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MATHEMATICAL SOLUTION OF FINGERING PHENOMENON IN VERTICAL DOWNWARD DIRECTION THROUGH HETEROGENEOUS POROUS MEDIUM

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ABSTRACT. Present study explores the Fingering (Instability) phenomenon's mathematical model that ensues during the process of secondary oil recovery where two not miscible fluids (water and oil) flow within a heterogeneous porous medium as water is injected vertically downwards. Variational iteration method with proper initial and boundary conditions is being used to determine approximate analytic solution for governing nonlinear second order partial differential equation. Whereas MATLAB is applied to acquire the solution's numerical findings and graphical representations.

1. INTRODUCTION

Crude oil appears to occur in underground porous rock formations of sandstone or carbonate. The process of oil recovery involves three phases naming, the process of primary oil recovery, the process of secondary oil recovery and the process of enhanced oil recovery. First stage-The primary oil recovery process is the process of draining the oil which flows naturally to the beneath of the oil well owing to the reservoir's pressure and gravity. On an average, in the first stage, only 10 percent of the oil below the surface is recovered, so secondary and tertiary measures are needed. Second Stage - Secondary oil recovery is the

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process where the reservoir is either water flooded or gas injected or chemical injected to maintain a pressure that continues to move oil toward the surface.

If another less viscous fluid displaces a fluid flowing across a porous media, protuberance takes place instead of usual movement of the entire front, which shoots at a relatively high speed through the porous medium. It results into the creation of different size and shapes of numerous fingers. Hence this phenomenon is either termed as fingering phenomenon or instability. Instability occurs because of injected fluid's force. Because of the variation in viscosities of flowing fluids, fingering (instability) phenomenon holds greater importance during secondary oil recovery process. As compared to the first stage, 25-30% more oil from the reservoir can be recovered in the second stage.

Several physicists studied and discussed this phenomenon with different viewpoints. To begin with, in the year 1969 Verma conferred the statistical behavior of instability with capillary pressure in heterogeneous porous medium [4]. Meher studied the phenomenon of instability using the Adomian method of decomposition in the year 2013 [9]. Patel et al. solved one dimensional phenomenon of instability occurring in inclined porous media using the method of Homotopy analysis in the year 2014 [7]. Borana et al. provided the numerical findings for the phenomenon of instability through inclined homogeneous porous medium which occurs in a double phase flow using Crank-Nicolson Scheme in the year 2016 [8]. Most Recently, in the year of 2017 Parikh investigated mathematical model for the phenomenon of instability in vertical downward direction by homogeneous porous medium using Generalized separable method [2].

Now, this particular study deals with fingering (instability) phenomenon resulting from the flow of two not miscible fluids (i.e. water and oil) via a heterogeneous porous medium when water is injected vertically downwards considering the two required conditions (I) mean capillary pressure and (II) gravitational effect.

The phenomenon's governing equation is a nonlinear second order partial differential equation that is being solved by applying Variational iteration method [5] with appropriate initial and boundary conditions. Many physicists have shown that Method of variational iteration is influential than both the Homotopy perturbation Method and the Adomian Decomposition Method [3]. But, so far, in heterogeneous porous medium, no researcher had studied this phenomenon in vertical downward direction.

2. ASSUMPTION AND MATHEMATICAL DESCRIPTION OF THE PROBLEM

The uniform injection of water into an oil-saturated heterogeneous porous matrix is assumed to occur vertically downwards. We have considered the vertical pipe shaped part of the heterogeneous porous matrix of the actual field to be surrounded by an impermeable surface with the exception of its two ends (uppermost which is identified as the common interface $(z = 0)$ and lowermost which is precisely linked to oil production) for the sake of mathematical study. The irregular fingers are formed in downward direction due to external injection force and gravitational impact, so that water flows vertically downward through the interconnected capillaries to pull oil towards the lowermost of the pipe shaped part of porous matrix as displayed in Figure 1. Schematic fingers are chosen as rectangular fingers of varying size and shape for mathematical construction, which is shown in Figure 2. However, it is challenging for any given time $t > 0$ to obtain saturation of the injected water at any depth z. Therefore, average cross section region of all rectangular fingers of average length is chosen. For present study, saturation of the injected water (S_w) is described as average cross section region employed through schematic fingers at depth z for given $t \geq 0$. The displacement phase is in the z-direction with injection time t. Saturation is therefore determined as $S_w(z, t)$ that is the function of depth z and time t.

