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TOTAL GRAPH OF $\mathbb{Z}_N$ AND ITS COMPLEMENT WITH RESPECT TO NIL IDEAL

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ABSTRACT. Let $\mathbb{Z}_n$ be a non-reduced commutative ring and let $N(\mathbb{Z}_n)$ denote the set of the nil elements of $\mathbb{Z}_n$. In this paper, we introduce the total graph of $\mathbb{Z}_n$ with respect to $N(\mathbb{Z}_n)$, denoted by $T(\Gamma_N(\mathbb{Z}_n))$, as a simple undirected graph with all the elements of $\mathbb{Z}_n$ as vertices and any two distinct vertices x and y are adjacent if and only if $x + y \in N(\mathbb{Z}_n)$. Some properties of $T(\Gamma_N(\mathbb{Z}_n))$ and its subgraphs $T_{N(\mathbb{Z}_n)}$ and $T_{\overline{N(\mathbb{Z}_n)}}$ are studied. Also, we study some properties associated to the graph $\overline{T(\Gamma_N(\mathbb{Z}_n))}$, the complement of $T(\Gamma_N(R))$.

1. INTRODUCTION

The idea of the total graph of a commutative ring R, denoted by $T(\Gamma(R))$, was first put forward by Anderson and Badawi [3] as a simple undirected graph having vertex set R and two distinct vertices x and y of $T(\Gamma(R))$ are adjacent if and only if $x + y \in Z(R)$, where $Z(R)$ denotes the set of all the zero-divisors of R. One can find detailed literature on total graphs in [3–5, 7, 8].

P. W. Chen [6], in the year 2003, introduced a special kind of graph structure of a commutative ring R whose vertex set contains all the elements of R and two distinct vertices x and y are adjacent if and only if $xy \in N(R)$, where $N(R)$ denotes the set of all the nil elements of the ring R. This concept was

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later modified by Ai-Hua Li and Qi-Sheng Li [2] who defined it as an undirected simple graph $\Gamma_N(R)$ with vertex set $Z_N(R)^* = \{x \in R^* \mid xy \in N(R)$ for some $y \in R^* = R - \{0\}\}$ and two distinct vertices x and y are adjacent if and only if $xy \in N(R)$ or $yx \in N(R)$.

In this paper, we take $R = \mathbb{Z}_n$. Throughout this paper, we shall use the notation $N(\mathbb{Z}_n)$ to denote the set of all the nil elements of the ring $\mathbb{Z}_n$. That is, $N(\mathbb{Z}_n) = \{x \in \mathbb{Z}_n : x^2 = 0\}$. For any commutative ring R, $N(R)$ is an ideal of R. We call this ideal a *nil ideal* of the ring R. We define the total graph of $\mathbb{Z}_n$ with respect to $N(\mathbb{Z}_n)$, denoted by $T(\Gamma_N(\mathbb{Z}_n))$, as a simple, undirected graph whose vertex set contains all the elements of $\mathbb{Z}_n$ and any two distinct vertices x and y of $T(\Gamma_N(\mathbb{Z}_n))$ are adjacent if and only if $x + y \in N(\mathbb{Z}_n)$. Let $T_{N(\mathbb{Z}_n)}$ and $T_{\overline{N(\mathbb{Z}_n)}}$ denote the induced subgraphs of $T(\Gamma_N(\mathbb{Z}_n))$ whose vertex sets are $N(\mathbb{Z}_n)$ and $\overline{N(\mathbb{Z}_n)}$ respectively, where $\overline{N(\mathbb{Z}_n)} = \mathbb{Z}_n - N(\mathbb{Z}_n)$. Also, the complement of the total graph $T(\Gamma_N(\mathbb{Z}_n))$, denoted by $\overline{T(\Gamma_N(\mathbb{Z}_n))}$, is the simple undirected graph whose vertex set is $\mathbb{Z}_n$ and two distinct vertices x and y are adjacent if and only if $x + y \in \mathbb{Z}_n - N(\mathbb{Z}_n)$.

