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SPIKING ACTIVITY OF LIFH NEURON MODEL WITH VARIABLE INPUT STIMULUS

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ABSTRACT. Neuronal information processing occurs in term of spikes. A neuron can emits various kinds of spiking patterns based on the applied input stimulus. In this article, we study the spiking pattern of LIFH neuron model in the presence of four different kinds of applied input stimulus, namely, constant input stimulus, uniformly distributed input stimulus, Gaussian distributed input stimulus and stochastic input stimulus. Here, we notice the tonic and semi-tonic spiking pattern for Gaussian distributed input stimulus and stochastic input stimulus.

1. INTRODUCTION

Spike forms the building block for neuronal information processing. A neuron encodes information in term of spike sequences in two ways as, in term of inter-spike intervals or spikes generated in unit interval of time. The former method is known as temporal encoding techniques whereas the later scheme is known a rate coding scheme. Probability distribution of inter-spike intervals provides inter-spike-interval distribution (ISI distribution), which can further be characterized by coefficient of correlation [1, 3, 4, 6–9].

There are a number of neuron models has been suggested in literature depending on the biological behavior of neurons. Threshold based neuron models

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includes an additional threshold value as the maximum membrane potential after that the neuron fires a spike [2]. Biological neuron models are based on biological phenomenon of a neuron and it does not include the additional threshold value. Cable equations assume that membrane potential propagates in a neuron like wave or electricity in a cable. Integrate-and-fire neuron model and Leaky integrate-and-fire neuron models are the best example of threshold based neuron model. Hodgkin-Huxley model and Morris-Lecar neuron models are classical example for biological neuron model. IF and LIF models are the first choice for studying the neuronal information processing. Karmeshu et. al. [9] has studied the LIF model with the previous values of membrane potential and suggested a distributed delay framework. Choudhary et. al [3, 4] has investigated the LIF neuron model in DDF with the hypo-exponential distributed delay kernel and proposed the LIFH neuron model (Leaky integrate-and-fire neuron model with hypo-exponential distributed delay kernel) and noticed that the LIFH neuron model is capable to generate approximately all kind of ISI distribution patterns as noticed in experimental studies. Baghela et. al. [10] has investigated the information processing mechanism of LIFH neuron model in DDF with refractory time period. We investigate the spiking activity of LIFH neuron model in presence of four different kind of variable input stimulus.

The article is divided into 4 sections. After a brief introduction in Section 1, Section 2 deals with the formulation LIFH neuron model. Section 3 contains the simulation based study of the LIFH neuron model. The last section 4 contains the findings, result analysis and conclusion.

2. LIFH Neuron Model

The LIF neuron model is most popular threshold based neuron model, here rate of change of membrane potential can be defined as

$$\frac{dV}{dt} = -\beta V(t) + I(t).$$

Here β is known as membrane potential decay constant and $I(t)$ is applied input stimulus.

Incorporation of the previous values of membrane potential includes the distributed delay kernel function, which the results as

$$\frac{dV}{dt} = -\beta \int_0^t K(t-\tau)V(\tau)d\tau + I(t). \tag{2.1}$$

Here $K(t)$ is a kernel function which works as a memory. Choudhary et. al. [3,4] investigated the LIF neuron in DDF with $K(t)$ as hypo-exponentially distributed. The model proposed by Choudhary et. al. [3,4] is known as LIFH neuron model (Leaky integrate-and-fire neuron model with hypo-exponential distributed kernel. Sum of two independent Poisson process results into hypo-exponential distribution. Incorporation of hypo-exponential distributed kernel (2.2) function in Eq. 2.1 results the LIFH neuron model as [3,4]

$$f(x) = -\frac{\lambda_E \lambda_I}{\lambda_E - \lambda_I}(e^{-\lambda_E t} - e^{-\lambda_I t}). \tag{2.2}$$

Incorporation of $f(x)$ as a kernel function in Eq. (2.1) results the LIFH neuron model as [3,4]

$$\frac{dV}{dt} = -\frac{\beta \lambda_E \lambda_I}{\lambda_E - \lambda_I} \int_0^t (e^{-\lambda_E (t-\tau)} - e^{-\lambda_I (t-\tau)})V(\tau)d\tau + I(t).$$