Due to the flow of water and oil in a heterogeneous porous medium, the law of Darcy applies to determine the velocity of injected water (wetting fluid) (V_w) and the velocity of native oil (non-wetting fluid) (V_o) for the low number of Reynolds. The porous medium is taken as heterogeneous. As a result, porosity (P) and permeability (K) are selected as variables. As water and oil flow vertically downwards, the gravitational impact plays a key role in raising the velocity of water and oil for an extra term ρg in the law of Darcy [6].

Let the lowermost of the vertical pipe shaped part of heterogeneous porous matrix be at $z = L$, measured at the top $z = 0$.

In present investigation, our particular interest is to determine the saturation of injecting water in well-developed fingers due to water injection which helps to push oil towards oil production well.

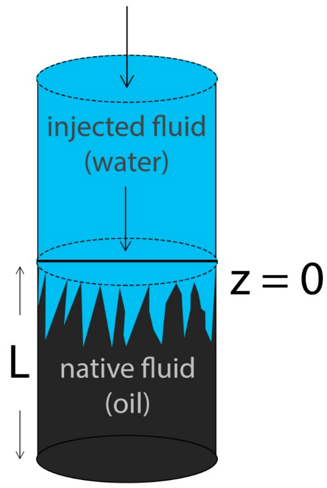

FIGURE 1. Fingers formation in pipe shaped part of porous matrix

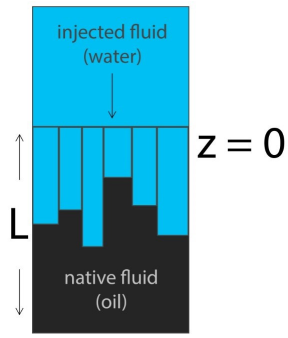

FIGURE 2. Schematic view of fingers

3. MATHEMATICAL CONSTRUCTION

Using the law of Darcy, the velocities of the injected water (wetting fluid) V_W and native oil (non-wetting fluid) V_o can be described as follows [6]:

$$V_w = -\left(\frac{K_w}{\mu_w}\right) K \left(\frac{\partial P_w}{\partial z} + \rho_w g\right), \tag{3.1}$$

$$V_o = -\left(\frac{K_o}{\mu_o}\right) K \left(\frac{\partial P_o}{\partial z} + \rho_o g\right). \tag{3.2}$$

The equation of continuity for flowing fluids is written according to the law of mass conservation for incompressible flow, respectively,

$$P\left(\frac{\partial S_w}{\partial t}\right) + \frac{\partial V_w}{\partial z} = 0, \tag{3.3}$$

$$P\left(\frac{\partial S_o}{\partial t}\right) + \frac{\partial V_o}{\partial z} = 0. \tag{3.4}$$

Using well known result for phase saturation [1],

$$S_w + S_o = 1. \tag{3.5}$$

Only because of capillary pressure P_c, fluid can flow through interconnected capillaries. Capillary pressure P_c is described as the difference in pressure of the flowing fluid through their common interface. It is a function of the injected fluid saturation. This can be noted down as [1]

$$P_c(S_w) = P_o - P_w. \tag{3.6}$$

The capillary pressure is a linear function of displacing fluid saturation. It is in the opposite direction [2]. Thus,

$$P_c = -\beta S_w. \tag{3.7}$$

Consider the relation between the saturation of the phase and relative permeability as stated below [1]:

