

## A MATHEMATICAL MODEL FOR STIMULATED HYPOTHALAMIC-PITUITARY ADRENAL REACTIVITY DYNAMIC CHANGES IN HEALTHY PATIENTS

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**ABSTRACT.** In this analysis, the two probability distributions of Weibull parameters are incorporated into a larger population, obtained by adding an additional parameter. To create a transmuted Weibull distribution, we generalize the two parameters Weibull distributions using Shaw et al's quadratic rank transmutation map [2]. We give a detailed explanation of the mathematical properties of the distribution as well as its reliability behaviour. Real information is shown how useful the transmuted Weibull distribution is for modelling reliability. Eventually, we come to the conclusion that medical solutions had obtained and evaluated the relevant mathematical findings. Ultimately, we conclude that the application part coincides with a mathematical model and the result is linked to the medical report. In the future, this paper will be very beneficial in the medicinal field.

### 1. INTRODUCTION

The consistency of the methods used in a statistical analysis depends heavily on the assumed probability model or distribution. As a result, substantial effort has been made to build broad classes of standard probability distributions along

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with new statistical methods. Nevertheless, there are still several big problems in the absence of any classical or modern probability model for the real facts. The distribution of Weibull is a popular one named after the Swedish physicist Waladdi Weibull. In 1939 he used this distribution to analyze the material's breaking power. Since then, lifetime data in reliability engineering are widely analyzed. It is a versatile distribution, which depends on the value of the form parameter, can take into account the properties of other types of distributions.

A random variable  $Y$  is stated to have the cumulative distribution function (cdf):

$$(1.1) \quad F(y) = (1 + \delta) G(y) - \delta G(y)^2 \quad |\delta| \leq 1,$$

where  $G(y)$  is the base distribution cdf. Note that at  $\delta = 0$ , the base random variable distribution is available. Aryal et al. [1] studied the transmuted distribution of Gumbel, and that the transmuted distribution of Gumbel could be used in the modeling of climate data. In this analysis, the transmuted Weibull distribution and other its properties are mathematically formulated. Possible applications will also be given.

## 2. MATHEMATICAL MODEL AND ASSUMPTIONS

Random variable  $Y$ , if the probability density function is given by the Weibull distribution with the parameter  $\theta > 0$  and  $\sigma > 0$ ,

$$g(y) = \frac{\theta}{\sigma} \left[ \frac{y}{\sigma} \right]^{\theta-1} \exp \left( - \left( \frac{y}{\sigma} \right)^\theta \right), \quad y > 0.$$

The cdf of  $Y$  is given by

$$(2.1) \quad G(y) = 1 - \exp \left( - \left( \frac{y}{\sigma} \right)^\theta \right).$$

Now using (1.1) and (2.1) we have the cdf of transmuted weibull distribution

$$F(y) = \left[ 1 - \exp \left( - \left( \frac{y}{\sigma} \right)^\theta \right) \right] \left[ 1 + \exp \left( - \left( \frac{y}{\sigma} \right)^\theta \right) \right].$$

Hence, the pdf of transmuted Weibull distribution with parameters  $\theta$ ,  $\sigma$  and  $\delta$  is

$$f(y) = \frac{\theta}{\sigma} \left[ \frac{y}{\sigma} \right]^{\theta-1} \exp \left( - \left( \frac{y}{\sigma} \right)^\theta \right) \left[ 1 - \delta + 2\delta \exp \left( - \left( \frac{y}{\sigma} \right)^\theta \right) \right].$$

In this section we shall present the moments and Quantiles for the transmuted Weibull distribution. The  $k^{th}$  order moments of a transmuted Weibull random variable  $Y$  in terms of gamma function  $\Gamma(\cdot)$ , is given by

$$E(Y^K) = \sigma^k \Gamma\left(1 + \frac{k}{\theta}\right) \left\{1 - \delta + \delta 2^{\frac{-k}{\theta}}\right\}.$$

