

COSMIC INFLATION IN BIANCHI TYPE IX SPACE WITH BULK VISCOSITY

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ABSTRACT. In present paper we have constructed the Bianchitype-IX inflationary universe under framework of the effect of bulk viscosity and flat potential. To developed inflationary model we have consider the supplementary criteria that shear coefficient is directly proportional to expansion scalar which provides an appropriate relation with coefficients of metric $b = a^n$ where n is non-negative constant other than one. We conclude that rate of Higgs field decreases with time and proper volume V is increasing function of time which indicates that universe is expanded continuously. To find solutions of fields equations, we assume $\xi\theta = \alpha$ (constant) as given by Brevik et al. where ξ is coefficient of bulk viscosity and θ expansion in model. The presence of bulk viscosity provides inflationary solution in current model. The model isotropize in special case. The cosmological parameters of model are also studied.

1. INTRODUCTION

In recent scenario, it is very curious to study the inflationary theory because it has astrophysical importance to understand the behavior of universe at early stage of evolution. The phenomenon of inflation makes us unable to solving the several problems like isotropy, homogeneity and flatness of observable universe.

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The bianchi classification is an essential tool for developing and classified cosmological model in which we have to find solutions of Einstein field equations which are nonlinear differential equations and the field of gravitation is coupled minimally to scalar field. The familiar solution like Robertson Walker model [1], Taub-Nut solution [2], the de- sitter universe [3] can be explained by bianchi IX space time. This model allowed not only expansion but rotation also. Guth [4] gives the basic idea of inflationary universe in context of grand unification theory (GUT). Inflationary scenario in physical universe exist in many version as studied by Linde [5]. Bali and Singh [6] derived LRS bianchi type I for stiff fluid in present of bulk viscosity. Weinberg [7] investigated the common formula for bulk as well as shear viscosity to describe cosmology rate production. Banerjee et al. [8] studied inflationary model for viscous fluid under various bianchi. Santos et al. [9] derived exact solution of Einstein field equation to obtained isotropize, homogenous inflationary universe in presence of bulk viscosity. On the basis of above situation we have investigated cosmic acceleration in bianchi type IX inflationary universe with bulk viscosity (ξ) and flat region where potential $V(\phi)$ is constant. The concept of higg's field with potential $V(\phi)$ play significant role in this discussion. To obtained inflationary model we assume $\xi\theta = \alpha$ (constant) as given by Brevik et al. [10] and condition that shear is proportional to expansion which leads to supplement condition between metric coefficient $a(t)$ and $b(t)$ where t is cosmic time. The inflation in the developed model start with big bang at $T=0$. The model isotropize for particular case. we find spatial volume increase with time and Higg's field ϕ decrease show inflationary scenario of universe.

2. METRIC AND FIELDS EQUATIONS

The Bianchi Type IX space- time can be describe the line element

$$(2.1) \quad ds^2 = -dt^2 + a^2 dx^2 + b^2 dy^2 + (b^2 \sin^2 y + a^2 \cos^2 y) dz^2 - 2a^2 \cos y dx dz,$$

where a and b are metric coefficients and function of cosmic time t . The action of gravitational field coupled minimally to a scalar field with potential $V(\phi)$ leads t ,

$$l = \int (R - \frac{1}{2} \phi_i \phi_j g_{ij} - V(\phi)) \sqrt{-g} dx^4,$$

which on variation of l w.r.t dynamic field provided Einstein field equations (in gravitational unit $c = 1, G = 1$) is given by

$$(2.2) \quad R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}.$$

The energy momentum tensor t_{ij} in presence of viscosity for scalar field is given by Guth [4]

$$T_{ij} = \phi_i \phi_j - \left(\frac{1}{2}\phi_l \phi^l + V(\phi)\right)g_{ij} - \xi\theta(g_{ij} + u_i u_j),$$

where the terms V, ϕ, ξ and θ represents potential, Higg's field, coefficient of bulk viscosity and scalar of expansion respectively. The co-moving coordinates is assumed such that $u_i = (0, 0, 0, 1)$. The conservation relation is given by

$$\frac{1}{\sqrt{-g}}\partial_i(\sqrt{-g}\phi_i) = -\frac{dV(\phi)}{d\phi},$$

where $\phi_i = \frac{\partial\phi}{\partial x_i}, \phi_j = \frac{\partial\phi}{\partial x_j}$ and $\phi^l = \frac{\partial\phi}{\partial x_l}$.