2. Preliminaries

For any graph G, the diameter of G, denoted by $diam(G)$ is given by $diam(G) = sup\{d(x, y)$: where x and y are distinct vertices of G$\}$ and $d(x, y)$ is the length of the shortest path joining x and y. The *girth* of a graph G, denoted by $gr(G)$, is the length of the shortest cycle in G. If G contains no cycles, then $gr(G) = \infty$. A graph G is said to be *Eulerian* if and only if the degree of each of its vertices is even. A non-empty subset S of the set of all the vertices V of a graph is called a dominating set if every vertex in $V - S$ is adjacent to at least one vertex in S. The *domination number* γ of a graph G is defined to be the minimum cardinality of a dominating set in G and the corresponding dominating set is called a $\gamma-$set of G.

A ring R is said to be *non-reduced* if it contains at least one non-zero nil element. Otherwise it is said to be *reduced*.

3. The basic structure of $T(\Gamma_N(\mathbb{Z}_n))$

For any non-reduced $\mathbb{Z}_n$, the total graph $T(\Gamma_N(\mathbb{Z}_n))$ of $\mathbb{Z}_n$ with respect to its nil ideal $N(\mathbb{Z}_n) = \{x \in \mathbb{Z}_n : x^2 \equiv 0 \pmod{n}\}$ is a simple, undirected graph having vertex set $\mathbb{Z}_n$ and any two distinct vertices x and y of $T(\Gamma_N(\mathbb{Z}_n))$ are adjacent if and only if $x+y \in N(\mathbb{Z}_n)$.

Proposition 3.1. *Let $\mathbb{Z}_n$ be non-reduced and let n be odd. Suppose that $\exists$ some $m \in \mathbb{Z}_n - N(\mathbb{Z}_n)$ such that $2m \in N(\mathbb{Z}_n)$. Then $2m = n_1$, for some $n_1 \in N(\mathbb{Z}_n)$,*

$$\Rightarrow m = \frac{n_1}{2} \begin{cases} \in N(\mathbb{Z}_n) & \text{if } n_1 \text{ is even} \\ \notin \mathbb{Z}_n & \text{if } n_1 \text{ is odd} \end{cases}.$$

In both the cases, we get a contradiction. Thus for any non-reduced $\mathbb{Z}_n$ and for any odd n, $\exists$ no $m \in \mathbb{Z}_n - N(\mathbb{Z}_n)$ such that $2m \in N(\mathbb{Z}_n)$.

Again, let $\mathbb{Z}_n$ be non-reduced and let n be even. Since $\mathbb{Z}_n$ is non-reduced, so either $n = 2^k$ for some $k > 1$, or $n = 2^r.p_1^{r_1}.p_2^{r_2}...p_s^{r_s}$, where at least one $r, r_i's > 1$ (since $\mathbb{Z}_n$ is non-reduced).

(i) *Let $n = 2^k$. Then $\exists$ some $m = 2^k - 2^{[\frac{k-1}{2}]} \in \mathbb{Z}_{2^k} - N(\mathbb{Z}_{2^k})$ such that $m + m = 2m \in N(\mathbb{Z}_{2^k})$.*

(ii) *Let $n = 2^r \cdot p_1^{r_1} \cdot p_2^{r_2} \cdot \cdots \cdot p_s^{r_s}$. Then $\exists$ some $m = 2^{[\frac{r_1+1}{2}]-1} \cdot p_1^{[\frac{r_2+1}{2}]} \cdot \cdots \cdot p_s^{[\frac{r_s+1}{2}]}$ $\in \mathbb{Z}_n - N(\mathbb{Z}_n)$ such that $m+m = 2m \in N(\mathbb{Z}_n)$.*

Thus for any non-reduced $\mathbb{Z}_n$ and for any even n, $\exists$ some $m \in \mathbb{Z}_n - N(\mathbb{Z}_n)$ such that $2m \in N(\mathbb{Z}_n)$.

4. Main Results

For $R = \mathbb{Z}_n$, the set $N(R)$ is an ideal of R. Since $N(R)$ is closed under addition, so for any distinct elements $x, y \in N(R)$, $x + y \in N(R)$.

Throughout this section, we use the notation $|N(\mathbb{Z}_n)| = \alpha$ and $|\mathbb{Z}_n - N(\mathbb{Z}_n)| = \beta$.

