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DIHEDRAL GROUP AS GENERALIZED CONJUGACY CLASS GRAPH AND ITS RELEVANT MATRICES

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ABSTRACT. In this paper, the generalized conjugacy class graph for dihedral group of order $2n$ is constructed to show the relation between the orbits and their cardinalities. The orbits of the set denoted by Ω must be computed first by using conjugation action. The elements in each orbit are all pairs of commuting elements in the form of (a, b) where a and b are elements of the dihedral group and the lowest common multiple of the order of the elements has to be two. Also here, some relevant matrices named as adjacency, incident and Laplacian matrices that can represent the graph are also constructed. Eigenvalues from those matrices are computed to give information on graph energies either energy, denoted by $\varepsilon(\Gamma_G)$ or Laplacian energy, denoted by $LE(\Gamma_G)$. Interestingly, we have found that the values for both $\varepsilon(\Gamma_G)$ and $LE(\Gamma_G)$ are equal.

1. INTRODUCTION

Group theory is a branch of abstract algebra developed to study and manipulate abstract concepts involving symmetry. Group theory studies algebraic structures such as groups, rings, fields and vector spaces. One of the most important algebraic structure is groups. Groups are widely used in many branches of physical sciences. This happened because groups are often form when an operation like multiplication or composition is applied to a set or system. Many types of groups

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are found such as group of matrices, dihedral group, symmetry group, cyclic group and others. Interestingly, graph is used to show the behaviour or properties of some groups.

Graph theory begins with very simple geometric ideas and has many powerful applications to solve real world problem. In chemistry, graph theory is used to solve molecular problems and the molecular structure can be represented as a graph where the atoms are the vertices and the edges represent the bond between the atoms. Normally, a graph $G = (V, E)$ consists of two sets V and E. The elements of V are called vertices(or nodes). Then, the elements of E are called edges. The sets V and E are the vertex set and edge set of G and are often denoted by $V(G)$ and $E(G)$, respectively (see [4]).

There are vast amount of studies have been done in order to show the relation between group theory and graph theory. For example, a graph related to conjugacy classes is introduced by Bertram et al.(1990) (see [1]). In 2013, Omer et al. (see [7]) constructed the conjugacy class graph using the probability of metacyclic 2-groups. Furthermore, Omer et al.(2015) (see [8]) introduced generalized conjugacy class graph where the vertices of the graphs are the non-central orbits under group action on a set and the edges are formed if the cardinalities of the orbits are not coprime. Recently, Zaid et al. (2018) (see [10]) constructed generalized conjugacy class graph based on the orbits of three nonabelian metabelian groups of order 12. The resulting graphs are complete graphs K_2, K_3 and K_6. They further used the orbits to find the probability that an element of the group fixes the set containing all pairs of commuting elements of size two.

The theory of graph energy has been used by chemists in approximating the energies related to π electron orbitals in conjugated hydrocarbon. Besides its chemical applications, there are a few applications in other field of science such as in graph entropies, modelling of properties of proteins and in the search for the genetic causes of Alzheimer disease. In mathematics, the energy of a graph of a group is basically the sum of the absolute values of the eigenvalues. Nowadays, many researchers have carried out their investigation on the energies of graphs related to adjacency matrix and Laplacian matrix. The concept of energy and Laplacian energy have been considered in Gutman and Bapat (see [3, 4]).

In this study, generalized conjugacy class graph of dihedral group of order $2n$, where $6 \leq n \leq 12$ is constructed to show the relation between the orbits and their cardinalities. Then, from the graphs formed, it can be represented as adjacency, incidence and Laplacian matrices.

2. PRELIMINARIES

Some basic definitions and steps used in this research are given in this section. In this research, there are six steps as follows:

STEP 1: Constructing the Cayley table to extract information on the commutativity property.

STEP 2: Determine the elements in Ω set that can be defined as below:

$$\Omega = \{(a,b) \in D_{2n} \times D_{2n} \mid ab = ba, a \neq b, lcm(\mid a \mid, \mid b \mid) = 2\}.$$

STEP 3: Identifying the orbits by using the concept of group action.

Definition 2.1 (Group Acts on Set (see [9])). *Let G be a group and X be a set. G acts on X if there is a function:*

$$* : G \times X \to X,$$

such that

i. $(g * h) * x = g * (h * x)$ *, for all* g *,* $h \in G$*,* $x \in X$*,*
ii. *there exists* $e \in G$ *such that* $e * x = x$ *for all* $x \in X$*.*

Definition 2.2 (Orbit (see [2])). *Suppose G is a finite group that act on a set Ω and $\omega \in \Omega$. The orbit of ω, denoted by*

$$O(\omega) = \{g \cdot \omega \mid g \in G, \omega \in \Omega\}.$$

If the group action is conjugation action, the orbit is written as

$$O(\omega) = \left\{g \cdot \omega \cdot g^{-1} | g \in G, \omega \in \Omega\right\}.$$

Hence, $O(\omega)$ is called the conjugacy classes of ω in G. We denote by K as the number of conjugacy classes in a group. Orbit of point ω in set Ω is the set of element of Ω to which ω can be moved by element in G.

STEP 4: Constructing the Generalized Conjugacy Class Graph, $\Gamma_G^{\Omega_c}$.

Definition 2.3 (Generalized Conjugacy Classes Graph (see [8])). *Let G be a finite non abelian group and let Ω be a set of G. If G acts on the set Ω, the vertices of generalized conjugacy class graph are $K(\Omega) - |A|$, where $K(\Omega)$ is the number of conjugacy class under group action on Ω and $A = \{\omega \in \Omega, \omega g = g\omega, g \in G\}$, the central orbits (or sometimes called the central conjugacy classes). Two vertices of $\Gamma_G^{\Omega_c}$ are connected by an edge if their cardinalities are not set-wise relatively prime.*

STEP 5: Finding the matrices of the Generalized Conjugacy Class Graph.

Definition 2.4 (Adjacency Matrix (see [6])). *Let G be a graph of order n and size m, where $V(G) = \{v_1, v_2, .., v_n\}$ and $E(G) = \{e_1, e_2, \ldots, e_m\}$. The* ***adjacency matrix*** *of G is $n \times n$ matrix $A(G) = [a_{ij}]$, where*

$$[a_{ij}] = \begin{cases} 1, \text{ if } v_i v_j \in E(G), \\ 0, \text{ if there is no edge between } v_i \text{ and } v_j, \end{cases}$$

where i is a row and j is a column.