Now the Einstein field equations (2.2) for metric (2.1) leads to

$$\begin{aligned} \frac{b_4^2}{b^2} + 2\frac{b_{44}^2}{b^2} + \frac{1}{b^2} - \frac{3a_4^2}{4b^4} &= -8\pi\left[\frac{\phi_4^2}{2} - V(\phi) - \xi\theta\right] \\ \frac{1}{4}\frac{a^2}{b^2} + \frac{a_4 b_4}{ab} + \frac{b_{44}}{b} + \frac{a_{44}}{a} &= -8\pi\left[\frac{\phi_4^2}{2} - V(\phi) - \xi\theta\right] \\ 2\frac{a_4 b_4}{ab} - \frac{1}{4}\frac{a^2}{b^4} + \frac{b_4^2}{b^2} + \frac{1}{b^2} &= -8\pi\left[\frac{-\phi_4^2}{2} - V(\phi) - 2\xi\theta\right] \\ \left[\frac{a_{44}}{a} + \frac{a_4 b_4}{ab} - \frac{b_{44}}{b} + \frac{a^2}{b^4} - \frac{b_4^2}{b^2} - \frac{1}{b^2}\right] \cos y &= 0. \end{aligned}$$

To construct inflationary model of the physical universe. We have consider $\xi\theta = \alpha$ (constant) from Brevik et.al. [10] because it have significant role to connect with existence of LR cosmology using FRW metric. Stein-Shabes investigated that Higg's field with flat potential has flat region evolves at slow rate but universe is expanding in exponentially manner under effect of energy of vacuum field. It is supposed that the scalar field would take appropriate time to crossed the flat region so the universe expanded to become isotropic and homogenous on the horizon scale. For $V(\phi) = \text{constant}$, the conservation relation becomes

$$\phi_{44} + \left(\frac{a_4}{a} + 2\frac{b_4}{b}\right)\phi_4 = 0,$$

where the subscript 4 indicates the ordinary differentiation w.r.t cosmic time t we are interested to find inflationary solution by solving non-linear differential equations, so we obtained

$$(2.3) \quad \frac{a_{44}}{a} + \frac{a_4 b_4}{ab} - \frac{b_{44}}{b} + \frac{a^2}{b^4} - \frac{b_4^2}{b^2} - \frac{1}{b^2} = 0.$$

We have assume that shear scalar is proportionate to expansion scalar which provides supplement conditions between metric coefficients such that $b = a^n$, where $n \neq 1$, the equation (2.3) leads to

$$(2.4) \quad 2a_{44} + 4n \frac{(a_{44})2}{a} = \frac{2}{1-n} [a^{1-2n} - a^{3-4n}].$$

We assume $a_4 = f(a)$, which gives $a_{44} = f f'$, in which $f' = \frac{df}{dt}$ equation (2.4) leads to

$$\frac{df^2}{dt} + \frac{4n}{a} f^2 = \frac{2}{(1-n)} [a^{1-2n} - a^{3-4n}].$$

On solving, it provides

$$f^2 = \left[\frac{a^{2(1-n)}}{1-n^2} + \frac{a^{4(1-n)}}{2(1-n)} + D a^{-4n} \right],$$

where D is constant of integration,

$$a_4 = \left[\frac{a^{2(1-n)}}{1-n^2} + \frac{a^{4(1-n)}}{2(1-n)} + D a^{-4n} \right]^{\frac{1}{2}},$$

which leads to

$$(2.5) \quad \int \left[\frac{a^{2(1-n)}}{1-n^2} + \frac{a^{4(1-n)}}{2(1-n)} + D a^{-4n} \right]^{\frac{1}{2}} da = \pm (t - t_0),$$

where t_0 is integration constant.

The space-time metric (2.1) reduces in to

$$ds^2 = - \left[\frac{T^{2(1-n)}}{1-n^2} + \frac{T^{4(1-n)}}{2(1-n)} + D T^{-4n} \right]^{\frac{1}{2}} dT^2 \\ + T^2 dX^2 + T^{2n} dY^2 + (T^{2n} \sin^2 Y + T^2 \cos^2 Y) dZ^2 - 2T^2 \cos Y dX dZ$$

where the transformation is given by $a = T$, $x = X$, $y = Y$ and $z = Z$.

3. PHYSICAL AND GEOMETRICAL FEATURES OF THE MODEL

The proper volume for model is given by

$$V = \sqrt{-g} = ab^2 \sin Y = T^{2n+1} \sin Y.$$

The rate of Higg's Field is obtained as

$$\phi_4 = \frac{\mu}{ab^2} = \frac{\mu}{T^{2n+1}},$$

which yields to $\phi = \int \frac{\mu}{T^{2n+1}} dT + C$, where C is the integrating constant.

The scalar of expansion (θ) is given by

$$\theta = (2n+1) \left[\frac{1}{(1-n^2)T^{2n}} + \frac{1}{2(1-n)T^{2(2n-1)}} \right] + DT^{-2(2n+1)} \Big]^\frac{1}{2}.$$

