

MATHEMATICAL MODEL FOR POROUS INFLUENCE THROUGH MILD STENOSIS THROMBOSIS OVER DELIVERY OF OXYGEN AND NUTRIENTS ARTERIES

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ABSTRACT. In order to understand the irregular flow conditions of blood in a locally constricted blood vessel, analytical results are obtained for a porous effect on oscillatory blood flow that acts as a Newtonian flow. Compared to the radius of the unconstricted tube, the surface roughness is presumed to be cosine-shaped and the maximum height of the roughness is very small. The main focus of investigation of the porous effect on oscillatory arterial blood flow with mild stenosis, a mathematical model has been developed.

1. INTRODUCTION

Due to its physiological and importance and significance, blood flow issues through a stenosed artery are of growing interest. A number of blood vessel disorders are covered by a general term called "stenosis," which is one of the current health threats. Stenosis refers partially or completely to the occlusion of the arterial lumen due to the deposition of a fatty substance. It is the most wide spread arterial diseases of blood vessel. Many researchers have presented theoretical and experimental models of arterial stenosis. Stenosis refers to the occlusion of the arterial lumen partly or fully due to the deposition of fatty substance. The flow of blood through an artery depends on the heart's pumping action, giving rise to a gradient of pressure that creates an oscillatory flow

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in the blood vessel. Under a simple harmonic pressure gradient, Womersely (1955) studied the oscillatory motion of a viscous fluid in a rigid tube and examined the influence of frequency on the instantaneous flow rate. Barnes et al (1971) studied the non-Newtonian fluid flow behaviour through a straight, rigid circular cross-section tube. In their analysis, the theoretical study findings were shown to be in strong agreement with the predictions of the experiment. A numerical study of pulsatile flow through canine femoral arteries with lumen constriction was considered by Daly (1976) to be numerical. By numerically overcoming the fluid dynamics equations for flow fields, Back et al (1977) discussed the issue of pulsatile blood flow through coronary arteries with multiple non-obstructive' plaques. In a rigid tube with stenosis, Newman et al (1979) analyzed the oscillatory flow numerically. Doffin and Chagneau (1981) investigated the oscillating flow between a stenosis and a clot model. Kumar et al (2005) explored computational techniques for flow with porous effects in blood vessels. Oscillatory arterial blood flow with moderate stenosis have become studied extensively by Bhardwaj and Kanodia (2007). Pulsatile flow of couple stress fluid through a porous medium with periodic body acceleration and magnetic field has been discussed by Rathod and Tanveer (2009). In the presence of a magnetic field, Sanyal and Biswas (2010) analysed pulsatile blood flow through an axi-symmetric artery. We discuss the implications of Bhardwaj and Kanodia (2007) with porous medium influence in the present paper.

2. MATHEMATICAL MODEL

Let us presume, through an artery with mild constriction, that the blood is Newtonian, viscous and incompressible in character. It is assumed that the flow is unsteady and axially symmetric. It is assumed that the density and viscosity are constant. The artery, preceding and following the stenosis, does have a constant radius. Due to any abnormal growth in the lumen of the artery, it is often assumed that the constriction occurs symmetrically. The idealistic stenosis geometry (Fig.1) is given by

$$\frac{R(x)}{R_0} = 1 - \frac{\epsilon}{2R_0} \left[1 + \cos \frac{x\pi}{d} \right],$$

where R_0 is the radius of the normal artery, $R(x)$ is the radius of the artery in the stenotic region, $2d$ is the length of stenosis and ϵ is the maximum height of

the stenosis such that $\epsilon/R \ll 1$. The equation of motion governing the flow field

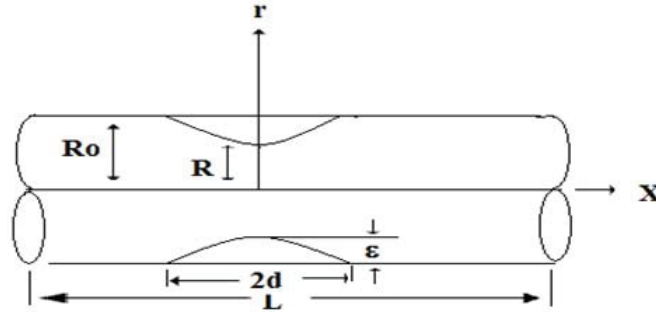


FIGURE 1. Geometrical Structure of Stenosis arteries

in the tube is

$$(2.1) \quad \rho \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 w}{\partial^2 t} + \frac{1}{r} \frac{\partial w}{\partial r} \right) - \frac{\mu}{k} w,$$

where p is the fluid pressure, ρ is the density and w is the velocity in the axial direction, μ is the viscosity, k is the permeability of the medium. The boundary conditions are

$$(2.2) \quad w = 0 \text{ on } r = R,$$

$$(2.3) \quad \frac{\partial w}{\partial r} = 0 \text{ on } r = 0.$$

3. NUMERICAL SOLUTION

In the section under the pressure gradient, which varies with time, the simple solution of the motion of a viscous fluid is obtained. Transformation is described by $y = r/R_0$ is implemented before continuing with the solution. The basic equation (2.1) becomes on using the boundary conditions (2.2) and (2.3):