Theorem 4.1. *[1] Let $R = \mathbb{Z}_n$ be non-reduced and $N(\mathbb{Z}_n)$ be the set of all the nil elements of $\mathbb{Z}_n$. Then $T_{N(\mathbb{Z}_n)}$ is a complete subgraph of $T(\Gamma_N(\mathbb{Z}_n))$ and $T_{N(\mathbb{Z}_n)}$ is disjoint from $T_{\overline{N(\mathbb{Z}_n)}}$.*

Theorem 4.2. *[1] Let $R = \mathbb{Z}_n$ and let $|N(R)| = \alpha$ and $|R - N(R)| = \beta$. Then*

(1) *If $|R|$ is odd, then $T_{\overline{N(R)}}$ is the disjoint union of $\frac{\beta}{2\alpha}$ complete bipartite graphs $K_{\alpha,\alpha}$.*

(2) *If $|R|$ is even, then $T_{\overline{N(R)}}$ is the disjoint union of the complete graph K_α and $\frac{\beta-\alpha}{2\alpha}$ complete bipartite graphs $K_{\alpha,\alpha}$.*

Theorem 4.3. *[1] Let $R = \mathbb{Z}_n$, $|N(R)| = \alpha$ and $|R - N(R)| = \beta$. Then*

(1) *If $|R|$ is odd, then $T(\Gamma_N(R))$ is the disjoint union of the complete graph K_α and $\frac{\beta}{2\alpha}$ complete bipartite graphs $K_{\alpha,\alpha}$.*
(2) *If $|R|$ is even, then $T(\Gamma_N(R))$ is the disjoint union of two complete graphs K_α and $\frac{\beta-\alpha}{2\alpha}$ complete bipartite graphs $K_{\alpha,\alpha}$.*

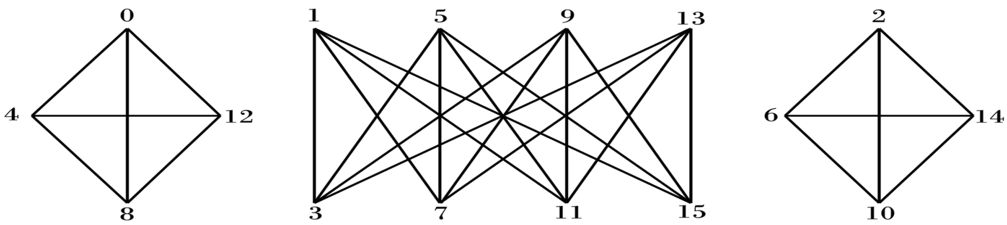

FIGURE 1. $T(\Gamma(Z_{16}))$

Theorem 4.4. *For $R = \mathbb{Z}_n$, let x be any vertex of $T(\Gamma_N(R))$. Then*

$$deg(x) = \begin{cases} \alpha & \text{if } x \in R - N(R) \text{ such that } 2x \notin N(R) \\ \alpha - 1 & \text{if } x \in N(R) \text{ or } x \in R - N(R) \text{ such that } 2x \in N(R) \end{cases}.$$

Proof. From Theorem 4.3, we have

$$T(\Gamma_N(R)) = \begin{cases} K_\alpha \cup (\frac{\beta-\alpha}{2\alpha})K_{\alpha,\alpha} \cup K_\alpha, & \text{if } |R| \text{ is even} \\ K_\alpha \cup (\frac{\beta}{2\alpha})K_{\alpha,\alpha}, & \text{if } |R| \text{ is odd} \end{cases},$$

where the unions are disjoint.

Let $x \in T(\Gamma_N(R))$. Clearly, two cases arise:

Case 1: $|R|$ is odd. If $x \in K_\alpha$, then $deg(x) = \alpha - 1$. If $x \in K_{\alpha,\alpha}$, then $deg(x) = \alpha$.

Case 2: $|R|$ is even. If $x \in K_\alpha$, then $deg(x) = \alpha - 1$. If $x \in K_{\alpha,\alpha}$, then $deg(x) = \alpha$. Since $x \in K_\alpha$, $\forall\, x \in T_{N(R)}$ or $\forall\, x \in R - N(R)$ such that $2x \in N(R)$ and since $x \in K_{\alpha,\alpha}$ $\forall\, x \in R - N(R)$ such that $2x \notin N(R)$, therefore

$$deg(x) = \begin{cases} \alpha & \text{if } x \in R - N(R) \text{ such that } 2x \notin N(R) \\ \alpha - 1 & \text{if } x \in N(R) \text{ or } x \in R - N(R) \text{ such that } 2x \in N(R) \end{cases}.$$