Definition 2.5 (Incidence Matrix (see [6])). *Let G be a graph of order n and size m, where $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $E(G) = \{e_1, e_2, \ldots, e_m\}$. The* ***incident matrix*** *of G is $n \times m$ matrix $I(G) = [b_{ij}]$, where*

$$[b_{ij}] = \begin{cases} 1, & \text{if } v_i \text{ is } \textbf{incident} \text{ with } e_j, \\ 0, & \text{if there is no edge incident on } v_i \ , \end{cases}$$

where i is a row and j is a column.

Definition 2.6 (Laplacian Matrix (see [5])). *Let $G = (V, E)$ be a finite simple graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G) = \{e_1, e_2, \ldots, e_m\}$. Denoting the degree of vertex i by $d_G(i)$, let*

$$D(G) = diag(d_G(1), d_G(2) \ldots, d_G(n))$$

be the diagonal matrix in which its non-zero entries are the degree of vertices in G. The ***Laplacian matrix*** *$L(G)$ of G is defined by $L(G)$=$D(G)$-$A(G)$, where $A(G)$ is the adjacency matrix of G.*

STEP 6: Computing some information from the matrices such as eigenvalues and energies.

Definition 2.7 (Energy (see [4])). *The energy of the graph Γ_G is defined as follows:*

$$\varepsilon(\Gamma_G) = \sum_{i=1}^{n} \mid \lambda_i \mid,$$

where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of the adjacency matrix of Γ_G.

Definition 2.8 (Laplacian Energy (see [3])). *Let G be a graph with n vertices and m edges. Let $\mu_1, \mu_2 \ldots \mu_n$ be the eigenvalues of the Laplacian matrix of G. The Laplacian energy is defined as follows:*

$$LE(\Gamma_G) = \sum_{i=1}^{n} \mid \mu_i - \frac{2m}{n} \mid .$$

3. Main Results

We assume $G = D_{2n}$ presented by $D_{2n} = \langle a, b \mid a^n = b^2 = 1, bab = a^{-1} \rangle$, the dihedral group of order $2n$, to be the focused group for this study. Here, we consider $n = 6, 7, 8, 9, 10, 11, 12$ in order to establish more general observations.

Earlier in this section, we show in details of two examples of the calculation of O_i, the orbits of generalized conjugacy class graph for D_{22} and D_{24}. Then, this discussion is followed by the construction of the generalized conjugacy class graph, $\Gamma_G^{\Omega_c}$, which depends on the number of different orbits obtained. Next, we represent the resulting graphs into relevant matrices named as adjacency, incidence and Laplacian matrices. In the end of this section, we use the information on the eigenvalues of some matrices constructed, to find graph energies denoted by $\varepsilon(\Gamma_G)$ and $LE(\Gamma_G)$.

3.1. **Calculation of Orbits of Generalized Conjugacy Class Graph.**

In this subsection, we display Table 1 which gives information on the total number of orbits of generalized conjugacy class graph, central and non-central orbits. The required calculations were carried out by hand and also with the help of MAPLE. Two examples of the calculations will follow after Table 1.

Table 1: The Number of Orbits of Generalized Conjugacy Class Graph

Dihedral Group, D_{2n}	Number of Orbits, $K(\Omega)$	Number of Central Orbits	Number of Non-central Orbits
D_{12}	12	2	10
D_{14}	2	0	2
D_{16}	12	2	10
D_{18}	2	0	2
D_{20}	12	2	10
D_{22}	2	0	2
D_{24}	12	2	10

Now, calculation in details is shown below for D_{22}.

Example 1. *Dihedral group of order 22, D_{22}.*

Let G be the dihedral group of order 22, $D_{22} = \langle a, b \mid a^{11} = b^2 = 1, bab = a^{-1}\rangle$ and Ω is a set where

$$\Omega = \{(x, y) \in D_{22} \times D_{22} \mid xy = yx, x \neq y, lcm(\mid x \mid, \mid y \mid) = 2\}.$$

Based on the Cayley table, the elements of the set Ω in D_{22} is found as follows:

$$\Omega = \{(1, b), (1, ab), (1, a^2b), (1, a^3b), (1, a^4b), (1, a^5b), (1, a^6b), (1, a^7b), (1, a^8b), (1, a^9b), (1, a^{10}b), (b, 1), (ab, 1), (a^2b, 1), (a^3b, 1), (a^4b, 1), (a^5b, 1), (a^6b, 1), (a^7b, 1), (a^8b, 1), (a^9b, 1), (a^{10}b, 1)\}$$

and giving the number of elements in the set Ω, $\mid \Omega \mid = 22$. Next, the orbits of the set Ω are calculated by using Definition 2.2 as follows:

When $\omega = (1, b)$, then $O(1, b) = \{g \cdot (1, b) \cdot g^{-1} \mid g \in D_{22}\}$. Substituting g with all elements of D_{22}, the following conditions are obtained:

if $g = 1$: $(1 \cdot (1, b) \cdot 1) = (1, b) \cdot 1 = (1, b)$;

if $g = a$: $(a \cdot (1, b) \cdot a^{10}) = (a, ab) \cdot a^{10} = (1, a^2b)$;