$$K_w = S_w\,, K_o = 1 - \alpha S_w. \tag{3.8}$$

For more definiteness, choose $\alpha \approx 1$,

$$K_o \approx 1 - S_w = S_o \quad (\because\ S_w + S_o = 1).$$

By the variation law, porosity and permeability of heterogeneous porous medium is described as function of variable z only [4],

$$\text{Porosity } P = P(z) = \frac{1}{a_1 - a_2 z} \tag{3.9}$$

$$\text{Permeability } K = K(z) = K_0(1 + bz).$$

As $P(z)$ cannot go beyond unity, we consider that $a_1 - a_2 z \geq 1$.

For simplicity, $K \propto P$ [10]

$$\text{So, } K = K_c P. \tag{3.10}$$

Substituting the values of V_w and V_o from equations (3.1) and (3.2) into equations (3.3) and (3.4) respectively,

$$P\left(\frac{\partial S_w}{\partial t}\right) = \frac{\partial}{\partial z}\left[\frac{K_w}{\mu_w}K\frac{\partial P_w}{\partial z}\right] - \frac{\partial}{\partial z}\left[\frac{K_w}{\mu_w}K\rho_w g\right], \tag{3.11}$$

$$P\left(\frac{\partial S_o}{\partial t}\right) = \frac{\partial}{\partial z}\left[\frac{K_o}{\mu_o}K\frac{\partial P_o}{\partial z}\right] - \frac{\partial}{\partial z}\left[\frac{K_o}{\mu_o}K\rho_o g\right]. \tag{3.12}$$

Substituting the value of P_w from (3.6) into (3.11),

$$P\left(\frac{\partial S_w}{\partial t}\right) = \frac{\partial}{\partial z}\left[\frac{K_w}{\mu_w}K\left(\frac{\partial P_o}{\partial z} - \frac{\partial P_c}{\partial z}\right)\right] - \frac{\partial}{\partial z}\left[\frac{K_w}{\mu_w}K\rho_w g\right]. \tag{3.13}$$

From equation (3.5),

$$\frac{\partial S_w}{\partial t} + \frac{\partial S_o}{\partial t} = 0.$$

Thus,

$$\frac{\partial S_w}{\partial t} = -\frac{\partial S_o}{\partial t}.$$

Replacing the values of $\frac{\partial S_o}{\partial t}$ in (3.12) and compare with (3.11),

$$\frac{\partial}{\partial z}\left[\left(\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}\right)K\frac{\partial P_o}{\partial z} - \frac{K_w}{\mu_w}K\frac{\partial P_c}{\partial z}\right] = \frac{\partial}{\partial z}\left[\left(\frac{K_w}{\mu_w}\rho_w + \frac{K_o}{\mu_o}\rho_o\right)Kg\right]. \tag{3.14}$$

Integrating equation (3.14) with respect to z,

$$\left(\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}\right)K\frac{\partial P_o}{\partial z} - \frac{K_w}{\mu_w}K\frac{\partial P_c}{\partial z} = \left(\frac{K_w}{\mu_w}\rho_w + \frac{K_o}{\mu_o}\rho_o\right)Kg + C(t). \tag{3.15}$$

By simplifying the above equation (3.15),

$$\frac{\partial P_o}{\partial z} = \frac{\frac{K_w}{\mu_w}\frac{\partial P_c}{\partial z} + \left(\frac{K_w}{\mu_w}\rho_w + \frac{K_o}{\mu_o}\rho_o\right)g + \frac{C(t)}{K}}{\left(\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}\right)}. \tag{3.16}$$

