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VARIOUS DEFUZZIFICATION METHODS FOR TRAPEZOIDAL DENSE FUZZY SETS

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ABSTRACT. This present study extends the dense fuzzy sets (FS) into the Trapezoidal Dense Fuzzy Sets (TpDFS) and it deals with various defuzzification methods based on α-cuts, area of compensation and graded mean integration method Here, Cauchy's sequence has been employed for all the defuzzification methods.

1. INTRODUCTION

Fuzzy set was introduced to deal with the vagueness and uncertainties [15]. To illustrate the actual nature of the fuzzy sets, Zimmermann [16] defined algebraic operations for the supports of fuzzy sets, but it was found to be inaccurate and rather unworkable since it involved too many calculations. To solve this inexact solution of Zimmermann, Dubois and Prade [9] introduced algebraic operations for infinite supports of fuzzy sets where the operations are found to be more rapid and exact. Kaufmann and Gupta [10]integrated the fuzzy set on its arithmetic operations and gave its arithmetic theory with its application. Linguistic variables are used as the input parameter in the decision making problem. To represent the value for the linguistic variables, fuzzy number was introduced. To improve the application of the arithmetic operator, Chen [5] established the function principle for the arithmetic operations of generalized

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fuzzy numbers (GFS). Dat et al., [7] developed a new arithmetic operation between generalized trapezoidal fuzzy numbers (GTrFN) as Chen's function principal caused the loss of information and it did not give exact results. The Vertex Method was introduced by Dong and Shah [12]to evaluate the value of the functions of interval variable and fuzzy variable. Ban and Coroianu [1] characterized the set of real parameters associated with the fuzzy number and it was found that for any given fuzzy number there existed at least one trapezoidal fuzzy number (TrFN) which preserved a fixed parameter.

A method for the construction of membership function without using α-cut was proposed by Chutia et al., [6]. Baruah [2] established that the concepts behind fuzziness and randomness and how they are linked. Generalized fuzzy sets, extended fuzzy sets, and generalized emended fuzzy sets were offered by Buckley [3]. To help in the ordering of fuzzy subsets, Yager [13] introduced the ordering of fuzzy subsets of unit interval which maps fuzzy number to real number with the help of function. Buckley and Chanas [4] proposed an easy and fast method to rank a fuzzy number. Wang, et.al [11] proposed a method on ranking to measure the L-R deviation degree of the fuzzy number. Yu, et.al [14] derived ranking fuzzy numbers that ensured full consideration for all information of fuzzy numbers. Dense fuzzy set (DFS) for the triangular dense fuzzy number (TDFN) and new defuzzification methods for triangular numbers were presented by De and Beg [8].

Thus, in this present study, a dense fuzzy set is considered and is extended to trapezoidal dense fuzzy sets based on trapezoidal fuzzy numbers. Then, different kinds of defuzzification are used namely defuzzification method (DM) based on α-cuts, DM based on area compensation and DM based on graded mean integration method. To get a converging point, Cauchy sequence is used for the DFS. This property helps to develop the TpDFS where the fuzziness is decreasing with time. This article is structured as follows: It begins with the introduction and followed by the preliminaries where some of the basic definitions are given in section two. And the third section viewed the various defuzzification methods through trapezoidal dense fuzzy sets and the existing technique. Finally, conclusion and the future directions are offered.

2. PRELIMINARIES

Definition 2.1. *Let $\tilde{A}$ be the fuzzy set (FS) where we consider the set of elements which are functions mapping from the natural number to the crisp number x. Now, as all the elements in the set $\tilde{A}$ converge to the crisp number x as n $\to \infty$ where n$\in \mathbb{N}$ is a natural number, then the considered fuzzy sets are called dense fuzzy set (DFS), i.e. $\mu_{\tilde{A}} : X \times \mathbb{N} \in [0,1]$. By representing it in the form of fuzzy set, it can be expressed as,*

$$\tilde{A} = \{((x,1), \mu_{\tilde{A}}(x,1)), ((x,2), \mu_{\tilde{A}}(x,2)), ((x,3), \mu_{\tilde{A}}(x,3)), \ldots, ((x,n), \mu_{\tilde{A}}(x,n))\}.$$