if $g = a^2$ *:* $(a^2 \cdot (1, b) \cdot a^9) = (a^2, a^2b) \cdot a^9 = (1, a^4b)$ *;*
if $g = a^3$ *:* $(a^3 \cdot (1, b) \cdot a^8) = (a^3, a^3b) \cdot a^8 = (1, a^6b)$ *;*
if $g = a^4$ *:* $(a^4 \cdot (1, b) \cdot a^7) = (a^4, a^4b) \cdot a^7 = (1, a^8b)$ *;*
if $g = a^5$ *:* $(a^5 \cdot (1, b) \cdot a^6) = (a^5, a^5b) \cdot a^6 = (1, a^{10}b)$ *;*
if $g = a^6$ *:* $(a^6 \cdot (1, b) \cdot a^5) = (a^6, a^6b) \cdot a^5 = (1, ab)$ *;*
if $g = a^7$ *:* $(a^7 \cdot (1, b) \cdot a^4) = (a^7, a^7b) \cdot a^4 = (1, a^3b)$ *;*
if $g = a^8$ *:* $(a^8 \cdot (1, b) \cdot a^3) = (a^8, a^8b) \cdot a^3 = (1, a^5b)$ *;*
if $g = a^9$ *:* $(a^9 \cdot (1, b) \cdot a^2) = (a^9, a^9b) \cdot a^2 = (1, a^7b)$ *;*
if $g = a^{10}$ *:* $(a^{10} \cdot (1, b) \cdot a) = (a^{10}, a^{10}b) \cdot a = (1, a^9b)$ *;*
if $g = b$ *:* $(b \cdot (1, b) \cdot b) = (b, 1) \cdot b = (1, b)$ *;*
if $g = ab$ *:* $(ab \cdot (1, b) \cdot ab) = (ab, a) \cdot ab = (1, a^2b)$ *;*
if $g = a^2b$ *:* $(a^2b \cdot (1, b) \cdot a^2b) = (a^2b, a^2) \cdot a^2b = (1, a^4b)$ *;*
if $g = a^3b$ *:* $(a^3b \cdot (1, b) \cdot a^3b) = (a^3b, a^3) \cdot a^3b = (1, a^6b)$ *;*
if $g = a^4b$ *:* $(a^4b \cdot (1, b) \cdot a^4b) = (a^4b, a^4) \cdot a^4b = (1, a^8b)$ *;*
if $g = a^5b$ *:* $(a^5b \cdot (1, b) \cdot a^5b) = (a^5b, a^5) \cdot a^5b = (1, a^{10}b)$ *;*
if $g = a^6b$ *:* $(a^6b \cdot (1, b) \cdot a^6b) = (a^6b, a^6) \cdot a^6b = (1, ab)$ *;*
if $g = a^7b$ *:* $(a^7b \cdot (1, b) \cdot a^7b) = (a^7b, a^7) \cdot a^7b = (1, a^3b)$ *;*
if $g = a^8b$ *:* $(a^8b \cdot (1, b) \cdot a^8b) = (a^8b, a^8) \cdot a^8b = (1, a^5b)$ *;*
if $g = a^9b$ *:* $(a^9b \cdot (1, b) \cdot a^9b) = (a^9b, a^9) \cdot a^9b = (1, a^7b)$ *; and*
if $g = a^{10}b$ *:* $(a^{10}b \cdot (1, b) \cdot a^{10}b) = (a^{10}b, a^{10}) \cdot a^{10}b = (1, a^9b)\cdot$

Therefore, we have found $O(1, b)$*, the orbit for* $(1, b)$ *as follows:*

1) $O(1, b) = \{(1, b), (1, ab), (1, a^2b), (1, a^3b), (1, a^4b), (1, a^5b), (1, a^6b), (1, a^7b), (1, a^8b), (1, a^9b), (1, a^{10}b)\}.$

This means the following:

$$O(1, b) = O(1, ab) = O(1, a^2b) = O(1, a^3b) = O(1, a^4b) = O(1, a^5b) = O(1, a^6b) = O(1, a^7b) = O(1, a^8b) = O(1, a^9b) = O(1, a^{10}b).$$

When $\omega = (b, 1)$*, then* $O(b, 1) = \{g \cdot (b, 1) \cdot g^{-1} \mid g \in D_{22}\}$*. Substituting* g *with all elements of* D_{22}*, the following conditions are obtained:*

if $g = 1$ *:* $(1 \cdot (b, 1) \cdot 1) = (b, 1) \cdot 1 = (b, 1)$ *;*
if $g = a$ *:* $(a \cdot (b, 1) \cdot a^{10}) = (ab, a) \cdot a^{10} = (a^2b, 1)$ *;*
if $g = a^2$ *:* $(a^2 \cdot (b, 1) \cdot a^9) = (a^2b, a^2) \cdot a^9 = (a^4b, 1)$ *;*
if $g = a^3$ *:* $(a^3 \cdot (b, 1) \cdot a^8) = (a^3b, a^3) \cdot a^8 = (a^6b, 1)$ *;*

if $g = a^4$ *:* $(a^4 \cdot (b, 1) \cdot a^7) = (a^4b, a^4) \cdot a^7 = (a^8b.1)$ *;*
if $g = a^5$ *:* $(a^5 \cdot (b, 1) \cdot a^6) = (a^5b, a^5) \cdot a^6 = (a^{10}b, 1)$ *;*
if $g = a^6$ *:* $(a^6 \cdot (b, 1) \cdot a^5) = (a^6b, a^6) \cdot a^5 = (ab, 1)$ *;*
if $g = a^7$ *:* $(a^7 \cdot (b, 1) \cdot a^4) = (a^7b, a^7) \cdot a^4 = (a^3b, 1)$ *;*
if $g = a^8$ *:* $(a^8 \cdot (b, 1) \cdot a^3) = (a^8b, a^8) \cdot a^3 = (a^5b, 1)$ *;*
if $g = a^9$ *:* $(a^9 \cdot (b, 1) \cdot a^2) = (a^9b, a^9) \cdot a^2 = (a^7b, 1)$ *;*
if $g = a^{10}$ *:* $(a^{10} \cdot (b, 1) \cdot a) = (a^{10}b, a^{10}) \cdot a = (a^9b, 1)$ *;*
if $g = b$ *:* $(b \cdot (b, 1) \cdot b) = (1, b) \cdot b = (b, 1)$ *;*
if $g = ab$ *:* $(ab \cdot (b, 1) \cdot ab) = (a, ab) \cdot ab = (a^2b, 1)$ *;*
if $g = a^2b$ *:* $(a^2b \cdot (b, 1) \cdot a^2b) = (a^2, a^2b) \cdot a^2b = (a^4b, 1)$ *;*
if $g = a^3b$ *:* $(a^3b \cdot (b, 1) \cdot a^3b) = (a^3, a^3b) \cdot a^3b = (a^6b, 1)$ *;*
if $g = a^4b$ *:* $(a^4b \cdot (b, 1) \cdot a^4b) = (a^4, a^4b) \cdot a^4b = (a^8b, 1)$ *;*
if $g = a^5b$ *:* $(a^5b \cdot (b, 1) \cdot a^5b) = (a^5, a^5b) \cdot a^5b = (a^{10}b, 1)$ *;*
if $g = a^6b$ *:* $(a^6b \cdot (b, 1) \cdot a^6b) = (a^6, a^6b) \cdot a^6b = (ab, 1)$ *;*
if $g = a^7b$ *:* $(a^7b \cdot (b, 1) \cdot a^7b) = (a^7, a^7b) \cdot a^7b = (a^3b, 1)$ *;*
if $g = a^8b$ *:* $(a^8b \cdot (b, 1) \cdot a^8b) = (a^8, a^8b) \cdot a^8b = (a^5b, 1)$ *;*
if $g = a^9b$ *:* $(a^9b \cdot (b, 1) \cdot a^9b) = (a^9, a^9b) \cdot a^9b = (a^7b, 1)$ *; and*
if $g = a^{10}b$ *:* $(a^{10}b \cdot (b, 1) \cdot a^{10}b) = (a^{10}, a^{10}b) \cdot a^{10}b = (a^9b, 1)\cdot$

