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EDGE TO EDGE *D*-DISTANCE AND RADIUS AND DIAMETER OF GRAPHS

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ABSTRACT. In a graph the distance between nodes can be extended to distance between a node and an edge, an edge to an edge. In this paper, we extend the D-distance ([2]) concept between edges and study the D-radius and D-diameter of few families of graphs using this distance.

1. INTRODUCTION

In study of graphs, distance concept plays a vital role. This is used for verifying isomorphism, operations on graphs, problems involving hamiltonicity, connectivity etc. Symmetry in graphs depends on distance concept.

In a previous article [2], the authors have proposed the D-distance concept between nodes of a graph. Using this concept, we defined edge-to-edge D-diatance, in an obvious way.

All through in this paper, all graphs we consider are *connected*.

In a graph G, let $p = \alpha\beta$ and $q = \gamma\delta$ be two edges or lines. Then the *edge-to-edge D-distance* is defined as $d^D(p,q) = \min\left\{d^D(\alpha,\gamma), d^D(\alpha,\delta), d^D(\beta,\gamma), d^D(\beta,\delta)\right\}$.

For any edge f in a graph G, the *edge-to-edge D-eccentricity* $e_3^D(f)$ is define as $e_3^D(f) = \max\left\{d^D(f,e) : e \in E(G)\right\}$.

The *D-radius* a graph G, using this distance, is defined as the minimum edge-to-edge D-eccentricity among all edges of G. This is denoted by $r_3^D(G)$.

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The D-*diameter* of a graph G, using this distance, is defined as the maximum edge-to-edge D-eccentricity among all edges of G and is denoted by $d_3^D(G)$.

Other valuable references are [1, 3–6].

2. The Results

In this section, using the above distance, we obtain the radius and diameter of some classes of graphs. First complete graphs will be dealt.

Theorem 2.1. *The radius and diameter of K_m, w.r.t. edge to edge D-distance, are* $r_3^D(K_m) = d_3^D(K_m) = 2m - 1$.

Proof. In the complete graph K_m, if two lines are adjacent then the D-distance between them is 0 and it is $2m - 1$ if they are not adjacent. Thus D-eccentricity of each line is $2m - 1$. Therefore $r_3^D(K_m) = d_3^D(K_m) = 2m - 1$. □

Next we consider the complete bipartite graph.

Theorem 2.2. *We have $r_3^D(K_{m,n}) = d_3^D(K_{m,n}) = m + n + 1$ for the complete bipartite graph $K_{m,n}$.*

Proof. In $K_{m,n}$ there are $m+n$ nodes and mn lines. The D-distance between two lines is 0, if they are adjacent and is $m+n+1$, if they are not adjacent. Thus the D-eccentricity of every line is $m+n+1$. Hence the D-radius and D-diameter of $K_{m,n}$ are $m+n+1$. □

Next we deal with star graph on $n + 1$ nodes.

Theorem 2.3. *In star graph, $St_{n,1}$, we have $r_3^D(st_{n,1}) = d_3^D(st_{n,1}) = 0$.*

Proof. In $St_{n,1}$, one node is adjacent to all others. Hence every line has one common node. Hence the D-distance between lines is 0. So the $e_3^D(e)$ is 0 for all lines. Therefore $r_3^D(St_{n,1})$ and $d_3^D(St_{n,1})$ is 0. □

Next we consider the cyclic graph.

Theorem 2.4. *For cycle graph, C_n, we have*

$$r_3^D(C_n) = d_3^D(C_n) = \begin{cases} \frac{3n-5}{2} & \text{if } n = 2m+1 \text{ is odd} \\ \frac{3n-2}{2} & \text{if } n = 2m \text{ is even.} \end{cases}$$

Proof. In C_n, degree of nodes is 2. Thus the distances between various lines are as follows.

Case (1): Suppose n is odd.

In this case the D-distances are as follows:

TABLE 1. Cyclic Graph, n odd

	e_1	e_2	$\cdots$	$e_{\frac{n-1}{2}}$	$e_{\frac{n+1}{2}}$	$e_{\frac{n+3}{2}}$	$e_{\frac{n+5}{2}}$	$\cdots$	e_n
e_1	0	0	$\cdots$	$\frac{3n-11}{2}$	$\frac{3n-5}{2}$	$\frac{3n-5}{2}$	$\frac{3n-11}{2}$	$\cdots$	0
e_2	0	0	$\cdots$	$\frac{3n-17}{2}$	$\frac{3n-11}{2}$	$\frac{3n-5}{2}$	$\frac{3n-5}{2}$	$\cdots$	5
e_3	5	0	$\cdots$	$\frac{3n-23}{2}$	$\frac{3n-17}{2}$	$\frac{3n-11}{2}$	$\frac{3n-5}{2}$	$\cdots$	8
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$e_{\frac{n-1}{2}}$	$\frac{3n-11}{2}$	$\frac{3n-17}{2}$	$\cdots$	0	0	5	8	$\cdots$	$\frac{3n-5}{2}$
$e_{\frac{n+1}{2}}$	$\frac{3n-5}{2}$	$\frac{3n-11}{2}$	$\cdots$	0	0	0	5	$\cdots$	$\frac{3n-5}{2}$
$e_{\frac{n+3}{2}}$	$\frac{3n-5}{2}$	$\frac{3n-5}{2}$	$\cdots$	5	0	0	0	$\cdots$	$\frac{3n-11}{2}$
$e_{\frac{n+5}{2}}$	$\frac{3n-11}{2}$	$\frac{3n-5}{2}$	$\cdots$	8	5	0	0	$\cdots$	$\frac{3n-17}{2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
e_{n-1}	5	0	$\cdots$	$\frac{3n-5}{2}$	$\frac{3n-11}{2}$	$\frac{3n-17}{2}$	$\frac{3n-23}{2}$	$\cdots$	0
e_n	0	5	$\cdots$	$\frac{3n-5}{2}$	$\frac{3n-5}{2}$	$\frac{3n-11}{2}$	$\frac{3n-17}{2}$	$\cdots$	0

