8. Temperature profile for different values of thermophoresis parameter N_t

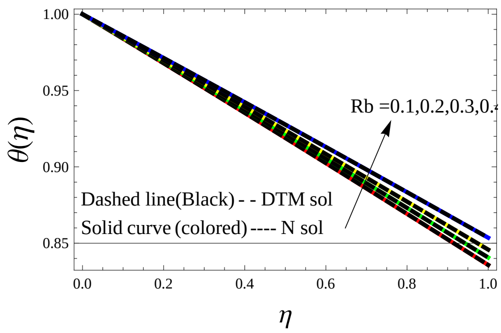

FIGURE 9. Temperature profile for different values of R_b

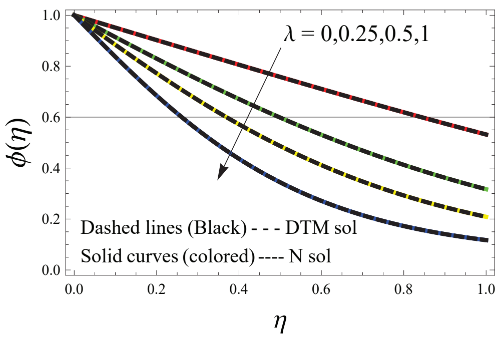

FIGURE 10. Concentration profile of the nano particles for different values of Slip parameters λ

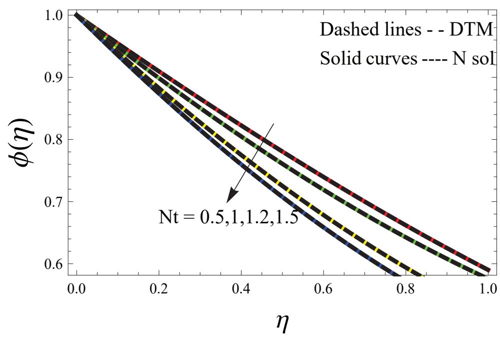

FIGURE 11. Concentration profile of the nano particles for different values of thermophoresis parameter N_t

5. CONCLUSION

The current study addresses semi analytic and numerical solution for bioconvection in MHD boundary layer flow, heat exchange of nanofluids and gyrotactic microorganisms over a linear stretching sheet. A Taylor's series solution is obtained for momentum, energy, concentration equation of nanofluids and density of gyrotactic microorganism's equations using Differential Transform method and explained graphically using DTM solution and numerical solution. Non-dimensional parameters are varied and the solutions are displayed through

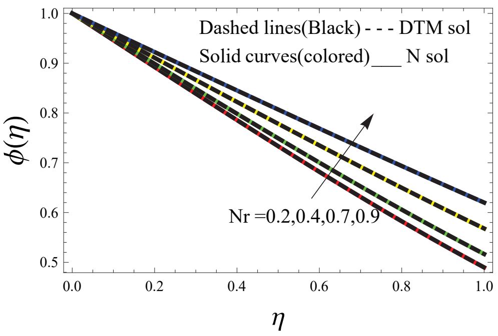

FIGURE 12. Concentration profile of the nano particles for different values of parameter N_r

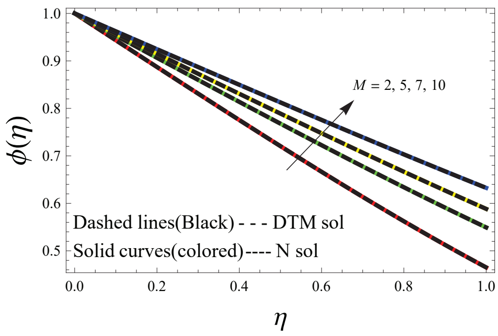

FIGURE 13. Concentration profile of the nano particles for different values of parameter M

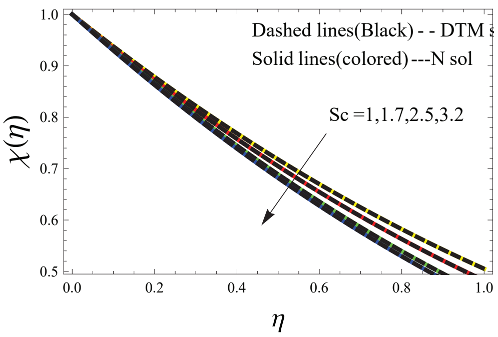

FIGURE 14. Concentration of microorganisms for different values of Schmidth number S_c

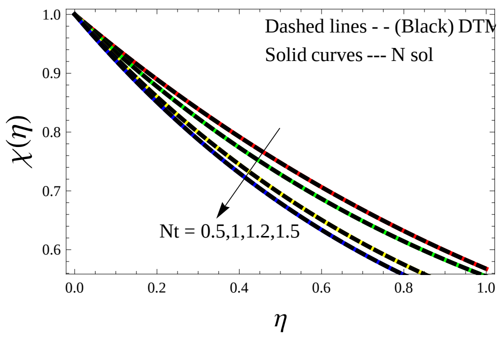

FIGURE 15. Effect of thermophoresis parameter on the density of gyrotactic microorganisms N_t

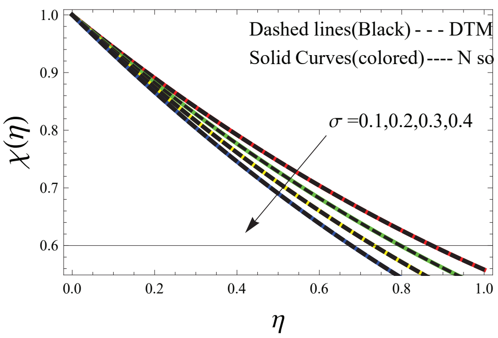

FIGURE 16. Concentration of microorganisms with different values of σ

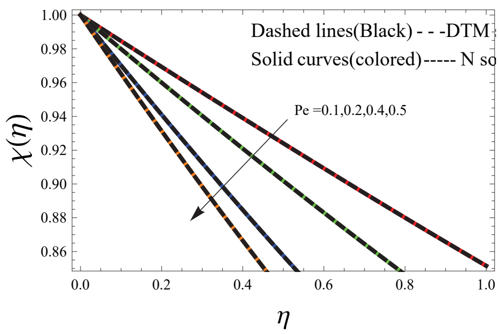

FIGURE 17. Effect of P_e on the density of gyrotactic microorganisms

graphs. Interesting features of the velocity of the stream and heat exchange are described.

(i) Velocity of Casson fluid increases against Casson fluid parameter is due to applied magnetic field normal to the flow.
(ii) Enhanced magnitude Schmidt number decreases the concentration of the mobile microorganisms.
(iii) Nano particles possesses unlike values of Brownian motion parameter(Nb) and thermophoresis parameter(Nt), hence concentration of nanofluids and heat flow increases for growing values of Nb and Nt.
(iv) Concentration of microorganisms reduces for an enhanced magnitude of Peclet parameter due to sensitivity of microorganisms and thinning of boundary layer thickness.
(v) Increase in stretching parameter influences friction near the surface of the sheet and hence temperature reduces.

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