Here, $V(t) = 0$ at $t = 0$ and λ_E and λ_I are excitatory membrane potential arrival rates and inhibitory membrane potentials arrival rates, respectively. The above equation is an integro-differential equation. It is too complex to easily obtain its analytical solution. Thus, the membrane potential evolution space can extended into so that we get a system of coupled differential equations, given as below

$$\begin{aligned} \frac{dV}{dt} &= -\frac{\beta \lambda_E \lambda_I}{\lambda_E - \lambda_I}(X - Y) + I(t), \\ \frac{dX(t)}{dt} &= -\lambda_E X + V, \\ \frac{dY(t)}{dt} &= -\lambda_I Y + V, \end{aligned} \tag{2.3}$$

with initial condition $V(t) = X(t) = Y(t)$ at $t = 0$. Choudhary *et al.* [2,3] has been studied this model with stochastic input stimulus and noticed that the model is capable to generate various kinds of spiking patterns and ISI distribution like uni-modal, bimodal, multi-modal with long tail behaviour.

In next section, we perform the simulation based study for LIFH neuron model given in Eq. (2.3) with four different kinds of input stimulus.

3. Simulation Based Study of LIFH Neuron Model

A neuron receives the input stimulus from other neurons or environment in various forms. The input stimulus can be categorized into four categories as (i) constant input (ii) input stimulus with certain constraints (iii) input stimulus without constraints and (iv) stochastic input stimulus. Here, we investigate the LIFH neuron model with four different kinds of input stimulus viz. constant input, uniformly distributed input, Gaussian distributed input and stochastic input stimulus. We apply the Euler-Maruyama numerical simulation technique to investigate the spiking activity of the LIFH neuron model defined in Eq. (2.3). In this technique the time interval $[0, t]$ is divided into equal length of n subinterval $[0, t_1]$, $[0, t_2]$, $\ldots$, $[t_(n-1), t_n]$ with step size $h = (t-0)/n$. If at time $t = t_i$, V_i is the membrane potential then in subsequent time $t = t_{i+1}$ the membrane potential can be computed as [8]

$$\begin{aligned} V_{i+1} &= V_i - (\frac{\beta \lambda_E \lambda_I}{\lambda_E - \lambda_I}(X_i - Y_i))h + I(t_i), \\ X_{i+1} &= X_i - (\lambda_E X_i - V_i)h, \\ \frac{dY(t)}{dt} &= Y_i - (\lambda_I Y_i - V_i)h. \end{aligned}$$

For $i = 1, 2, 3, \ldots, n$ with initial values $V_0 = 0$, $X_0 = 0$ and $Y_0 = 0$. We

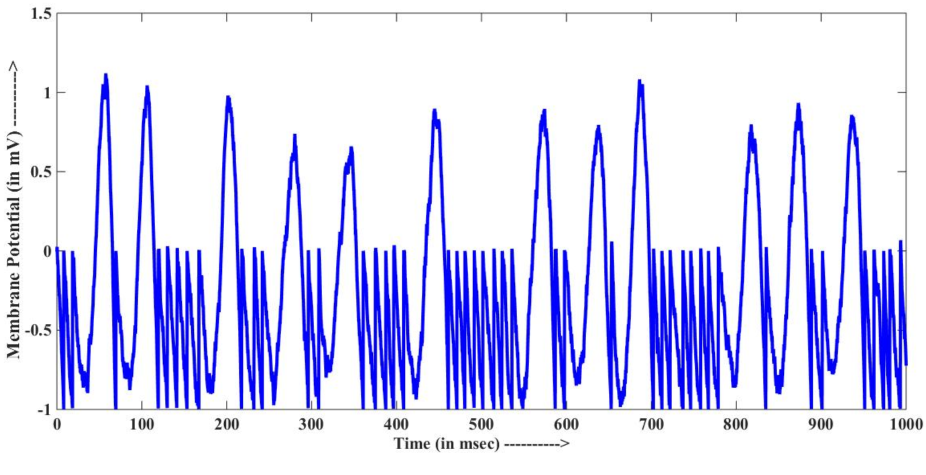

Figure 1. Spiking Activity of LIFH neuron with stochastic input and $\beta = 0.5$, $\lambda_E = 0.1$ $\lambda_I = 0.09$, Threshold $= 1$