$$\begin{aligned} \frac{\partial^2 w}{\partial^2 y} + \frac{1}{y} \frac{\partial w}{\partial y} - \rho \frac{R_0^2}{\mu} \frac{\partial w}{\partial t} - \left(\frac{R_0^2}{k} \right) w &= \frac{R_0^2}{\mu} \frac{\partial p}{\partial x} \\ w &= 0 \text{ on } y = \frac{R}{R_0}, \\ \frac{\partial w}{\partial y} &= 0 \end{aligned}$$

$$\begin{aligned}
 w(y, t) &= w(y) e^{i \omega t} - \frac{\partial p}{\partial x} = p e^{i \omega t} \\
 \frac{\partial^2 w}{\partial^2 y} + \frac{1}{y} \frac{\partial w}{\partial y} - \beta^2 w &= \frac{R_0^2}{\mu} p \\
 \beta^2 &= i \rho \frac{R_0^2 \omega}{\mu} + \frac{R_0^2}{k} \\
 (3.1) \quad W(y) &= \frac{P R_0^2}{\mu \beta^2} \left[1 - \frac{J_0(\beta y)}{J_0(\beta \frac{R}{R_0})} \right],
 \end{aligned}$$

where J_0 is the Bessel function of order zero. Then the resulting expression for the axial velocity in the tube is given by

$$W(r, t) = \frac{P R_0^2}{\mu \beta^2} \left[1 - \frac{J_0(\beta \frac{r}{R_0})}{J_0(\beta \frac{R}{R_0})} \right] \cdot e^{i \omega t}.$$

The volumetric flow rate Q as given below

$$(3.2) \quad Q = 2\pi \int_0^R w r \, dr,$$

which gives on integration, at once

$$\begin{aligned}
 Q &= \frac{\pi P R_0^4}{\mu \beta^2} \left(\frac{R}{R_0} \right) \left[\frac{R}{R_0} - \frac{2 J_0(\beta \frac{R}{R_0})}{J_0(\beta \frac{R}{R_0})} \right] \cdot e^{i \omega t} \\
 \frac{\partial p}{\partial x} &= -Q \mu \beta^2 \left[\frac{R}{R_0} - \frac{2 J_1(\beta \frac{R}{R_0})}{J_0(\beta \frac{R}{R_0})} \right]^{-1} \frac{1}{\pi R_0^4 \frac{R}{R_0}}.
 \end{aligned}$$

The shear stress at the wall $r = R$ is given by

$$\tau_R = \mu \left(\frac{\partial w}{\partial r} \right)_{r=R}.$$

Substituting expression (3.1) for w into above equation and using the relation (3.2) for Q , one obtains as

$$\frac{\tau_R}{Q} = \frac{-\mu \beta^2 J_1(\beta \frac{R}{R_0})}{\pi R_0^3 \left[\beta \left(\frac{R}{R_0} \right)^2 J_0(\beta \frac{R}{R_0}) - 2 \left(\frac{R}{R_0} \right) J_1(\beta \frac{R}{R_0}) \right]}.$$

If τ_N is normalized with steady flow solution given by

$$\tau_N = -\frac{R_0}{2} \left(\frac{\partial p}{\partial x} \right)_0$$

$$|\bar{\tau}| = \left| \frac{\tau_R}{\tau_n} \right| = \frac{(R(\frac{\partial p}{\partial x})_0) J_1(\beta \frac{R}{R_0}) \frac{R}{R_0} - \frac{2J_1(\beta \frac{R}{R_0})}{J_0(\beta \frac{r}{r_0})}}{-\frac{R_0}{2}(\frac{\partial p}{\partial x})_0 [\beta(\frac{R}{R_0})^2 J_0(\beta \frac{R}{R_0}) - 2(\frac{R}{R_0}) J_1(\beta \frac{R}{R_0})]}$$

$$|\bar{\tau}| = -2(\frac{R}{R_0}) \frac{J_1(\beta \frac{R}{R_0}) \frac{R}{R_0} - \frac{2J_1(\beta \frac{R}{R_0})}{J_0(\beta \frac{R}{R_0})}}{[\beta(\frac{R}{R_0})^2 J_0(\beta \frac{R}{R_0}) - 2(\frac{R}{R_0}) J_1(\beta \frac{R}{R_0})]}.$$

The resistance impedance to the flow is defined by in which the right hand side is known and can be obtained from equation (3.2)

$$Z = -(\frac{\partial p}{Q}).$$

4. RESULTS AND DISCUSSION

Axial velocity profiles in the tube have been seen in fig 2 and 3 with three graphs for $Ro = 1$, $P = 1$, $\mu = 0.2$, $\rho = 0.4$, $\omega = 4$, $\sigma = 1$, $\tau\omega = 0$ and different values for w/R_0 (height of stenosis) and k (Porous medium Parameter). From figures 2 and 3, it is observed that all axial velocity graphs decrease to $r = 1$, then each graph begins to decrease after velocity and tends to zero with the increase in y . It is also observed from Fig.2 and 3 that with the increase in k and \in / Ro , velocity increases, i.e. blood flow increases in the artery stenosis region.