Therefore, we have found $O(b, 1)$*, the orbit for* $(b, 1)$ *as follows:*

2) $O(b, 1) = \{(b, 1), (ab, 1), (a^2b, 1), (a^3b, 1), (a^4b, 1), (a^5b, 1), (a^6b, 1), (a^7b, 1), (a^8b, 1), (a^9b, 1), (a^{10}b, 1)\}$

This means the following:

$$O(b, 1) = O(ab, 1) = O(a^2b, 1) = O(a^3b, 1) = O(a^4b, 1) = O(a^5b, 1) = O(a^6b, 1)$$
$$= O(a^7b, 1) = O(a^8b, 1) = O(a^9b, 1) = O(a^{10}b, 1).$$

Based on above calculation we list all the orbits of the set Ω *for* D_{22} *and relabel them with* O_i *where* $i = 1, \ldots, K(\Omega)$ *for convenience as follows:*

$$O_1 = O(1, b) = \{(1, b), (1, ab), (1, a^2b), (1, a^3b), (1, a^4b), (1, a^5b), (1, a^6b), (1, a^7b), (1, a^8b), (1, a^9b), (1, a^{10}b)\},$$
$$O_2 = O(b, 1) = \{(b, 1), (ab, 1), (a^2b, 1), (a^3b, 1), (a^4b, 1), (a^5b, 1), (a^6b, 1), (a^7b, 1), (a^8b, 1), (a^9b, 1), (a^{10}b, 1)\}.$$

According to Definition 2.2, the number of orbits in D_{22} is $K(\Omega) = 2$.

Next, calculation in details for D_{24} is shown below where we notice a significant difference in terms of the existence of central-orbits.

Example 2. *Dihedral group of order 24, D_{24}.*

Let G be the dihedral group of order 24, $D_{24} = \langle a, b \mid a^{12} = b^2 = 1, bab = a^{-1}\rangle$, and Ω is a set where

$$\Omega = \{(x, y) \in D_{24} \times D_{24} \mid xy = yx, x \neq y, lcm(\mid x \mid, \mid y \mid) = 2\}.$$

Based on the Cayley table, the elements of the set Ω in D_{24} is found as follows:

$$\begin{aligned}\Omega =\{&(1, a^6), (1, b), (1, ab), (1, a^2b), (1, a^3b), (1, a^4b), (1, a^5b),\\ &(1, a^6b), (1, a^7b), (1, a^8b), (1, a^9b), (1, a^{10}b), (1, a^{11}b),\\ &(a^6, 1), (a^6, b), (a^6, ab), (a^6, a^2b), (a^6, a^3b), (a^6, a^4b),\\ &(a^6, a^5b), (a^6, a^6b), (a^6, a^7b), (a^6, a^8b), (a^6, a^9b),\\ &(a^6, a^{10}b), (a^6, a^{11}b), (b, 1), (b, a^6), (b, a^6b), (ab, 1),\\ &(ab, a^6), (ab, a^7b), (a^2b, 1), (a^2b, a^6), (a^2b, a^8b), (a^3b, 1),\\ &(a^3b, a^6), (a^3b, a^9b), (a^4b, 1), (a^4b, a^6), (a^4b, a^{10}b),\\ &(a^5b, 1), (a^5b, a^6), (a^5b, a^{11}b), (a^6b, 1), (a^6b, a^6),\\ &(a^6b, b), (a^7b, 1), (a^7b, a^6), (a^7b, ab), (a^8b, 1), (a^8b, a^6),\\ &(a^8b, a^2b), (a^9b, 1), (a^9b, a^6), (a^9b, a^3b), (a^{10}b, 1),\\ &(a^{10}b, a^6), (a^{10}b, a^4b), (a^{11}b, 1), (a^{11}b, a^6), (a^{11}b, a^5b)\},\end{aligned}$$

and giving the number of elements in the set Ω, $\mid \Omega \mid= 62$. Next, the orbit of the set Ω are calculated by using Definition 2.2 as follows:

When $\omega = (1, a^6)$, then $O(1, a^6) = \{g \cdot (1, a^6) \cdot g^{-1} \mid g \in D_{24}\}$. Substituting g with all elements of D_{24}, the following conditions are obtained:

if $g = 1$ *:* $(1 \cdot (1, a^6) \cdot 1) = (1, a^6) \cdot 1 = (1, a^6)$ *;*
if $g = a$ *:* $(a \cdot (1, a^6) \cdot a^{11}) = (a, a^7) \cdot a^{11} = (1, a^6)$ *;*
if $g = a^2$ *:* $(a^2 \cdot (1, a^6) \cdot a^{10}) = (a^2, a^8) \cdot a^{10} = (1, a^6)$ *;*
if $g = a^3$ *:* $(a^3 \cdot (1, a^6) \cdot a^9) = (a^3, a^9) \cdot a^9 = (1, a^6)$ *;*
if $g = a^4$ *:* $(a^4 \cdot (1, a^6) \cdot a^8) = (a^4, a^{10}) \cdot a^8 = (1, a^6)$ *;*
if $g = a^5$ *:* $(a^5 \cdot (1, a^6) \cdot a^7) = (a^5, a^{11}) \cdot a^7 = (1, a^6)$ *;*