Definition 2.2. *Let $\tilde{A} = \{a,b,c,d\}$ be the trapezoidal fuzzy number with $a = bf_n, b = b, c = c, cg_n$ where f_n and g_n are sequence of function. If f_n and g_n are functions which converges to 1 as n $\to \infty$ where $n \in \mathbb{N}$ is a natural number, then the fuzzy set $\tilde{A} = \{a,b,c,d\} \to [b,c]$. Then set $\tilde{A} = \{a,b,c,d\}$ is known as trapezoidal dense fuzzy set (TpDFS). This TpDFS is clearly explained in the Example 1 and 2.*

Definition 2.3. *(TpDFS based on Cartesian product of two sets) Let $\tilde{A}$ be the TpFS having elements of $\mathbb{R} \times \mathbb{N}$, with the membership grade satisfying the functional relation $\mu_{\tilde{A}} : \mathbb{R} \times \mathbb{N} \to [0,1]$. Now as $n \to \infty$ if $\mu(x,n) \to 1$ for some $x \in \mathbb{R}$ is a real number and $n \in \mathbb{N}$ is a natural number, then we can call the set $\tilde{A}$ as TpDFS. Now, the set is called normalized trapezoidal dense fuzzy set (NTpDFS) when the membership degree attains its highest degree 1 for some n in $\mathbb{N}$.*

Example 1. *As per the definitions 2.1-2.2, TpDFS are as follows:*

$$\tilde{A} = \{a = bf_n, b = b, c = c, cg_n\}$$

$$\tilde{A} = \{b(1 - \frac{\rho}{1+n}), b, c, c(1 + \frac{\sigma}{1+n})\}, \qquad \text{for} \quad 0 < \rho,\ \sigma < 1. \tag{2.1}$$

The membership function for $0 \le n$ is defined as follows:

$$\begin{cases} 0 & \text{if } x < b(1-\frac{\rho}{1+n}) \\ \{\frac{x - b(1-\frac{\rho}{1+n})}{\frac{\rho b}{1+n}}\} & \text{if } b(1-\frac{\rho}{1+n}) \le x \le b \\ 1 & \text{if } b \le c \\ \{\frac{c(1+\frac{\sigma}{1+n}) - x}{\frac{\sigma c}{1+n}}\} & \text{if } c \le x \le c(1+\frac{\sigma}{1+n}) \\ 0 & \text{if } x > c(1+\frac{\sigma}{1+n}) \end{cases}$$

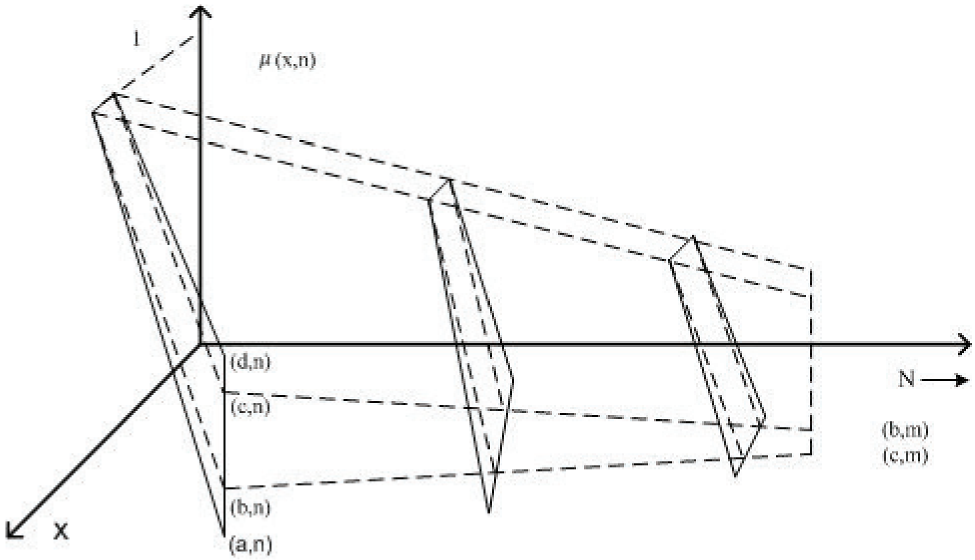

FIGURE 1. Membership function of NTpDFS.