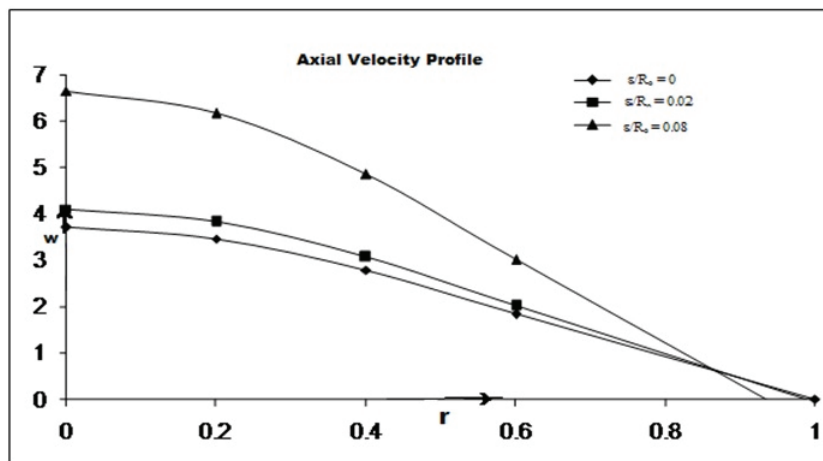


FIGURE 2. Different value k of Axial Velocity profile

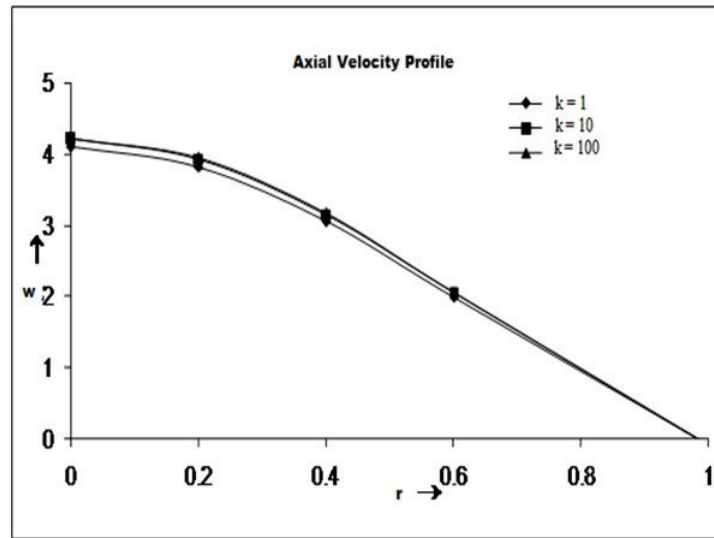


FIGURE 3. Different value k of Axial Velocity profile

REFERENCES

- [1] H.A. BARNERS, P. TOWNSEND, K. WALTERS: *On Pulsatile Flow of non-Newtonian Liquids*, Rheo. Acta., **10** (1971), 517-527.
- [2] L.D. BACK, J.R. RADBILL, D.W. CRAWFORD: *Analysis of Pulsatile, Viscous Blood Flow through Diseased Coronary Arteries of Man*, J. Biomechanics, **10**(5-6) (1977), 339-353.
- [3] K. BHARDWAJ, K.K. KANOJIA: *Oscillatory Arterial Blood Flow With Mild Stenosis*, Acta Ciencia Indica, XXXIII, No. 1, 71, (2007).
- [4] B.J. DALY: *A Numerical Study of Pulsatile Flow through Stenosed Canine Femoral Arteries*, J. Biomechanics, **9**(7) (1976), 465-475.
- [5] D.L. NEWMAN, N. WESTERHOF, P. SIPKEMA: *Modeling of Aortic Stenosis*, J. Biomechanics, **12** (3) (1979), 229-235.
- [6] V.P. RATHOD, S. TANVEER: *Pulsatile Flow of Couple Stress Fluid through a Porous Medium with Periodic Body Acceleration and Magnetic Field*, Bull. Malays. Math. Sci. Soc., **32**(2) (2009), 245-259.
- [7] D.C. SANYAL, A. BISWAS: *Pulsatile Motion of Blood through an Axi-Symmetric Artery in Presence of Magnetic Field*, Assam Univ. Journal of Science and Technology: Physical Sciences and Technology, **5**(2) (2010), 12-20.
- [8] J.R. WOMERSLEY: *Oscillatory Motions of a Viscous Liquids in a Thin-walled Elastic Tube*, Phil. Mag., **46**(373) (1955), 199-221.
- [9] A. KUMAR, C.L. VARSHNEY, G.C. SHARMA: *Computational technique for flow in blood vessels with porous effects*, International Journal of Applied Mathematics and Mechanics (English Edition) (Springer Publishing) China, **26**(1) (2005), 63-72.

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