Substituting the values of $\frac{\partial P_o}{\partial z}$ from equation (3.16) into (3.13),

$$\frac{\partial S_w}{\partial t} = \frac{\partial}{\partial z}\left\{\frac{\frac{K_w}{\mu_w}K\left[\left(\frac{K_w}{\mu_w}\rho_w + \frac{K_o}{\mu_o}\rho_o\right)g + \frac{C(t)}{K} - \frac{\partial P_c}{\partial z}\left(\frac{K_o}{\mu_o}\right)\right]}{\left(\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}\right)}\right\} - \frac{\partial}{\partial z}\left(\frac{K_w}{\mu_w}K\rho_w g\right). \tag{3.17}$$

The native oil pressure (P_o) can be defined as [1],

$$P_o = \frac{P_o - P_w}{2} + \frac{P_o + P_w}{2} = \frac{1}{2}P_c + \bar{P}. \tag{3.18}$$

By differentiating (3.18) with respect to z,

$$\frac{\partial P_o}{\partial z} = \frac{1}{2}\frac{\partial P_c}{\partial z}. \tag{3.19}$$

Eliminating $\frac{\partial P_o}{\partial z}$ from (3.19) to (3.16),

$$\frac{1}{2}\frac{\partial P_o}{\partial z}\left(\frac{K_o}{\mu_o} - \frac{K_w}{\mu_w}\right) - \left(\frac{K_w}{\mu_w}\rho_w + \frac{K_o}{\mu_o}\rho_o\right) g = \frac{C\left(t\right)}{K}. \tag{3.20}$$

Now, substituting the value of $\frac{C(t)}{K}$ from equation (3.20) into equation (3.17),

$$P\left(\frac{\partial S_w}{\partial t}\right) = \frac{\partial}{\partial z}\left[\frac{K_w}{\mu_w}K\left(-\frac{1}{2}\frac{\partial P_c}{\partial z}\right)\right] - \frac{\partial}{\partial z}\left[\frac{K_w}{\mu_w}K\rho_w g\right]. \tag{3.21}$$

Using standard relation (3.7), (3.8) and (3.10) in equation (3.21),

$$\frac{\partial S_w}{\partial t} = \frac{\beta K_c}{2\mu_w}\left[\frac{\partial}{\partial z}\left(S_w\frac{\partial S_w}{\partial z}\right) + S_w\left(\frac{\partial S_w}{\partial z}\right)\frac{a_2}{a_1}\right] - \frac{K_w}{\mu_w}\rho_w g\left[\frac{\partial S_w}{\partial z} + S_w\frac{a_2}{a_1}\right], \tag{3.22}$$

$\left(\because \frac{1}{P}\frac{\partial P}{\partial z} = \frac{\partial(\log P)}{\partial z} = \frac{\partial}{\partial z}\left(-\log a_1 + \frac{a_2}{a_1}z\right) = \frac{a_2}{a_1} \quad (3.9)\right)$
(neglecting higher order terms of z)

Equation (3.22) represents governing nonlinear second order partial differential equation for the fingering phenomenon that occurs during the process of secondary oil recovery in vertical downward direction through heterogeneous porous medium.

For the sake of simplicity, selecting dimensionless variables

$$Z = \frac{z}{L},\ T = \frac{K_c\beta t}{2\mu_w L^2}, \quad 0 \le Z \le 1,\ 0 \le T \le 1.$$

Using above variables, the equation (3.22) can be reduced into dimensionless form as

$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial z}\left(S_w\frac{\partial S_w}{\partial z}\right) + BS_w\frac{\partial S_w}{\partial z} - A\frac{\partial S_w}{\partial z} - ABS_w, \tag{3.23}$$

where $A = \frac{2L\rho_w g}{\beta},\ B = \frac{a_2}{a_1}L$ and $S_w\left(z,t\right) = S_w\left(Z,T\right)$.

Equation (3.23) is a nonlinear second order partial differential equation for the phenomenon of fingering.