Moreover if  $\frac{k}{\theta} = s$  is a positive integer then

$$E(Y^K) = \sigma^k r! \left\{1 - \delta + \delta 2^{-s}\right\}.$$

Therefore the expected value  $E(Y)$  and variance  $\text{Var}(Y)$  of a transmuted Weibull random variable  $Y$  are respectively, given by

$$\begin{aligned} E(Y) &= \sigma \Gamma\left(1 + \frac{1}{\theta}\right) \left\{1 - \delta + \delta 2^{\frac{-1}{\theta}}\right\}, \\ \text{Var}(Y) &= \sigma^2 \left\{ \Gamma\left(1 + \frac{2}{\theta}\right) \left\{1 - \delta + \delta 2^{\frac{-2}{\theta}}\right\} - \Gamma^2\left(1 + \frac{1}{\theta}\right) \left(1 - \delta + \delta 2^{\frac{-1}{\theta}}\right)^2 \right\}. \end{aligned}$$

### 3. APPLICATIONS

The CRH challenge test is representative of pituitary adrenal reactivity dynamic changes [10]. A lower dose dexamethasone stimulation experiment [5] was conducted in a particular day to check the input tolerance of the hypothalamic pituitary adrenal (HPA) axis.

Figure 1 displays concentrations of salivary cortisol for patients and control upon waking. Clients had significantly lower levels of cortisol than that of the control group at the time of waking, although this disparity did not achieve any value. Figure 2 demonstrates salivary basal cortisol levels of treatment and control groups. Patients began with considerably lower cortisol levels as predicted through the morning cortisol measurements. Figure 3 substantially increased ACTH plasma levels in both groups 20 minutes after CRH injection.

Figure 4 and 5 indicates the levels of saliva cortisol during the test of CRH.

Comparing plasma ACTH levels and plasma and saliva cortisol levels with basal morning cortisol values during CRH test, similar results can be found, with clinicians seeing a less marked rise. It indicates a downward driven tolerance or a shift in the number of CRH receptors in patients at the pituitary level. In major depression [3, 6, 7], anxiety disorders [8, 10], and eating disorders [4, 9],

a blunted ACTH reaction to CRH was found, while in all these tests, cortisol responses were usual.