if $g = a^6$ *:* $(a^6 \cdot (1, a^6) \cdot a^6) = (a^6, 1) \cdot a^6 = (1, a^6)$ *;*
if $g = a^7$ *:* $(a^7 \cdot (1, a^6) \cdot a^5) = (a^7, a) \cdot a^5 = (1, a^6)$ *;*
if $g = a^8$ *:* $(a^8 \cdot (1, a^6) \cdot a^4) = (a^8, a^2) \cdot a^4 = (1, a^6)$
if $g = a^9$ *:* $(a^9 \cdot (1, a^6) \cdot a^3) = (a^9, a^3) \cdot a^3 = (1, a^6)$ *;*
if $g = a^{10}$ *:* $(a^{10} \cdot (1, a^6) \cdot a^2) = (a^{10}, a^4) \cdot a^2 = (1, a^5)$ *;*
if $g = a^{11}$ *:* $(a^{11} \cdot (1, a^6) \cdot a) = (a^{11}, a^5) \cdot a = (1, a^6)$ *;*
if $g = b$ *:* $(b \cdot (1, a^6) \cdot b) = (b, a^6 b) \cdot b = (1, a^6)$ *;*
if $g = ab$ *:* $(ab \cdot (1, a^6) \cdot ab) = (ab, a^7 b) \cdot ab = (1, a^6)$ *;*
if $g = a^2 b$ *:* $(a^2 b \cdot (1, a^6) \cdot a^2 b) = (a^2 b, a^8 b) \cdot a^2 b = (1, a^6)$ *;*
if $g = a^3 b$ *:* $(a^3 b \cdot (1, a^6) \cdot a^3 b) = (a^3 b, a^9 b) \cdot a^3 b = (1, a^6)$ *;*
if $g = a^4 b$ *:* $(a^4 b \cdot (1, a^6) \cdot a^4 b) = (a^4 b, a^{10} b) \cdot a^4 b = (1, a^6)$ *;*
if $g = a^5 b$ *:* $(a^5 b \cdot (1, a^6) \cdot a^5 b) = (a^5 b, a^{11} b) \cdot a^5 b = (1, a^6)$ *;*
if $g = a^6 b$ *:* $(a^6 b \cdot (1, a^6) \cdot a^6 b) = (a^6 b, b) \cdot a^6 b = (1, a^6)$ *;*
if $g = a^7 b$ *:* $(a^7 b \cdot (1, a^6) \cdot a^7 b) = (a^7 b, ab) \cdot a^7 b = (1, a^6)$ *;*
if $g = a^8 b$ *:* $(a^8 b \cdot (1, a^6) \cdot a^8 b) = (a^8 b, a^2 b) \cdot a^8 b = (1, a^6)$ *;*
if $g = a^9 b$ *:* $(a^9 b \cdot (1, a^6) \cdot a^9 b) = (a^9 b, a^3 b) \cdot a^9 b = (1, a^6)$ *;*
if $g = a^{10} b$ *:* $(a^{10} b \cdot (1, a^6) \cdot a^{10} b) = (a^{10} b, a^4 b) \cdot a^{10} b = (1, a^6)$ *; and*
if $g = a^{11} b$ *:* $(a^{11} b \cdot (1, a^6) \cdot a^{11} b) = (a^{11} b, a^5 b) \cdot a^{11} b = (1, a^6) \cdot$.

Therefore, we have found the following two central orbits:

1) $O(1, a^6) = \{(1, a^6)\}$,
2) $O(a^6, 1) = \{(a^6, 1)\}$.

When $\omega = (1, b)$, *then* $O(1, b) = \{g \cdot (1, b) \cdot g^{-1} \mid g \in D_{24}\}$. *Substituting* g *with all elements of* D_{24}, *the following conditions are obtained:*

if $g = 1$ *:* $(1 \cdot (1, b) \cdot 1) = (1, b) \cdot 1 = (1, b)$ *;*
if $g = a$ *:* $(a \cdot (1, b) \cdot a^{11}) = (a, ab) \cdot a^{11} = (1, a^2 b)$ *;*
if $g = a^2$ *:* $(a^2 \cdot (1, b) \cdot a^{10}) = (a^2, a^2 b) \cdot a^{10} = (1, a^4 b)$ *;*
if $g = a^3$ *:* $(a^3 \cdot (1, b) \cdot a^9) = (a^3, a^3 b) \cdot a^9 = (1, a^6 b)$ *;*
if $g = a^4$ *:* $(a^4 \cdot (1, b) \cdot a^8) = (a^4, a^4 b) \cdot a^8 = (1, a^8 b)$ *;*
if $g = a^5$ *:* $(a^5 \cdot (1, b) \cdot a^7) = (a^5, a^5 b) \cdot a^7 = (1, a^{10} b)$ *;*
if $g = a^6$ *:* $(a^6 \cdot (1, b) \cdot a^6) = (a^6, a^6 b) \cdot a^6 = (1, b)$ *;*
if $g = a^7$ *:* $(a^7 \cdot (1, b) \cdot a^5) = (a^7, a^7 b) \cdot a^5 = (1, a^2 b)$ *;*
if $g = a^8$ *:* $(a^8 \cdot (1, b) \cdot a^4) = (a^8, a^8 b) \cdot a^4 = (1, a^4 b)$ *;*
if $g = a^9$ *:* $(a^9 \cdot (1, b) \cdot a^3) = (a^9, a^9 b) \cdot a^3 = (1, a^6 b)$ *;*
if $g = a^{10}$ *:* $(a^{10} \cdot (1, b) \cdot a^2) = (a^{10}, a^{10} b) \cdot a^2 = (1, a^8 b)$ *;*

if $g = a^{11}$ *:* $(a^{11} \cdot (1, b) \cdot a) = (a^{11}, a^{11}b) \cdot a = (1, a^{10}b)$ *;*
if $g = b$ *:* $(b \cdot (1, b) \cdot b) = (b, 1) \cdot b = (1, b)$ *;*
if $g = ab$ *:* $(ab \cdot (1, b) \cdot ab) = (ab, a) \cdot ab = (1, a^2b)$ *;*
if $g = a^2b$ *:* $(a^2b \cdot (1, b) \cdot a^2b) = (a^2b, a^2) \cdot a^2b = (1, a^4b)$ *;*
if $g = a^3b$ *:* $(a^3b \cdot (1, b) \cdot a^3b) = (a^3b, a^3) \cdot a^3b = (1, a^6b)$ *;*
if $g = a^4b$ *:* $(a^4b \cdot (1, b) \cdot a^4b) = (a^4b, a^4) \cdot a^4b = (1, a^8b)$ *;*
if $g = a^5b$ *:* $(a^5b \cdot (1, b) \cdot a^5b) = (a^5b, a^5) \cdot a^5b = (1, a^{10}b)$ *;*
if $g = a^6b$ *:* $(a^6b \cdot (1, b) \cdot a^6b) = (a^6b, a^6) \cdot a^6b = (1, b)$ *;*
if $g = a^7b$ *:* $(a^7b \cdot (1, b) \cdot a^7b) = (a^7b, a^7) \cdot a^7b = (1, a^2b)$ *;*
if $g = a^8b$ *:* $(a^8b \cdot (1, b) \cdot a^8b) = (a^8b, a^8) \cdot a^8b = (1, a^4b)$ *;*
if $g = a^9b$ *:* $(a^9b \cdot (1, b) \cdot a^9b) = (a^9b, a^9) \cdot a^9b = (1, a^6b)$ *;*
if $g = a^{10}b$ *:* $(a^{10}b \cdot (1, b) \cdot a^{10}b) = (a^{10}b, a^{10}) \cdot a^{10}b = (1, a^8b)$ *; and*
if $g = a^{11}b$ *:* $(a^{11}b \cdot (1, b) \cdot a^{11}b) = (a^{11}b, a^{11}) \cdot a^{11}b = (1, a^{10}b)\cdot$