Example 2. *A fuzzy set* $\tilde{A}$ *is considered and let* $\tilde{A} = \{0.2, 0.4, 0.6, 0.8\}$. *Let us check whether the given set is a dense fuzzy set (DFS).*

Solution: *The DFS can be written as* $\tilde{A} = \{b(1 - \frac{\rho}{1+n}), b, c, c(1 + \frac{\sigma}{1+n})\}$,

$$n = 1; (0.34, 0.4, 0.6, 0.75), \qquad n = 4; (0.376, 0.4, 0.6, 0.66),$$

$$n = 9; (0.388, 0.4, 0.6, 0.63), \qquad n = 14; (0.392, 0.4, 0.6, 0.62).$$

This will proceed as $n \to \infty$. *From these values, TpFS* $\tilde{A}$ *is formed then the fuzzy set* $\tilde{A} = \{a, b, c, d\}$ *converges to sets* $\{b\}$ *and* $\{c\}$. *Thus, all the points converge to* $\{0.4\}$*and* $\{0.6\}$. *Therefore the fuzzy set* $\tilde{A} = \{a, b, c, d\}$ *is a TpDFS.*

3. DEFUZZIFICATION

In this section, three important and known types of defuzzification methods (DM) are extended using TpDFS. Defuzzification is the method of changing a fuzzy quantity into a single crisp value with respect to a FS.

3.1. **Different Defuzzification method.**

The following are some of the important and known types of defuzzification methods:

(i) Defuzzification Method based on α-cuts.

(ii) Defuzzification based on area combination.
(iii) Defuzzification based on Graded Mean Integration Method (GMIM).

3.1.1. ***Defuzzification Method based on α-cuts:*** For this defuzzification method based on α-cuts, the old defuzzification for the TpFS is,

$$I_{old}(\tilde{A}) = \frac{1}{2}\int_0^1 [L^{-1}(\alpha,n) + R^{-1}(\alpha,n)]d\alpha = \frac{a+b+c+d}{4}.$$

Now for the new defuzzification based on α-cuts for TrDFS,

$$I_{new}(\tilde{A}) = \frac{1}{2N}\sum_{n=0}^{N}\int_0^1 [L^{-1}(\alpha,n) + R^{-1}(\alpha,n)]d\alpha. \tag{3.1}$$

L-R α-cut of a TpDFS $\tilde{A} = \mu(x,n)$ are $L^{-1}(\alpha,n)$ and $R^{-1}(\alpha,n)$.

$$\alpha \in \frac{x - b(1 - \frac{\rho}{1+n})}{\frac{\rho b}{1+n}}$$

$$L^{-1}(\alpha,n) = b[1 - \frac{\rho}{1+n} + \frac{\alpha\rho}{1+n}] \tag{3.2}$$

$$\alpha \in \frac{c(1 + \frac{\rho}{1+n}) - x}{\frac{\sigma c}{1+n}}$$

$$R^{-1}(\alpha,n) = c[1 + \frac{\sigma}{1+n} - \frac{\alpha\sigma}{1+n}]. \tag{3.3}$$

From (3.2) and (3.3),

$$L^{-1}(\alpha,n) + R^{-1}(\alpha,n) = b + c + \frac{\sigma c - \rho b}{1+n} + \frac{\alpha(\rho b - \sigma c)}{1+n}, \tag{3.4}$$

$$\int_0^1 [L^{-1}(\alpha,n) + R^{-1}(\alpha,n)]d\alpha = b + c + \frac{\sigma c - \rho b}{1+n} + \frac{(\rho b - \sigma c)}{2(1+n)}. \tag{3.5}$$

By substituting (3.5) in (3.1),

$$I(\tilde{A}) = \frac{b+c}{2} + \frac{\sigma c - \rho b}{N}\left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{N} + \frac{1}{1+N}\right) + \frac{(\rho b - \sigma c)}{4N}\left(\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{N} + \frac{1}{1+N}\right).$$

Obviously, $N \to \infty$, $I(\tilde{A}) \to \frac{b+c}{2}$.