investigate the spiking activity of LIFH neuron model with four different kind

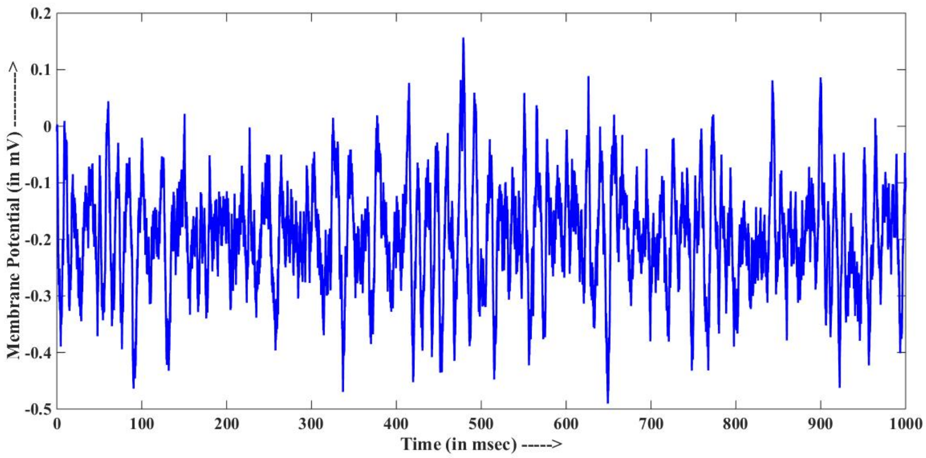

FIGURE 2. Spiking Activity of LIFH neuron with stochastic input and $\beta = 0.5$, $\lambda_E = 0.5$ $\lambda_I = 1$, Threshold $= 1$

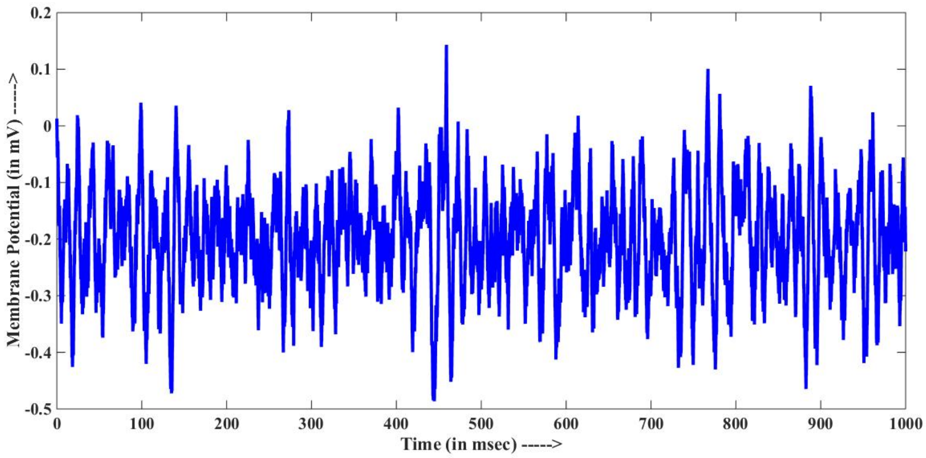

FIGURE 3. Spiking Activity of LIFH neuron with stochastic input and $\beta = 0.5$, $\lambda_E = 0.5$ $\lambda_I = 1$, Threshold $= 1$

of input stimulus with three set of parameters. Fig. 1 to Fig. 3 represents the spiking activity for stochastic input, Fig. 4 to Fig. 6 show the spiking activity for constant input stimulus, Fig. 7 to Fig. 9 illustrate the spiking activity for uniformly distributed input stimulus where as Fig. 10 to Fig. 12 represent the spiking activity of LIFH neuron model for Gaussian distributed input stimulus. The spiking activity shown in Fig. 1 to Fig. 3 are obtained for stochastic input stimulus with mean value of 0.1 and noise value of 0.05. Fig. 1 represents the spiking activity when excitatory and inhibitory membrane potential arrival rates are very small. This represents a tonic spiking pattern. Fig. 2 is obtained for higher excitatory arrival rate as compared with inhibitory arrival rates, whereas Fig. 3 is obtained for smaller excitatory arrival rates as compared wit inhibitory arrival rates. Here, we notice the noisy spiking patterns. Fig. 4 to Fig. 6 are spiking patterns of LIFH neuron model for constant input stimulus. Fig. 4 is obtained for 0.1 input stimulus value whereas Fig. 5 and Fig. 6 are obtained for 0.3 input stimulus value. In Fig. 4, excitatory and inhibitory rates are very small, thus, we notice the tonic piking pattern in LIFH neuron, where as for Fig. 5 and Fig. 6, it is excitatory and inhibitory arrival rates which cause to neuron not to fire,. Thus no spiking activity is noticed in Fig. 5 and Fig. 6. Fig.