The following are the associated initial and boundary conditions:

$$S_w\left(Z,0\right) = S_{w_0}\left(Z\right), \quad \text{where} \quad Z > 0,$$

$$S_w(0,T) = S_{w_1}(T), \quad \text{where } T > 0,$$
$$S_w(1,T) = S_{w_2}(T), \quad \text{where } T > 0.$$

4. Solution by variational iteration method

Following Variational iteration method, we can derive the correction functional for equation (3.23) as below,

$$\begin{aligned} S_{w_{n+1}}(Z,T) = S_{w_n}(Z,T) + \int_0^T \lambda(T) \Bigg[&\frac{\partial S_{w_n}(Z,\tau)}{\partial \tau} \\ &- \frac{\partial}{\partial z}\left(\tilde{S}_{w_n}(Z,\tau)\frac{\partial \tilde{S}_{w_n}(Z,\tau)}{\partial z}\right) \\ &- B\tilde{S}_{w_n}(Z,\tau)\frac{\partial \tilde{S}_{w_n}(Z,\tau)}{\partial z} + A\frac{\partial \tilde{S}_{w_n}(Z,\tau)}{\partial z} + AB\tilde{S}_{w_n}(Z,\tau)\Bigg] d\tau. \end{aligned}$$

Here $\tilde{S}_{w_n}$ is deliberated as restricted variation, i.e. $\delta\tilde{S}_{w_n} = 0$.

Computing variation with respect to S_{w_n}, noticing that $\delta\tilde{S}_{w_n}(0) = 0$, yields

$$\begin{aligned} \delta S_{w_{n+1}}(Z,T) =& \delta S_{w_n}(Z,T) + \delta \int_0^T \lambda(T) \Bigg[\frac{\partial S_{w_n}(Z,\tau)}{\partial \tau} \\ &- \frac{\partial}{\partial z}\left(\tilde{S}_{w_n}(Z,\tau)\frac{\partial \tilde{S}_{w_n}(Z,\tau)}{\partial z}\right) \\ &- B\tilde{S}_{w_n}(Z,\tau)\frac{\partial \tilde{S}_{w_n}(Z,\tau)}{\partial z} + A\frac{\partial \tilde{S}_{w_n}(Z,\tau)}{\partial z} + AB\tilde{S}_{w_n}(Z,\tau)\Bigg] d\tau \\ \delta S_{w_{n+1}}(Z,T) =& \delta S_{w_n}(Z,T) + \left[\lambda(z)\,\delta S_{w_n}(Z,T)\right]_{\tau=T} \\ &- \int_0^T \lambda'(T)\,\delta S_{w_n}(Z,T)\,d\tau + \int_0^T \lambda AB\delta S_{w_n}(Z,T)\,d\tau. \end{aligned}$$

This gives following stationary conditions,

$$\begin{aligned} \left[1 + \lambda(T)\right]_{\tau=T} =& 0, \\ \left[-\lambda'(T) + AB\lambda(T)\right]_{\tau=T} =& 0. \end{aligned}$$

So, Lagrange's multiplier $\lambda = -e^{AB(\tau - T)}$. As a result, iteration formula can be found as follows,

$$S_{w_{n+1}}(Z,T) = S_{w_n}(Z,T) - \int_0^T e^{AB(\tau-T)}\left[\frac{\partial S_{w_n}(Z,\tau)}{\partial\tau}\right.$$

$$(4.1) \qquad -\frac{\partial}{\partial z}\left(\tilde{S}_{w_n}(Z,\tau)\frac{\partial\tilde{S}_{w_n}(Z,\tau)}{\partial z}\right)$$

$$\left. -B\tilde{S}_{w_n}(Z,\tau)\frac{\partial\tilde{S}_{w_n}(Z,\tau)}{\partial z} + A\frac{\partial\tilde{S}_{w_n}(Z,\tau)}{\partial z} + AB\tilde{S}_{w_n}(Z,\tau)\right]d\tau$$