Therefore, we have found the following two orbits:

$$3)\ O(1,b) = \{(1,b),(1,a^2b),(1,a^4b),(1,a^6b),(1,a^8b),(1,a^{10}b)\}$$
$$= O(1,a^2b) = O(1,a^4b) = O(1,a^6b) = O(1,a^8b) = O(1,a^{10}b),$$

$$4)\ O(b,1) = \{(b,1),(a^2b,1),(a^4b,1),(a^6b,1),(a^8b,1),(a^{10}b,1)\}$$
$$= O(a^2b,1) = O(a^4b,1) = O(a^6b,1) = O(a^8b,1) = O(a^{10}b,1).$$

When $\omega = (1, ab)$*, then* $O(1, ab) = \{g \cdot (1, ab) \cdot g^{-1} \mid g \in D_{24}\}$*. Substituting* g *with all elements of* D_{24}*, the following conditions are obtained:*
if $g = 1$ *:* $(1 \cdot (1, ab) \cdot 1) = (1, ab) \cdot 1 = (1, ab)$*;*
if $g = a$ *:* $(a \cdot (1, ab) \cdot a^{11}) = (a, a^2b) \cdot a^{11} = (1, a^3b)$*;*
if $g = a^2$ *:* $(a^2 \cdot (1, ab) \cdot a^{10}) = (a^2, a^3b) \cdot a^{10} = (1, a^5b)$ *;*
if $g = a^3$ *:* $(a^3 \cdot (1, ab) \cdot a^9) = (a^3, a^4b) \cdot a^9 = (1, a^7b)$*;*
if $g = a^4$ *:* $(a^4 \cdot (1, ab) \cdot a^8) = (a^4, a^5b) \cdot a^8 = (1, a^9b)$*;*
if $g = a^5$ *:* $(a^5 \cdot (1, ab) \cdot a^7) = (a^5, a^6b) \cdot a^7 = (1, a^{11}b)$ *;*
if $g = a^6$ *:* $(a^6 \cdot (1, ab) \cdot a^6) = (a^6, a^7b) \cdot a^6 = (1, ab)$*;*
if $g = a^7$ *:* $(a^7 \cdot (1, ab) \cdot a^5) = (a^7, a^8b) \cdot a^5 = (1, a^3b)$*;*
if $g = a^8$ *:* $(a^8 \cdot (1, ab) \cdot a^4) = (a^8, a^9b) \cdot a^4 = (1, a^5b)$*;*
if $g = a^9$ *:* $(a^9 \cdot (1, ab) \cdot a^3) = (a^9, a^{10}b) \cdot a^3 = (1, a^7b)$*;*
if $g = a^{10}$ *:* $(a^{10} \cdot (1, ab) \cdot a^2) = (a^{10}, a^{11}b) \cdot a^2 = (1, a^9b)$*;*
if $g = a^{11}$ *:* $(a^{11} \cdot (1, ab) \cdot a) = (a^{11}, b) \cdot a = (1, a^{11}b)$*;*

if $g = b$ *:* $(b \cdot (1, ab) \cdot b) = (b, a^11) \cdot b = (1, a^{11}b)$*;*
if $g = ab$ *:* $(ab \cdot (1, ab) \cdot ab) = (ab, 1) \cdot ab = (1, ab)$*;*
if $g = a^2b$ *:* $(a^2b \cdot (1, ab) \cdot a^2b) = (a^2b, a) \cdot a^2b = (1, a^3b)$ *;*
if $g = a^3b$ *:* $(a^3b \cdot (1, ab) \cdot a^3b) = (a^3b, a^2) \cdot a^3b = (1, a^5b)$ *;*
if $g = a^4b$ *:* $(a^4b \cdot (1, ab) \cdot a^4b) = (a^4b, a^3) \cdot a^4b = (1, a^7b)$ *;*
if $g = a^5b$ *:* $(a^5b \cdot (1, ab) \cdot a^5b) = (a^5b, a^4) \cdot a^5b = (1, a^9b)$ *;*
if $g = a^6b$ *:* $(a^6b \cdot (1, ab) \cdot a^6b) = (a^6b, a^5) \cdot a^6b = (1, a^{11}b)$ *;*
if $g = a^7b$ *:* $(a^7b \cdot (1, ab) \cdot a^7b) = (a^7b, a^6) \cdot a^7b = (1, ab)$*;*
if $g = a^8b$ *:* $(a^8b \cdot (1, ab) \cdot a^8b) = (a^8b, a^7) \cdot a^8b = (1, a^3b)$ *;*
if $g = a^9b$ *:* $(a^9b \cdot (1, ab) \cdot a^9b) = (a^9b, a^8) \cdot a^9b = (1, a^5b)$ *;*
if $g = a^{10}b$ *:* $(a^{10}b \cdot (1, ab) \cdot a^{10}b) = (a^{10}b, a^9) \cdot a^{10}b = (1, a^7b)$ *; and*
if $g = a^{11}b$ *:* $(a^{11}b \cdot (1, ab) \cdot a^{11}b) = (a^{11}b, a^{10}) \cdot a^{11}b = (1, a^9b)\cdot$

Therefore, we have found the following orbits:

$$\begin{aligned} 5)\ O(1, ab) &= \big\{(1, ab), (1, a^3b), (1, a^5b), (1, a^7b), (1, a^9b), (1, a^{11}b)\big\} \\ &= O(1, a^3b) = O(1, a^5b) = O(1, a^7b) = O(1, a^9b) = O(1, a^{11}b), \end{aligned}$$

$$\begin{aligned} 6)\ O(ab, 1) &= \big\{(ab, 1), (a^3b, 1), (a^5b, 1), (a^7b, 1), (a^9b, 1), (a^{11}b, 1)\big\} \\ &= O(a^3b, 1) = O(a^5b, 1) = O(a^7b, 1) = O(a^9b, 1) = O(a^{11}b, 1). \end{aligned}$$