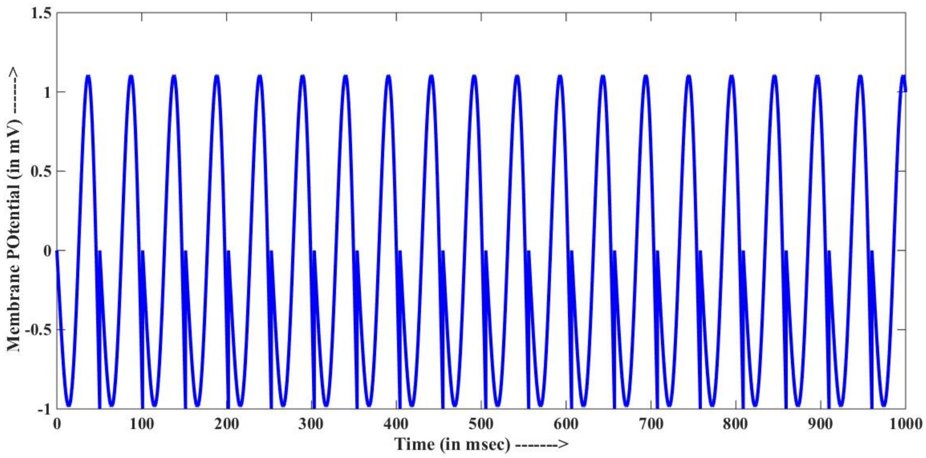

FIGURE 4. Spiking Activity of LIFH neuron with constant input and $\beta = 0.5$, $\lambda_E = 0.1$ $\lambda_I = 0.09$, Threshold $= 1$

7 to Fig. 9 show the spiking activity of LIFH neuron for uniformly distributed input stimulus in range 0 to 1. Here, all three spiking patterns are busty in nature and there is no effect of change in the excitatory and inhibitory arrival rates. Fig. 10 to Fig. 12 illustrates the spiking activity for LIFH neuron with Gaussian distributed input stimulus in range of 0 to 1. Fig. 10 represents the spiking activity for very small excitatory and inhibitory arrival rates and we notice partial tonic and partial noisy spiking patterns. Fig. 11 and Fig. 12 are obtained for larger values for excitatory and inhibitory arrival rates. Here

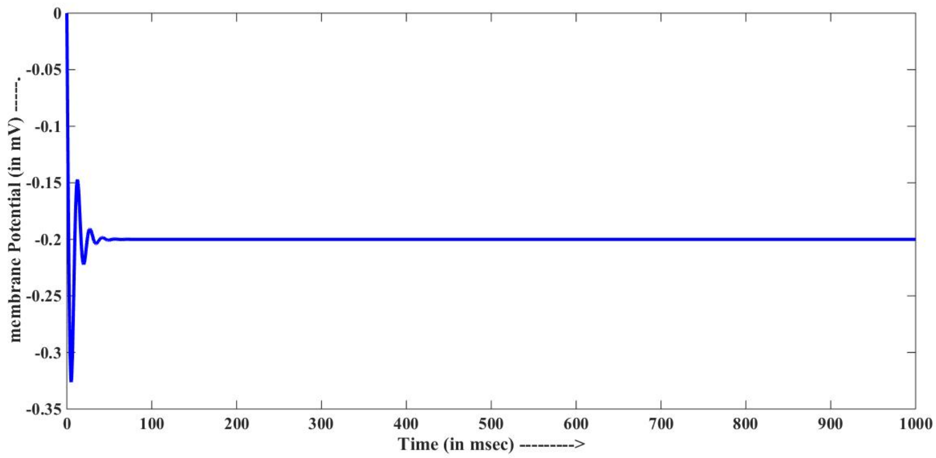

FIGURE 5. Spiking Activity of LIFH neuron with constant input and $\beta = 0.5$, $\lambda_E = 1$ $\lambda_I = 0.5$, Threshold $= 1$