For $n = 0$,

$$S_{w_1}(Z,T) = S_{w_0}(Z,T) - \int_0^T e^{AB(\tau-T)}\left[\frac{\partial S_{w_0}(Z,\tau)}{\partial\tau} - \frac{\partial}{\partial z}\left(\tilde{S}_{w_0}(Z,\tau)\frac{\partial\tilde{S}_{w_0}(Z,\tau)}{\partial z}\right)\right.$$

$$\left. - B\tilde{S}_{w_0}(Z,\tau)\frac{\partial\tilde{S}_{w_0}(Z,\tau)}{\partial z} + A\frac{\partial\tilde{S}_{w_0}(Z,\tau)}{\partial z} + AB\tilde{S}_{w_0}(Z,\tau)\right]d\tau.$$

Choose the initial approximation [2],

$$(4.2) \qquad S_{w_0} = e^{-z}.$$

By substituting this initial approximation in the iterative formula, we get first approximation

$$S_{w_1} = e^{-z} + \left(\frac{2e^{-2z}}{AB} - \frac{e^{-2z}}{A} + \frac{e^{-z}}{B} - e^{-z}\right)\left(1 - e^{-ABT}\right),$$

$$S_{w_2} = e^{-z} + \left(\frac{2e^{-2z}}{AB} - \frac{e^{-2z}}{A} + \frac{e^{-z}}{B} - e^{-z}\right)\left(1 - e^{-ABT}\right)$$

$$-\left[\left(\frac{18e^{-3z}}{AB} - \frac{9e^{-3z}}{A} + \frac{4e^{-2z}}{B} - 4e^{-2z}\right)\right.$$

$$- B\left(\frac{6e^{-3z}}{AB} - \frac{3e^{-3z}}{A} + \frac{2e^{-2z}}{B} - 2e^{-2z}\right)$$

$$\left. + A\left(\frac{4e^{-2z}}{AB} - \frac{2e^{-2z}}{A} + \frac{e^{-z}}{B} - e^{-z}\right)\right]\left(e^{-ABT}T - \frac{1}{AB} + \frac{e^{-ABT}}{AB}\right)$$

$$
\begin{aligned}
&-\Bigg[\left(\frac{2e^{-2z}}{AB}-\frac{e^{-2z}}{A}+\frac{e^{-z}}{B}-e^{-z}\right)\left(\frac{8e^{-2z}}{AB}-\frac{4e^{-2z}}{A}+\frac{e^{-z}}{B}-e^{-z}\right)\\
&+\left(\frac{4e^{-2z}}{AB}-\frac{2e^{-2z}}{A}+\frac{e^{-z}}{B}-e^{-z}\right)^2\\
&-B\left(\frac{2e^{-2z}}{AB}-\frac{e^{-2z}}{A}+\frac{e^{-z}}{B}-e^{-z}\right)\left(\frac{4e^{-2z}}{AB}-\frac{2e^{-2z}}{A}+\frac{e^{-z}}{B}-e^{-z}\right)\Bigg]\\
&\left(2e^{-ABT}T-\frac{1}{AB}+\frac{e^{-ABT}}{AB}\right).
\end{aligned}
\tag{4.3}
$$

Similarly, the more iterations can be obtained using iterative formula (4.1). Equation (4.3) is the approximate analytic solution of the governing nonlinear second order partial differential equation (3.23).

5. Numerical findings and graphical representation

From standard literature the values of certain constants are taken as follows: Length $(L) = 1$, Density $(\rho_w) = 0.1$, Gravity $(g) = 9.8$,$\beta = 2 \Rightarrow A \approx 1$.

Using MATLAB, numerical results and graphical representations of the solution (4.3) are carried out. Table 1 represents the numerical results for $S_w(Z, T)$ against T for given fixed time $T = 0.02, 0.04, 0.06, 0.08, 0.1$ and Figure 3 displays the graphs of $S_w(Z, T)$ against Z for given fixed time $T = 0.02, 0.04, 0.06, 0.08, 0.1$. Figure 4 and Figure 5 show the graphs of $S_w(Z, T)$ against T for given fixed depth $Z = 0.2, 0.4$ respectively.