□

Theorem 4.5. *The number of edges of $T(\Gamma_N(\mathbb{Z}_n))$ are*

$$\begin{cases} \frac{\alpha(\alpha+\beta-1)}{2} & \textit{if } n \textit{ is odd} \\ \frac{\alpha(\alpha+\beta-2)}{2} & \textit{if } n \textit{ is even} \end{cases}.$$

Proof. Let n be odd. By Theorem 4.2, $T(\Gamma_N(\mathbb{Z}_n))$ is the disjoint union of $1\ K_\alpha$ and $\frac{\beta}{2\alpha}\ K_{\alpha,\alpha}$'s. Therefore, by the Sum of Degrees of Vertices Theorem, $\alpha(\alpha-1)+\alpha\beta = 2|E|$, where $|E|$ denotes the number of edges, $\Rightarrow |E| = \frac{\alpha(\alpha+\beta-1)}{2}$.

Next, let n be even. Then $T(\Gamma_N(\mathbb{Z}_n))$ is the disjoint union of $2\ K_\alpha$'s and $\frac{\beta-\alpha}{2\alpha}$ $K_{\alpha,\alpha}$'s. Therefore, $\alpha(\alpha-1)+\alpha(\alpha-1)+\alpha(\beta-\alpha) = 2|E| \Rightarrow |E| = \frac{\alpha(\alpha+\beta-2)}{2}$. □

Theorem 4.6. *The graph $T(\Gamma_N(\mathbb{Z}_n))$ is non-Eulerian $\forall\ n \in \mathbb{N}$.*

Proof. From Theorem 4.4, for any $x \in T(\Gamma_N(\mathbb{Z}_n))$,

$$deg(x) = \begin{cases} \alpha & \text{if } x \in R - N(R) \text{ such that } 2x \notin N(R) \\ \alpha - 1 & \text{if } x \in N(R) \text{ or } x \in R - N(R) \text{ such that } 2x \in N(R) \end{cases}.$$

So the graph $T(\Gamma_N(\mathbb{Z}_n))$ contains vertices of degree α as well as $\alpha - 1$, which clearly have different parities. So the degree of each vertex of $T(\Gamma_N(\mathbb{Z}_n))$ is not even and therefore $T(\Gamma_N(\mathbb{Z}_n))$ is not an Eulerian graph. □

Theorem 4.7. *For any $m_1, m_2 \in \mathbb{Z}_n - N(\mathbb{Z}_n)$, m_1 is adjacent to m_2 if and only if every element of the coset $m_1 + N(\mathbb{Z}_n)$ is adjacent to every element of the coset $m_2 + N(\mathbb{Z}_n)$.*

Proof. that m_1 is adjacent to m_2. Then $m_1 + m_2 \in N(\mathbb{Z}_n)$ and thus $m_2 = z_i - m_1$, $z_i \in N(\mathbb{Z}_n)$. The elements of the coset $m_1 + N(\mathbb{Z}_n)$ are adjacent to the elements of the coset $(z_i - m_1) + N(\mathbb{Z}_n)$ since for some $n_1, n_2 \in N(\mathbb{Z}_n)$, $(m_1 + n_1) + (z_i - m_1 + n_2) = z_i + (n_1 + n_2) \in N(\mathbb{Z}_n)$. Conversely, let each element of the coset $m_1 + N(\mathbb{Z}_n)$ be adjacent to each element of $m_2 + N(\mathbb{Z}_n)$. Then for some $n_1, n_2 \in N(\mathbb{Z}_n)$, $(m_1 + n_1) + (m_2 + n_2) \in N(\mathbb{Z}_n) \Rightarrow (m_1 + m_2) + (n_1 + n_2) \in N(\mathbb{Z}_n) \Rightarrow m_1 + m_2 \in N(\mathbb{Z}_n)$. Therefore, m_1 is adjacent to m_2. □

Theorem 4.8. *Let R be a non-reduced commutative ring with unity. Then the following conditions hold:*

(1) *Let G be an induced subgraph of $T_{\overline{N(R)}}$ and let $m_1, m_2 \in G$ such that $m_1 \neq m_2$ and let m_1 and m_2 be connected by a path in G. Then $diam(T_{\overline{N(R)}}) \leq 2$.*

(2) *Let* $R - N(R) \neq \phi$. *If* $T_{\overline{N(R)}}$ *is connected and contains a cycle, then* $gr(T_{\overline{N(R)}}) = 3$ *or* 4.