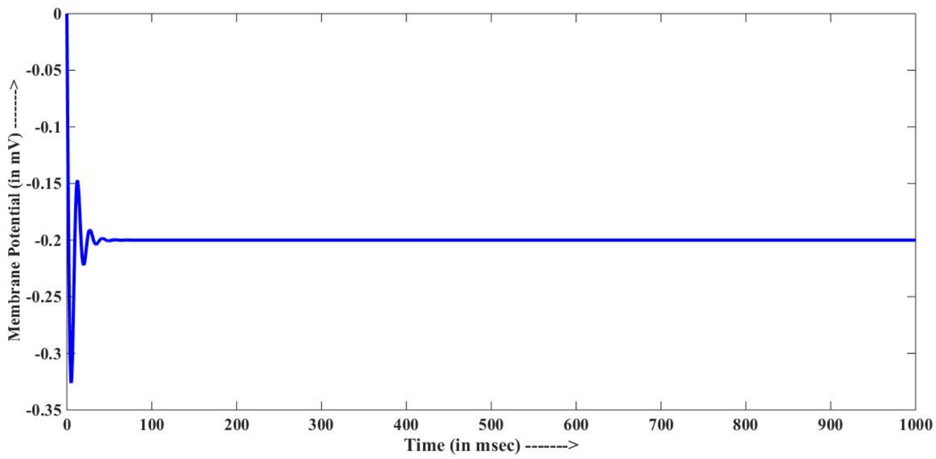

FIGURE 6. Spiking Activity of LIFH neuron with constant input and $\beta = 0.5$, $\lambda_E = 0.5$ $\lambda_I = 1$, Threshold $= 1$

we notice the noisy spiking activity. The noisy pattern in spiking patterns occurs due to the large fluctuation in excitatory and / or inhibitory membrane potential arrivals.

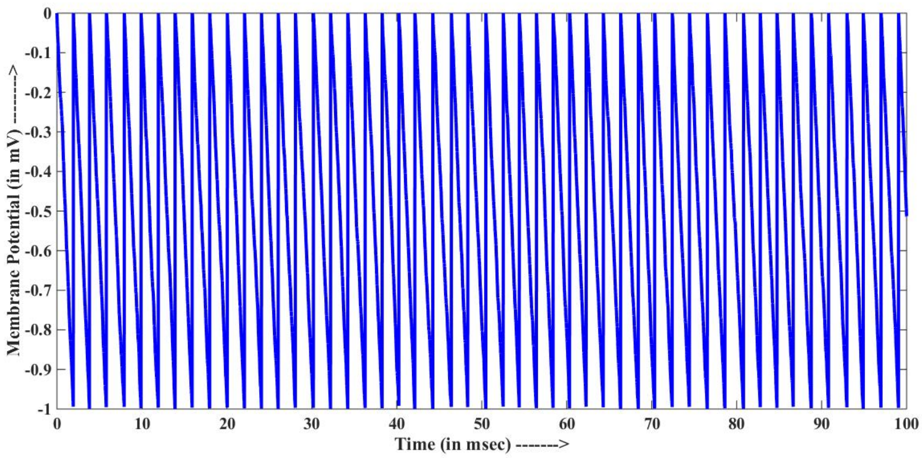

FIGURE 7. Spiking Activity of LIFH neuron with uniformly distributed input and $\beta = 0.5$, $\lambda_E = 0.1$ $\lambda_I = 0.09$, Threshold $= 1$