When $\omega = (b, a^6)$*, then* $O(b, a^6) = \{g \cdot (b, a^6) \cdot g^{-1} \mid g \in D_{24}\}$*. Substituting* g *with all elements of* D_{24}*, the following conditions are obtained:*
if $g = 1$ *:* $(1 \cdot (b, a^6) \cdot 1) = (b, a^6) \cdot 1 = (b, a^6)$ *;*
if $g = a$ *:* $(a \cdot (b, a^6) \cdot a^{11}) = (ab, a^7) \cdot a^{11} = (a^2b, a^6)$ *;*
if $g = a^2$ *:* $(a^2 \cdot (b, a^6) \cdot a^{10}) = (a^2b, a^8) \cdot a^{10} = (a^4b, a^6)$ *;*
if $g = a^3$ *:* $(a^3 \cdot (b, a^6) \cdot a^9) = (a^3b, a^9) \cdot a^9 = (a^6b, a^6)$ *;*
if $g = a^4$ *:* $(a^4 \cdot (b, a^6) \cdot a^8) = (a^4b, a^{10}) \cdot a^8 = (a^8b, a^6)$ *;*
if $g = a^5$ *:* $(a^5 \cdot (b, a^6) \cdot a^7) = (a^5b, a^{11}) \cdot a^7 = (a^10b, a^6)$ *;*
if $g = a^6$ *:* $(a^6 \cdot (b, a^6) \cdot a^6) = (a^6b, 1) \cdot a^6 = (1, a^6)$ *;*
if $g = a^7$ *:* $(a^7 \cdot (b, a^6) \cdot a^5) = (a^7b, a) \cdot a^5 = (a^2b, a^6)$ *;*
if $g = a^8$ *:* $(a^8 \cdot (b, a^6) \cdot a^4) = (a^8b, a^2) \cdot a^4 = (a^4b, a^6)$ *;*
if $g = a^9$ *:* $(a^9 \cdot (b, a^6) \cdot a^3) = (a^9b, a^3) \cdot a^3 = (a^6b, a^6)$ *;*
if $g = a^{10}$ *:* $(a^{10} \cdot (b, a^6) \cdot a^2) = (a^{10}b, a^4) \cdot a^2 = (a^8b, a^6)$ *;*
if $g = a^{11}$ *:* $(a^{11} \cdot (b, a^6) \cdot a) = (a^{11}b, a^5) \cdot a = (a^{10}b, a^6)$ *;*
if $g = b$ *:* $(b \cdot (b, a^6) \cdot b) = (1, a^6b) \cdot b = (b, a^6)$ *;*

if $g = ab$ *:* $(ab \cdot (b, a^6) \cdot ab) = (a, a^7b) \cdot ab = (a^2b, a^6)$ *;*
if $g = a^2b$ *:* $(a^2b \cdot (b, a^6) \cdot a^2b) = (a^2, a^8b) \cdot a^2b = (a^4b, a^6)$ *;*
if $g = a^3b$ *:* $(a^3b \cdot (b, a^6) \cdot a^3b) = (a^3, a^9b) \cdot a^3b = (a^6b, a^6)$ *;*
if $g = a^4b$ *:* $(a^4b \cdot (b, a^6) \cdot a^4b) = (a^4, a^{10}b) \cdot a^4b = (a^8b, a^6)$ *;*
if $g = a^5b$ *:* $(a^5b \cdot (b, a^6) \cdot a^5b) = (a^5, a^{11}b) \cdot a^5b = (a^{10}b, a^6)$ *;*
if $g = a^6b$ *:* $(a^6b \cdot (b, a^6) \cdot a^6b) = (a^6, b) \cdot a^6b = (b, a^6)$ *;*
if $g = a^7b$ *:* $(a^7b \cdot (b, a^6) \cdot a^7b) = (a^7, ab) \cdot a^7b = (a^2b, a^6)$ *;*
if $g = a^8b$ *:* $(a^8b \cdot (b, a^6) \cdot a^8b) = (a^8, a^2b) \cdot a^8b = (a^4b, a^6)$ *;*
if $g = a^9b$ *:* $(a^9b \cdot (b, a^6) \cdot a^9b) = (a^9, a^3b) \cdot a^9b = (a^6b, a^5b)$ *;*
if $g = a^{10}b$ *:* $(a^{10}b \cdot (b, a^6) \cdot a^{10}b) = (a^{10}, a^4b) \cdot a^{10}b = (a^8b, a^6)$ *; and*
if $g = a^{11}b$ *:* $(a^{11}b \cdot (b, a^6) \cdot a^{11}b) = (a^{11}, a^5b) \cdot a^{11}b = (a^10b, a^6)\cdot$

Then, we have found another two orbits as follows:

$$7)\ O(b, a^6) = \{(b, a^6), (a^2b, a^6), (a^4b, a^6), (a^6b, a^6), (a^8b, a^6), (a^{10}b, a^6)\}$$
$$= O(a^2b, a^6) = O(a^4b, a^6) = O(a^6b, a^6) = O(a^8b, a^6) = O(a^{10}b, a^6),$$

$$8)\ O(a^6, b) = \{(a^6, b), (a^6, a^2b), (a^6, a^4b), (a^6, a^6b), (a^6, a^8b), (a^6, a^{10}b)\}$$
$$= O(a^6, a^2b) = O(a^6, a^4b) = O(a^6, a^6b) = O(a^6, a^8b) = O(a^6, a^{10}b).$$

When $\omega = (ab, a^6)$*, then* $O(ab, a^6) = \{g \cdot (ab, a^6) \cdot g^{-1} \mid g \in D_{24}\}$*. Substituting* g *with all elements of* D_{24}*, the following conditions are obtained:*