Proof.

(1) If m_1 is adjacent to m_2, then $d(m_1, m_2) = 1$. Let $d(m_1, m_2) > 1$ and let $m_1 - -a_1 - -a_2 - -m_2$ be a path in G between m_1 and m_2. Then $m_1 + a_1, a_1 + a_2, a_2 + m_2 \in N(R)$. Now, $m_1 + m_2 = (m_1 + a_1) - (a_1 + a_2) - (a_2 + m_2) \in N(R)$, since $N(R)$ is an ideal of R. Hence, m_1 is connected to m_2 by a path of length 2. Thus, $diam(T_{\overline{N(R)}}) \leq 2$.

(2) The result easily follows from Theorem 4.2 since $gr(K_\alpha) = 3$ for $\alpha \geq 3$ and $gr(K_{\alpha,\alpha}) = 4$. □

Theorem 4.9. *Let* $\mathbb{Z}_n$ *be non-reduced and* $N(\mathbb{Z}_n)$ *be the set of all the nil elements of* $\mathbb{Z}_n$. *Then* $diam(T(\Gamma_N(\mathbb{Z}_n))) = 2$.

Proof. Since the diameter of any disconnected graph is equal to the maximum diameter of its connected components, so using Theorem 4.3, since $T(\Gamma_N(\mathbb{Z}_n))$ is the disjoint union of complete and complete bipartite graphs, so $diam(T(\Gamma_N(\mathbb{Z}_n)))$ $= diam(K_{\alpha,\alpha})$. Also, $\mathbb{Z}_n$, being non-reduced, $|N(\mathbb{Z}_n)| = \alpha \geq 2$. Consequently, $diam(T(\Gamma_N(\mathbb{Z}_n))) = 2$. □

Theorem 4.10. *Let* $f : R_1 \longrightarrow R_2$ *be a homomorphism. For any* $m_1, m_1' \in R_1$, *if the coset* $m_1 + N(R_1)$ *is adjacent to each element of* $m_1' + N(R_1)$, *then* $f(m_1) + N(R_2)$ *is adjacent to each element of* $f(m_1') + N(R_2)$.

Proof. Let $m_1 + N(R_1)$ be adjacent to $m_1' + N(R_1)$. Then for some $r_1, r_1' \in N(R_1)$, $(m_1 + r_1) + (m_1' + r_1') \in N(R_1) \Rightarrow (m_1 + m_1') + (r_1 + r_1') \in N(R_1) \Rightarrow m_1 + m_1' \in N(R_1)$. f, being a homomorphism, preserves adjacency and, thus, $f(m_1)$ is adjacent to $f(m_1')$ in R_2. That is, $f(m_1) + f(m_1') \in N(R_2)$. So for some $n_1, n_1' \in N(R_2)$, $(f(m_1) + n_1) + (f(m_1') + n_1') \in N(R_2) \Rightarrow f(m_1) + N(R_2)$ is adjacent to each element of $f(m_1') + N(R_2)$. □

5. Some properties associated to $\overline{T(\Gamma_N(\mathbb{Z}_n))}$, the complement of $T(\Gamma_N(\mathbb{Z}_n))$.

Being a complement of $T(\Gamma_N(\mathbb{Z}_n))$, the graph $\overline{T(\Gamma_N(\mathbb{Z}_n))}$ contains all the elements of $\mathbb{Z}_n$ as vertices and any two distinct vertices x and y of $\overline{T(\Gamma_N(\mathbb{Z}_n))}$ are adjacent if and only if $x + y \in \mathbb{Z}_n - N(\mathbb{Z}_n)$.