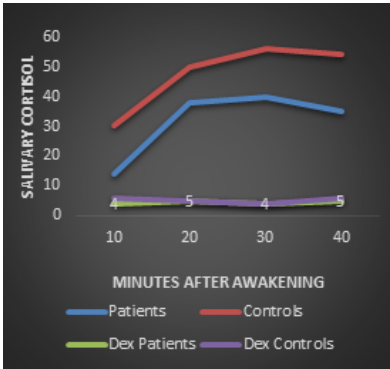


FIGURE 1. Cortisol After Awakening

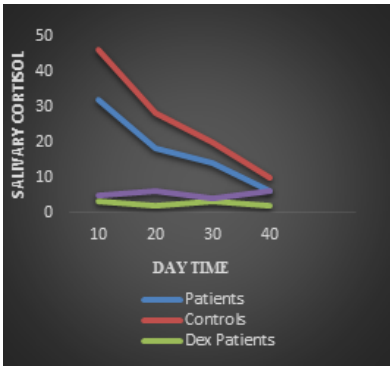


FIGURE 2. Diurnal Salivary Cortisol

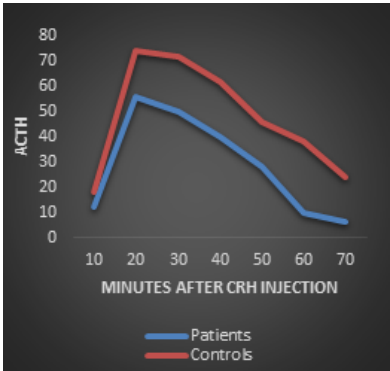


FIGURE 3. Plasma ACTH After CRH Challenge

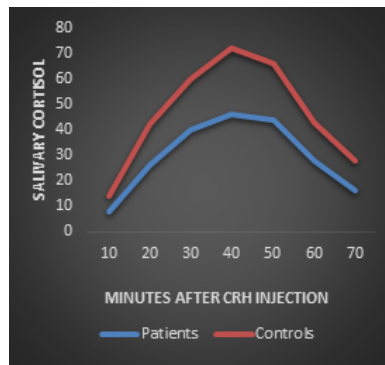


FIGURE 4. Salivary Cortisol After CRH Challenge

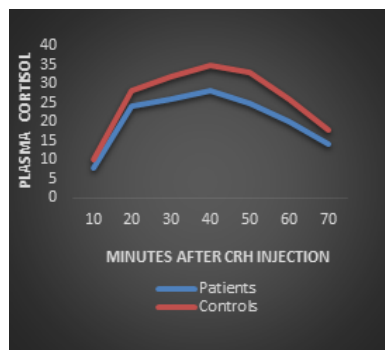


FIGURE 5. Plasma Cortisol After CRH Challenge

#### 4. MATHEMATICAL RESULTS

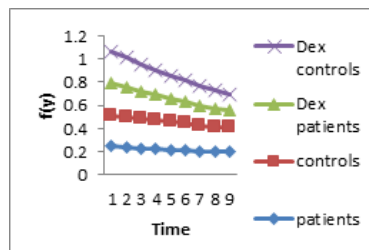


FIGURE 6. Salivary Cortisol after awakening.

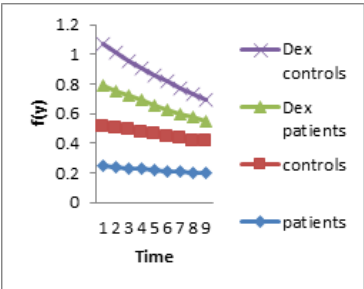


FIGURE 7. Diurnal Salivary Cortisol

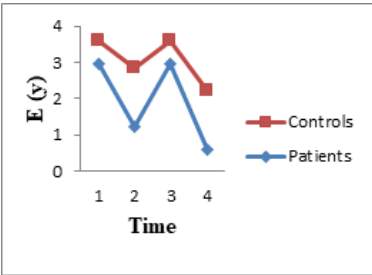


FIGURE 8. Plasma ACTH after CRH Challenge

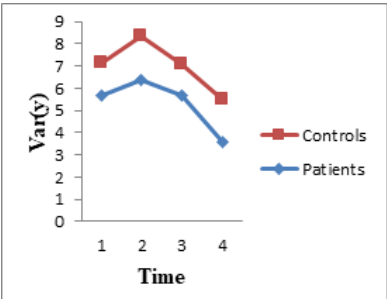


FIGURE 9. Salivary Cortisol after CRH Challenge

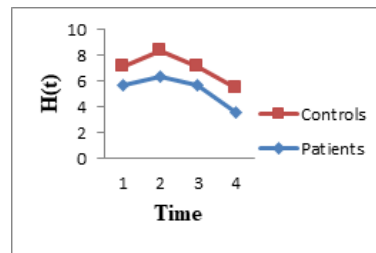


FIGURE 10. Plasma Cortisol after CRH Challenge

## 5. CONCLUSION

In this paper, the weibull distribution called the transmuted weibull distribution was generalized. Using the quadrat meter rank transmutation map, the subject distribution is generated by means of the two parameter weibull distribution. Some mathematical characteristics are addressed together with estimation problems. The hazard function and the reliability behavior of the transmuted weibull distribution show that the distribution of the subject can be used to model the reliability data. The utility of the transmitted weibull distribution and a relation to exponential weibull distribution have been investigated by two data sets published in the literature. Eventually, we come to the conclusion that medical solutions had obtained and evaluated the relevant mathematical findings.

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