6. Conclusion

In this study, we have discussed problem of fingering phenomenon in vertical downward direction through heterogeneous porous medium. Equation (3.23) describes the governing nonlinear second order partial differential equation for the phenomenon of fingering. The solution of equation (3.23) is found with the initial condition (4.2) in equation (4.3) using Variational iteration method. Table 1 reflects numerical results for different time T of saturation of the injected water for the phenomenon of fingering at different depth Z. It has been observed from Table 1, that saturation of injected water rises with respect to time T and reduces with respect to depth Z. This, in turn, reveals that, as time increases, the fingers will raise and move oil from the oil-formed area to the

TABLE 1. Saturation of injected water for the phenomenon of fingering

$T \longrightarrow$	0.02	0.04	0.06	0.08	0.1
$Z \downarrow$	$S_w(Z,T)$				
0.1	0.9210	0.9369	0.9525	0.9678	0.9827
0.2	0.8320	0.8450	0.8578	0.8703	0.8825
0.3	0.7517	0.7623	0.7728	0.7830	0.7930
0.4	0.6792	0.6879	0.6965	0.7049	0.7131
0.5	0.6138	0.6210	0.6280	0.6348	0.6415
0.6	0.5548	0.5606	0.5664	0.5720	0.5775
0.7	0.5015	0.5063	0.5109	0.5155	0.5201
0.8	0.4533	0.4572	0.4611	0.4649	0.4685
0.9	0.4098	0.4131	0.4162	0.4193	0.4223
1.0	0.3706	0.3732	0.3758	0.3783	0.3808

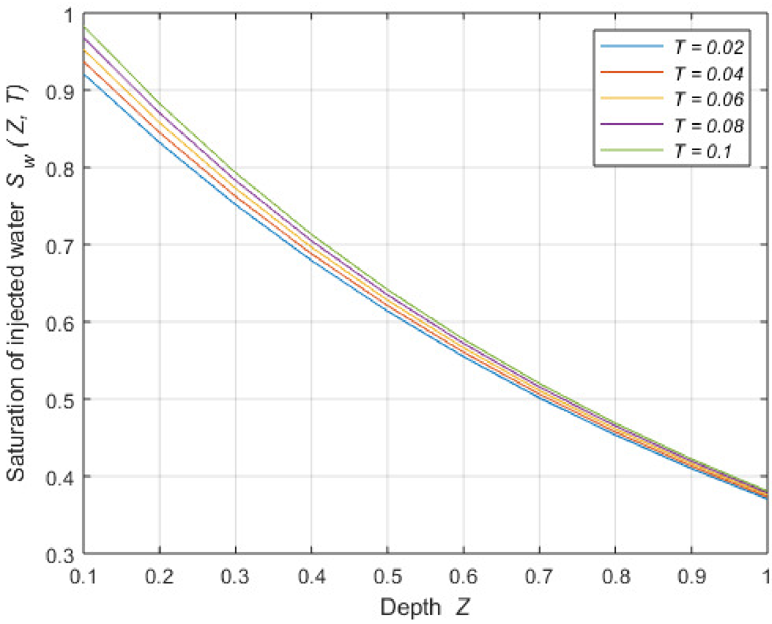

FIGURE 3. 2-dimension graph represents saturation of injected water $S_w(Z,T)$ against depth Z for fixed time $T = 0.02, 0.04, 0.06, 0.08, 0.1$