The non-vanishing component of shear σ_{ij} is given by

$$\begin{aligned} \sigma_{11} &= -\frac{2}{3}(n-1)T^2 \left[\frac{1}{(1-n^2)T^{2n}} + \frac{1}{2(1-n)T^{2(2n-1)}} \right] + DT^{-2(2n+1)} \Big]^\frac{1}{2} \\ \sigma_{22} &= \frac{1}{3}(n-1)T^2 \left[\frac{1}{(1-n^2)T^{2n}} + \frac{1}{2(1-n)T^{2(2n-1)}} \right] + DT^{-2(2n+1)} \Big]^\frac{1}{2} \\ \sigma_{33} &= \frac{1}{3}(n-1)[T^{2n} \sin^2 Y - 2T \cos^2 Y] \left[\frac{1}{(1-n^2)T^{2n}} + \frac{1}{2(1-n)T^{2(2n-1)}} \right] \\ &\quad + DT^{-2(2n+1)} \Big]^\frac{1}{2} \\ \sigma_{44} &= \frac{2}{3}(n-1) \left[\frac{1}{(1-n^2)T^{2n}} + \frac{1}{2(1-n)T^{2(2n-1)}} \right] + DT^{-2(2n+1)} \Big]^\frac{1}{2} \\ \sigma_{13} &= \frac{2}{3}(n-1)T^2 \cos Y \left[\frac{1}{(1-n^2)T^{2n}} + \frac{1}{2(1-n)T^{2(2n-1)}} \right] + DT^{-2(2n+1)} \Big]^\frac{1}{2}, \end{aligned}$$

hence

$$(3.1) \quad \sigma^2 = \frac{1}{2}(n-1)^2 \left[\frac{1}{(1-n^2)T^{2n}} + \frac{1}{2(1-n)T^{2(2n-1)}} \right] + DT^{-2(2n+1)} \Big]^\frac{1}{2}.$$

The Hubble parameter is given by

$$(3.2) \quad H = \frac{1}{3}(2n+1) \left[\frac{1}{(1-n^2)T^{2n}} + \frac{1}{2(1-n)T^{2(2n-1)}} \right] + DT^{-2(2n+1)} \Big]^\frac{1}{2}$$

4. GRAPHICAL REPRESENTATION OF THE MODEL

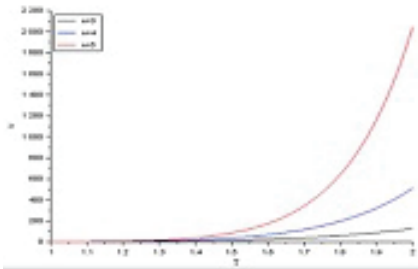


FIGURE 1. volume versus time

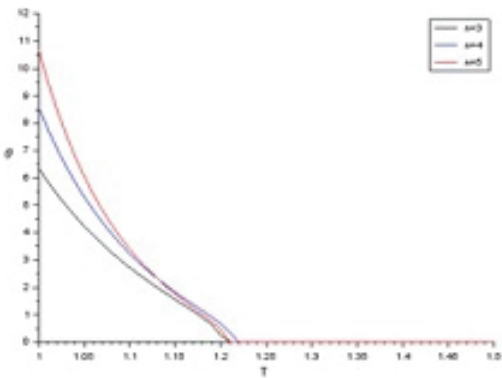


FIGURE 2. coefficient of expansion versus time

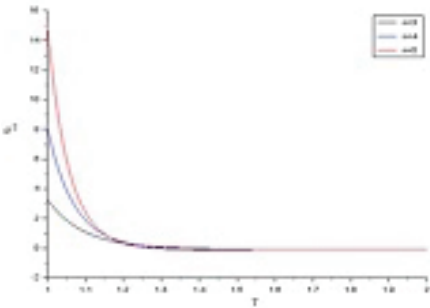


FIGURE 3. coefficient of shear verses time

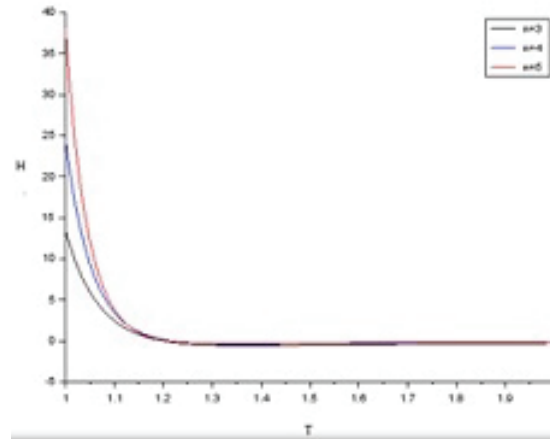


FIGURE 4. Hubble parameter verses time

5. CONCLUSION AND DISCUSSION

In the present paper we have studied the cosmic expansion of the universe in existence of massless scalar field has flat potential with bulk viscosity. From equation (3.1) it has been shown that the scalar of expansion decrease as T increases and it vanish at $T = \infty$ and $n = -\frac{1}{2}$. The proper volume for model (2.5) is increasing function of T which favorable to inflationary scenario exist in model. Since $\frac{\sigma}{\theta} \neq 0$ it shows that model (2.5) remain anisotropic for large value of T . The inflation in model start with big-bang at $T=0$. The hubble factor (H) is large at initial stage but become finite at late T . The Higgs field evolves at slow rate with increase time and universe expands due to vacuum field energy. The presence of bulk viscosity provides the inflationary phase of the current model.

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