For continuous case,

$$I(\tilde{A}) = \frac{1}{2N}\int_0^N \int_0^1 [L^{-1}(\alpha,n) + R^{-1}(\alpha,n)]d\alpha.dn,$$

$$I(\tilde{A}) = \frac{b+c}{2} + \frac{\sigma c - \rho b}{2N}Log(1+N) + \frac{(\rho b - \sigma c)}{4N}Log(1+N).$$

As $N \to \infty, I(\tilde{A}) \to \frac{b+c}{2}$ in both discrete and continuous case.

3.1.2. ***Defuzzification Method based on area combination:*** For GTpFN, the formula is defined as,

$$R_{old} = \frac{\int_a^b x\mu(x)dx + \int_b^c x\mu(x)dx + \int_c^d x\mu(x)dx}{\int_a^b \mu(x)dx + \int_b^c \mu(x)dx + \int_c^d \mu(x)dx} = \frac{c^2+d^2-a^2-b^2-ab+cd}{3(c+d-b-a)}.$$

For DFS, the new DM for the TpDFS is given by,

$$R_{new} = \sum_{n=0}^{N} \frac{\int_a^b x\mu(x,n)dx + \int_b^c x\mu(x,n)dx + \int_c^d x\mu(x,n)dx}{\int_a^b \mu(x,n)dx + \int_b^c \mu(x,n)dx + \int_c^d \mu(x,n)dx},$$

$$P = \sum_{n=0}^{N} \frac{[c^2+d^2-a^2-b^2-ab+cd]}{6},$$

$$Q = \sum_{n=0}^{N} \frac{[c+d-b-a]}{2}.$$

Substituting the corresponding value of a,b,c and d from (2.1) in P and Q and for R_{new} we get,

$$\frac{P}{Q} = \frac{N}{6}\left[[3c^2-3b^2] + \frac{(\sigma^2b^2-\rho^2b^2)}{N}\left(\frac{1}{1}+\frac{1}{4}+\cdots+\frac{1}{N^2}+\frac{1}{(1+N)^2}\right) + \frac{3\sigma c^2+3\rho b^2}{2}\left(\frac{1}{1}+\frac{1}{4}+\cdots+\frac{1}{N}+\frac{1}{(1+N)}\right)\right] \Bigg\backslash \frac{N}{2}\left(2(c-b) + \frac{b\rho + c\sigma}{N}\left(\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{N}+\frac{1}{1+N}\right)\right).$$

Obviously as $N \to \infty$, and so, $R_{new} \to \frac{b+c}{2}$.

3.1.3. ***Defuzzification Method based on Graded Mean Integration Method (GMIM):***. The graded mean method for the trapezoidal number which is defined as,

$$P_{old} = \frac{\frac{1}{2}\int_0^1 \alpha((L_A^{-1}(\alpha) + (R_A^{-1}(\alpha))d\alpha}{\int_0^1 \alpha d\alpha} = \frac{[a+2b+2c+d]}{6}$$

Now the graded mean integration method based on TpDFS is defined as,

$$P_{new} = \frac{\frac{1}{2}\sum_{n=0}^{N}\{\int_0^1 \alpha((L_A^{-1}(\alpha,n) + (R_A^{-1}(\alpha))d\alpha}{N\int_0^1 \alpha d\alpha}. \tag{3.6}$$

Now substitute the (3.4) in equation (3.6),

$$P_{new} = \left(\frac{b+c}{2} + \left[\frac{\sigma c - \rho b}{6N}\right]\sum_{n=0}^{N}\frac{1}{1+n}\right).$$

So, from the Cauchy sequence we get that as $N \to \infty$, and so, $P_{new} \to \frac{b+c}{2}$.

4. Conclusion

The basic purpose of the fuzzy set is to point towards a user-defined environment and to incorporate the measurement of exactness and the degree of truth for any particular event. In this piece of work, the dense fuzzy is extended to a trapezoidal dense fuzzy set (TpDFS) and it is explained with examples. To convert the fuzzy quantity into a crisp one, the defuzzification methods would be used. Therefore, three existing DM have been extended by considering the function of TpFN. The defuzzification methods based on TpFN are given for both TpFS and TpDFS as old and new respectively. In a nutshell, this article can be further extended to different shapes of the fuzzy numbers like pentagon, hexagon, heptagon, octagon, nonagon, decagon, etc., and also the defuzzification methods for Intuitionistic dense fuzzy sets can also be derived.

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