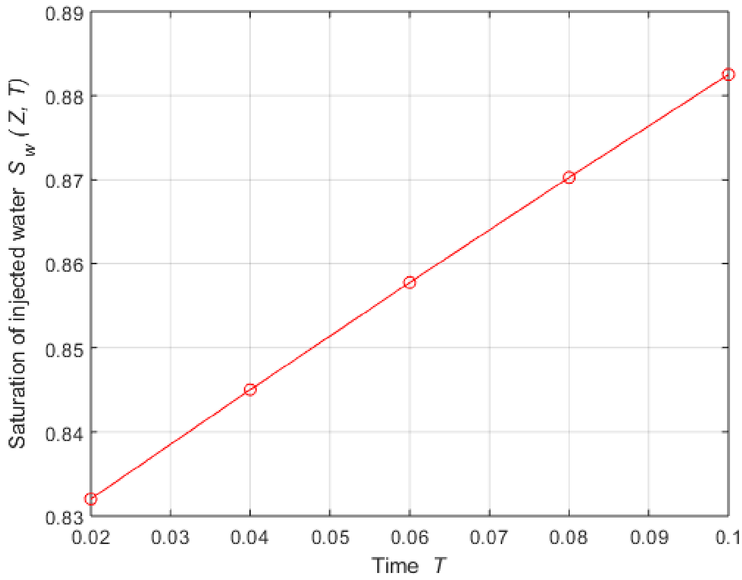

FIGURE 4. 2-dimension graph represents saturation of injected water $S_w(Z,T)$ against time T for fixed depth $Z = 0.2$

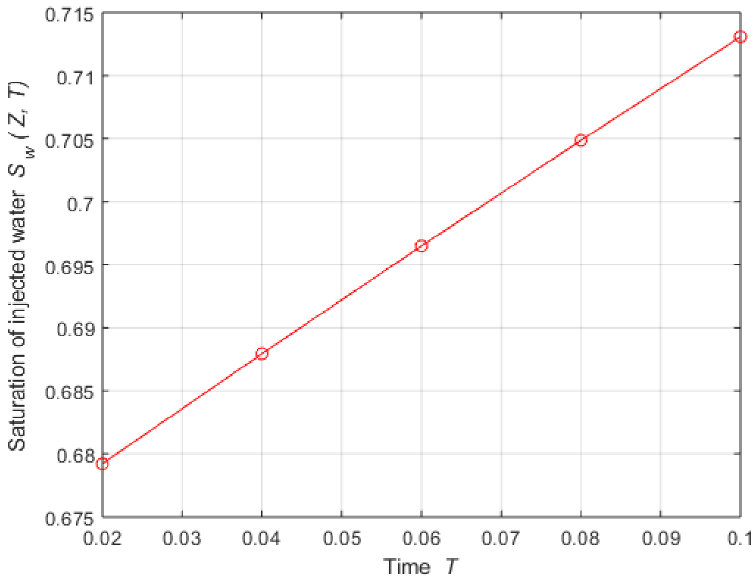

FIGURE 5. 2-dimension graph represents saturation of injected water $S_w(Z,T)$ against time T for fixed depth $Z = 0.4$

lowermost of the pipe shaped part of porous matrix. As a result, the remaining oil can be moved to the bottom during the process of secondary oil recovery. Figure 3 represents the solution graphically with respect to depth. Figure 4 and Figure 5 display the solution graphically with respect to time. Figure 3 shows that saturation of the injected water decreases with respect to depth Z, that is coherent with the physical nature of the problem.

NOMENCLATURE

V_w	Velocity of injected water	P	Porosity
V_o	Velocity of native oil	P_c	Capillary pressure
K	Variable permeability	β, K_c	Constant of proportionality
μ_w	Kinematic viscosity of injected water	a_1, a_2, K_0, b	Positive constants
μ_o	Kinematic viscosity of native oil	C	Arbitrary constant
ρ_w	Density of injected water	P_o	Pressure of oil
ρ_o	Density of native oil	$\bar{P}$	Constant mean pressure
g	Acceleration due to gravity	L	Length of pipe shaped porous matrix
K_w	Relative permeability of injected water	λ	Lagrange's multiplier
K_o	Relative permeability of native oil	$\tilde{S}_{w_n}$	Restricted variation
S_w	Saturation of injected water	T	Time
S_o	Saturation of native oil	Z	Depth

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