Theorem 5.1. *Let $R = \mathbb{Z}_n$. For $x \in N(R)$ and $y \in R - N(R)$ such that $2y \in N(R)$, $\{x + N(R)\} \cup \{y + N(R)\}$ forms a complete bipartite graph in $\overline{T(\Gamma_N(R))}$.*

Proof. In the graph $\overline{T(\Gamma_N(R))}$, each element of the coset $x + N(R)$ is adjacent to each element of the coset $y + N(R)$ since $(x + n_1) + (y + n_2) = (x + y) + (n_1 + n_2) \in R - N(R)$, for some $n_1, n_2 \in N(R)$, since $x + y \in R - N(R)$. Also the elements of the coset $x + N(R)$ are not adjacent to each other because for some $n_1, n_2 \in N(R)$, $(x + n_1) + (x + n_2) = 2x + (n_1 + n_2) \in N(R)$. Also since $2y + (n_1 + n_2)) \in N(R)$, for some $n_1, n_2 \in N(R)$, so the elements of the coset $y + N(R)$ are not adjacent to each other. Consequently, $\{x + N(R)\} \cup \{y + N(R)\}$ forms a complete bipartite graph in $\overline{T(\Gamma_N(R))}$. ☐

Theorem 5.2. *Let $R = \mathbb{Z}_n$ and x be any vertex of $\overline{T(\Gamma_N(R))}$. Then*

$$deg(x) = \begin{cases} n - \alpha & \text{if } x \in N(R) \text{ or } x \in R - N(R) \text{ such that } 2x \in N(R) \\ n - \alpha - 1 & \text{if } x \in R - N(R) \text{ such that } 2x \in R - N(R) \end{cases}.$$

The proof follows directly from Theorem 4.4.

Theorem 5.3. *$\overline{T(\Gamma_N(\mathbb{Z}_n))}$ is never Eulerian.*

The proof is straightforward since the degrees $n - \alpha$ and $n - \alpha - 1$ are of opposite parities.

Theorem 5.4. *Let $R = \mathbb{Z}_n$. Then the following statements hold:*

(a) *If R is a field such that $|R| > 2$, then $T(\Gamma_N(R))$ contains an isolated vertex.*
(b) *$T(\Gamma_N(R))$ contains no vertex of degree $n - 1$.*
(c) *$\overline{T(\Gamma_N(R))}$ contains no isolated vertex.*
(d) *$\overline{T(\Gamma_N(R))}$ contains a vertex of degree $n - 1$ if R is a field.*

Proof.

(a) Since R is a field, so $N(R) = \{0\}$. Thus for each $x \in R$, $\exists$ a unique $y \in R$ such that $x + y = 0 \in N(R)$, i.e. $x = -y$. This gives us $(\frac{n-1}{2})$ pairs of complete graphs K_2 and an isolated vertex 0.

(b) For any $R = \mathbb{Z}_n$, since $1, (n - 1) \in R - N(R)$, so $|N(R)| = \alpha \leq n - 2$. For any $x \in N(R)$ or $x \in R - N(R)$ such that $2x \in N(R)$, by Theorem 4.4, $deg(x) = \alpha - 1 \leq n - 3$. For any $x \in R - N(R)$ such that $2x \in R - N(R)$, $deg(x) = \alpha \leq n - 2$. So in either case, the vertices of $T(\Gamma_N(R))$ have degree less than $n - 1$.

(c) Let $\overline{T(\Gamma_N(R))}$ contain an isolated vertex x. Then in $T(\Gamma_N(R))$, $deg(x) = n-1$. But that contradicts (b). Hence $\overline{T(\Gamma_N(R))}$ contains no isolated vertex.

(d) Let $R = \mathbb{Z}_n$ be a field. By result (a), since $T(\Gamma_N(R))$ contains an isolated vertex, say x, thus, in $\overline{T(\Gamma_N(R))}$, $deg(x) = n-1$. Hence the result follows. □

Theorem 5.5. *For any $n > 1$ and non-reduced $\mathbb{Z}_n$, $\overline{T(\Gamma_N(\mathbb{Z}_n))}$ is always connected.*

Proof. $N(\mathbb{Z}_n)$, being an ideal of $\mathbb{Z}_n$, all the vertices of $N(\mathbb{Z}_n)$ are adjacent to each other in $T(\Gamma_N(\mathbb{Z}_n))$ and therefore in the corresponding graph $\overline{T(\Gamma_N(\mathbb{Z}_n))}$, each $x_i \in N(\mathbb{Z}_n)$ is adjacent to each $y_i \in \mathbb{Z}_n - N(\mathbb{Z}_n)$. So the graph $\overline{T(\Gamma_N(\mathbb{Z}_n))}$ is connected. □

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