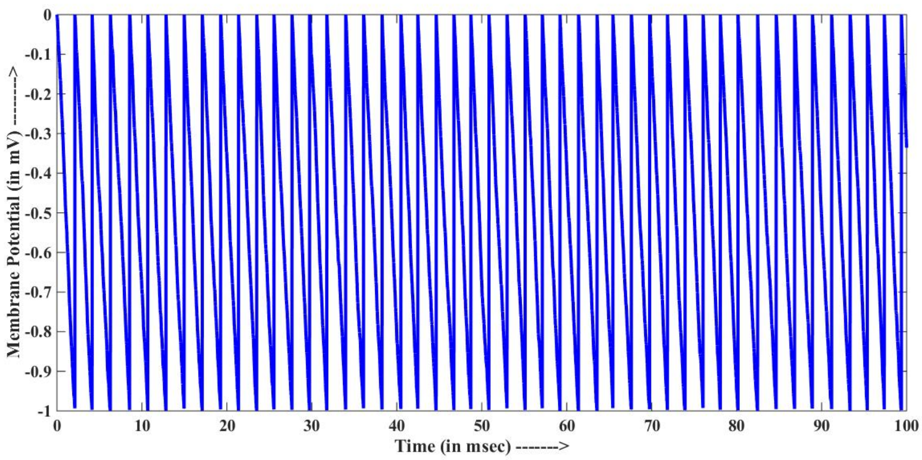

FIGURE 8. Spiking Activity of LIFH neuron with uniformly distributed input and $\beta = 0.5$, $\lambda_E = 1$ $\lambda_I = 0.5$, Threshold $= 1$

4. RESULT ANALYSIS AND CONCLUSION

Distributed delay framework provides a mechanism to study the current membrane potential evolution in the presence of its past values. LIFH neuron model has been derived from LIF model in DDF with hypo-exponential distributed delay kernel function. This model is capable to mimic nearly all kinds of ISI

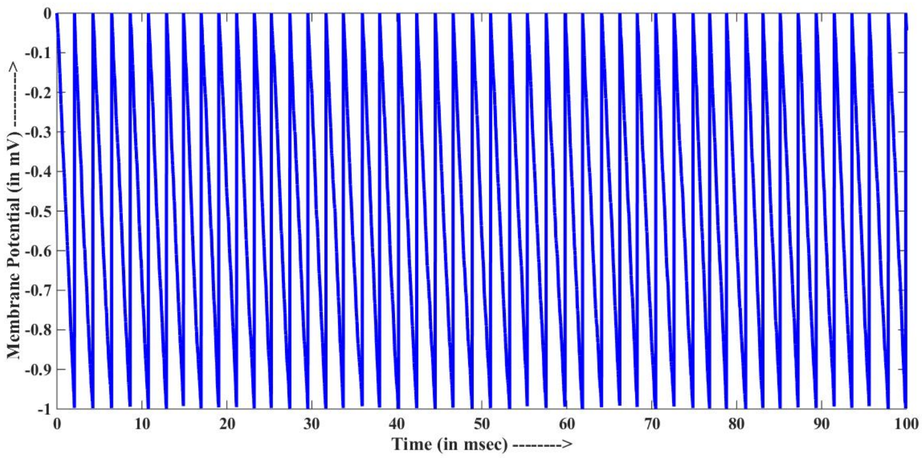

FIGURE 9. Spiking Activity of LIFH neuron with uniformly distributed input and $\beta = 0.5$, $\lambda_E = 0.5$ $\lambda_I = 1$, Threshold $= 1$

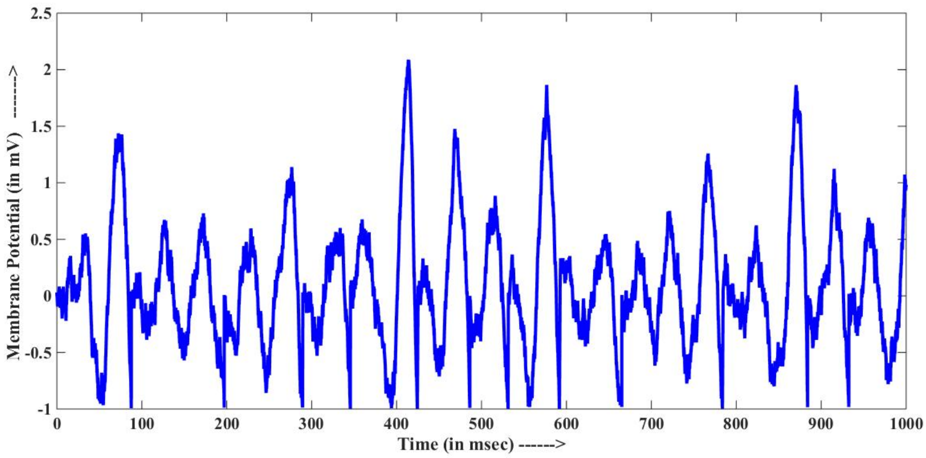

FIGURE 10. Spiking Activity of LIFH neuron with Gaussian distributed input and $\beta = 0.5$, $\lambda_E = 0.1$ $\lambda_I = 0.09$, Threshold $= 1$