From the table 1, we can see that the D-eccentricity of every line is $\frac{3n-5}{2}$. Hence $r_3^D(C_n)$ and $d_3^D(C_n)$ are $\frac{3n-5}{2}$ when n is odd.

Case (2): Suppose n is even.

Like above we can prove that $r_3^D(C_n)$ and $d_3^D(C_n)$ are $\frac{3n-2}{2}$ when n is even. Thus

$$r_3^D(C_n) = d_3^D(C_n) = \begin{cases} \frac{3n-5}{2} & \text{if } n = 2m+1 \text{ is odd} \\ \frac{3n-2}{2} & \text{if } n = 2m \text{ is even.} \end{cases}$$

□

Theorem 2.5. *For path graph P_n, we have*

$$r_3^D(P_n) = \begin{cases} \frac{3n-8}{2} & \text{if } n = 2m \text{ is even} \\ \frac{3n-5}{2} & \text{if } n = 2m+1 \text{ is odd} \end{cases},$$

and

$$d_3^D(P_n) = 3n - 7 \ \forall \ n > 3.$$

Further $r_3^D(P_3) = d_3^D(P_3) = 0$.

Proof. In the path graph on n nodes degree of each end node is 1 and remaining $n-2$ nodes is 2. The D-distances between lines are given below.

For $n = 2$, $d^D(e_1, e_1) = 0$ because we have only one line. Thus the D-eccentricity of each line is zero. Hence $r_3^D(P_2)$ and $d_3^D(P_2)$ are zero.

For $n = 3$, $d^D(e_1, e_2) = 0$ because we have one common node between the lines e_1 and e_2. Thus the D-eccentricity of P_3 is zero. Hence $r_3^D(P_3)$ and $d_3^D(P_3)$ are zero.

Next consider P_n with $n \geq 4$. We consider n even and odd cases separately.

Case (1): Suppose n is even.

In this case the D-distances are

TABLE 2. Path graph, n even

	e_1	e_2	$\cdots$	$e_{\frac{n}{2}-1}$	$e_{\frac{n}{2}}$	$e_{\frac{n}{2}+1}$	$\cdots$	e_{n-1}
e_1	0	0	$\cdots$	$\frac{3n-14}{2}$	$\frac{3n-8}{2}$	$\frac{3n-2}{2}$	$\cdots$	$3n-7$
e_2	0	0	$\cdots$	$\frac{3n-20}{2}$	$\frac{3n-14}{2}$	$\frac{3n-8}{2}$	$\cdots$	$3n-10$
e_3	5	0	$\cdots$	$\frac{3n-26}{2}$	$\frac{3n-20}{2}$	$\frac{3n-14}{2}$	$\cdots$	$3n-13$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$e_{\frac{n}{2}-1}$	$\frac{3n-14}{2}$	$\frac{3n-20}{2}$	$\cdots$	0	0	5	$\cdots$	$\frac{3n+2}{2}$
$e_{\frac{n}{2}}$	$\frac{3n-8}{2}$	$\frac{3n-14}{2}$	$\cdots$	0	0	0	$\cdots$	$\frac{3n-8}{2}$
$e_{\frac{n}{2}+1}$	$\frac{3n-2}{2}$	$\frac{3n-8}{2}$	$\cdots$	5	0	0	$\cdots$	$\frac{3n-14}{2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
e_{n-2}	$3n-10$	$3n-13$	$\cdots$	$\frac{3n-8}{2}$	$\frac{3n-14}{2}$	$\frac{3n-20}{2}$	$\cdots$	0
e_{n-1}	$3n-7$	$3n-10$	$\cdots$	$\frac{3n+2}{2}$	$\frac{3n-8}{2}$	$\frac{3n-14}{2}$	$\cdots$	0

From the table 2, we can see that the D-eccentricities are $\{3n-7,\ 3n-10,\ 3n-13,\ \cdots,\ \frac{3n+2}{2},\ \frac{3n-8}{2},\ \frac{3n-2}{2},\ \cdots,\ 3n-10,\ 3n-7\}$. Thus the minimum D-eccentricity of P_n is $\frac{3n-8}{2}$ and the maximum D-eccentricity of P_n is $3n-7$, if n is even.

Case (2): suppose n is odd.

Like above, we can prove that $r_3^D(P_n) = \frac{3n-5}{2}$ and $d_3^D(P_n) = 3n - 7 \; \forall \; n \geq 4$. Thus

$$r_3^D(P_n) = \begin{cases} \frac{3n-8}{2} & \text{if } n \text{ is even} \\ \frac{3n-5}{2} & \text{if } n \text{ is odd} \end{cases},$$

and

$$d_3^D(P_n) = 3n - 7 \; \forall \; n > 3.$$

□

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