distribution patterns as noticed during experimental studies. We have investigated the spiking pattern of LIFH neuron model in the presence of variable input stimulus. In presence of uniformly distributed input stimulus, the LIFH neuron model illustrates the busty spiking pattern. For constant input stimulus, the excitatory and inhibitory arrival rates are important and we need to find always a constant input values for which the LIFH neuron can emits spikes. For Gaussian

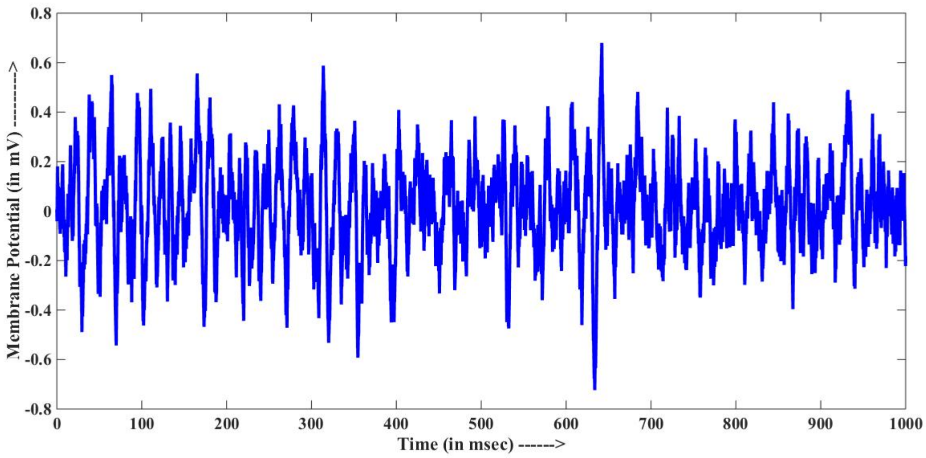

FIGURE 11. Spiking Activity of LIFH neuron with Gaussian distributed input and $\beta = 0.5$, $\lambda_E = 1$ $\lambda_I = 0.5$, Threshold $= 1$

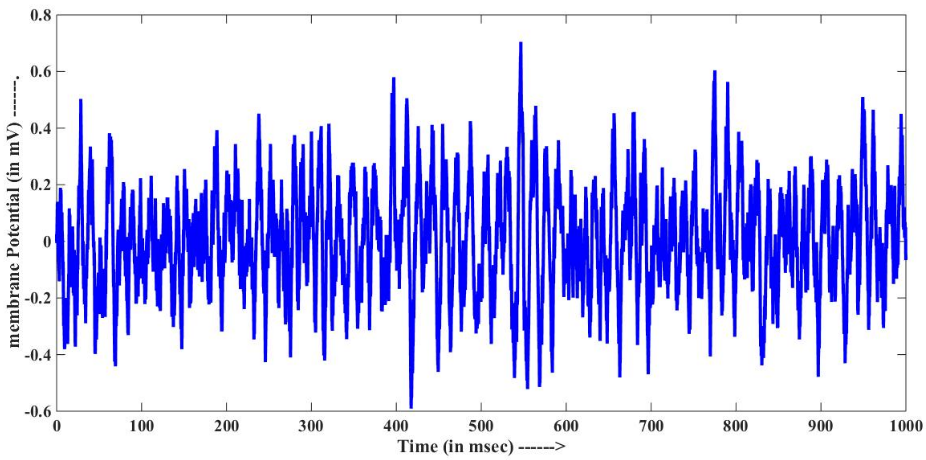

FIGURE 12. Spiking Activity of LIFH neuron with Gaussian distributed input and $\beta = 0.5$, $\lambda_E = 0.5$ $\lambda_I = 1$, Threshold $= 1$

distributed input stimulus and stochastic input stimulus, we notice tonic and semi tonic spiking patterns. It will interesting to investigate the ISI distribution patterns for LIFH neuron model in presence of such kind of variable input stimulus.

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