if $g = 1$ *:* $(1 \cdot (ab, a^6) \cdot 1) = (ab, a^6) \cdot 1 = (ab, a^6)$ *;*
if $g = a$ *:* $(a \cdot (ab, a^6) \cdot a^{11}) = (a^2b, a^7) \cdot a^{11} = (a^3b, a^6)$ *;*
if $g = a^2$ *:* $(a^2 \cdot (ab, a^6) \cdot a^{10}) = (a^3b, a^8) \cdot a^{10} = (a^5b, a^6)$ *;*
if $g = a^3$ *:* $(a^3 \cdot (ab, a^6) \cdot a^9) = (a^4b, a^9) \cdot a^9 = (a^7b, a^6)$ *;*
if $g = a^4$ *:* $(a^4 \cdot (ab, a^6) \cdot a^8) = (a^5b, a^{10}) \cdot a^8 = (a^9b, a^6)$ *;*
if $g = a^5$ *:* $(a^5 \cdot (ab, a^6) \cdot a^7) = (a^6b, a^{11}) \cdot a^7 = (a^11b, a^6)$ *;*
if $g = a^6$ *:* $(a^6 \cdot (ab, a^6) \cdot a^6) = (a^7b, 1) \cdot a^6 = (ab, a^6)$ *;*
if $g = a^7$ *:* $(a^7 \cdot (ab, a^6) \cdot a^5) = (a^8b, a) \cdot a^5 = (a^3b, a^6)$ *;*
if $g = a^8$ *:* $(a^8 \cdot (ab, a^6) \cdot a^4) = (a^9b, a^2) \cdot a^4 = (a^5b, a^6)$ *;*
if $g = a^9$ *:* $(a^9 \cdot (ab, a^6) \cdot a^3) = (a^{10}b, a^3) \cdot a^3 = (a^7b, a^6)$ *;*
if $g = a^{10}$ *:* $(a^{10} \cdot (ab, a^6) \cdot a^2) = (a^{10}b, a^2) \cdot a^2 = (a^9b, a^6)$ *;*
if $g = a^{11}$ *:* $(a^{11} \cdot (ab, a^6) \cdot a) = (a^{11}b, a) \cdot a = (a^{11}b, a^6)$ *;*
if $g = b$ *:* $(b \cdot (ab, a^6) \cdot b) = (a^{11}, a^6b) \cdot b = (a^{11}b, a^6)$ *;*
if $g = ab$ *:* $(ab \cdot (ab, a^6) \cdot .ab) = (1, a^7b) \cdot ab = (ab, a^6)$ *;*

if $g = a^2b$ *:* $(a^2b \cdot (ab, a^6) \cdot a^2b) = (a, a^8b) \cdot a^2b = (a^3b, a^6)$ *;*
if $g = a^3b$ *:* $(a^3b \cdot (ab, a^6) \cdot a^3b) = (a^2, a^9b) \cdot a^3b = (a^5b, a^6)$ *;*
if $g = a^4b$ *:* $(a^4b \cdot (ab, a^6) \cdot a^4b) = (a^3, a^{10}b) \cdot a^4b = (a^7b, a^6)$ *;*
if $g = a^5b$ *:* $(a^5b \cdot (ab, a^6) \cdot a^5b) = (a^4, a^{11}b) \cdot a^5b = (a^9b, a^6)$ *;*
if $g = a^6b$ *:* $(a^6b \cdot (ab, a^6) \cdot a^6b) = (a^5, b) \cdot a^6b = (a^{11}b, a^6)$ *;*
if $g = a^7b$ *:* $(a^7b \cdot (ab, a^6) \cdot a^7b) = (a^6, ab) \cdot a^7b = (ab, a^6)$ *;*
if $g = a^8b$ *:* $(a^8b \cdot (ab, a^6) \cdot a^8b) = (a^7, a^2b) \cdot a^8b = (a^3b, a^6)$ *;*
if $g = a^9b$ *:* $(a^9b \cdot (ab, a^6) \cdot a^9b) = (a^8, a^3b) \cdot a^9b = (a^5b, a^6)$ *;*
if $g = a^{10}b$ *:* $(a^{10}b \cdot (ab, a^6) \cdot a^{10}b) = (a^9, a^4b) \cdot a^{10}b = (a^7b, a^6)$ *;and*
if $g = a^{11}b$ *:* $(a^{11}b \cdot (ab, a^6) \cdot a^{11}b) = (a^{10}, a^5b) \cdot a^{11}b = (a^9b, a^6) \cdot$.

Therefore, we have found the following two orbits:

$$9)\ O(ab, a^6) = \{(ab, a^6), (a^3b, a^6), (a^5b, a^6), (a^7b, a^6), (a^9b, a^6), (a^{11}b, a^6)\}$$
$$= O(a^3b, a^6) = O(a^5b, a^6) = O(a^7b, a^6) = O(a^9b, a^6) = O(a^{11}b, a^6),$$

$$10)\ O(a^6, ab) = \{(a^6, ab), (a^3b, a^6), (a^5b, a^6), (a^7b, a^6), (a^9b, a^6), (a^{11}b, a^6)\}$$
$$= O(a^3b, a^6) = O(a^5b, a^6) = O(a^7b, a^6) = O(a^9b, a^6) = O(a^{11}b, a^6).$$

When $\omega = (b, a^6b)$*, then* $O(b, a^6b) = \{g \cdot (b, a^6b) \cdot g^{-1} \mid g \in D_{24}\}$*. Substituting* g *with all elements of* D_{24}*, the following conditions are obtained:*
if $g = 1$ *:* $(1 \cdot (b, a^6b) \cdot 1) = (b, a^6b) \cdot 1 = (b, a^6b)$ *;*
if $g = a$ *:* $(a \cdot (b, a^6b) \cdot a^{11}) = (ab, a^7b) \cdot a^{11} = (a^2b, a^8b)$ *;*
if $g = a^2$ *:* $(a^2 \cdot (b, a^6b) \cdot a^{10}) = (a^2b, a^8b) \cdot a^{10} = (a^4b, a^{10}b)$ *;*
if $g = a^3$ *:* $(a^3 \cdot (b, a^6b) \cdot a^9) = (a^3b, a^9b) \cdot a^9 = (a^6b, b)$ *;*
if $g = a^4$ *:* $(a^4 \cdot (b, a^6b) \cdot a^8) = (a^4b, a^{10}b) \cdot a^8 = (a^8b, a^2b)$ *;*
if $g = a^5$ *:* $(a^5 \cdot (b, a^6b) \cdot a^7) = (a^5b, a^{11}b) \cdot a^7 = (a^10b, a^4b)$ *;*
if $g = a^6$ *:* $(a^6 \cdot (b, a^6b) \cdot a^6) = (a^6b, b) \cdot a^6 = (b, a^6b)$ *;*
if $g = a^7$ *:* $(a^7 \cdot (b, a^6b) \cdot a^5) = (a^7b, ab) \cdot a^5 = (a^2b, a^8b)$ *;*
if $g = a^8$ *:* $(a^8 \cdot (b, a^6b) \cdot a^4) = (a^8b, a^2b) \cdot a^4 = (a^4b, a^{10}b)$ *;*
if $g = a^9$ *:* $(a^9 \cdot (b, a^6b) \cdot a^3) = (a^9b, a^3b) \cdot a^3 = (a^6b, b)$ *;*
if $g = a^{10}$ *:* $(a^{10} \cdot (b, a^6b) \cdot a^2) = (a^{10}b, a^4b) \cdot a^2 = (a^8b, a^2b)$ *;*
if $g = a^{11}$ *:* $(a^{11} \cdot (b, a^6b) \cdot a) = (a^{11}b, a^5b) \cdot a = (a^{10}b, a^4b)$ *;*
if $g = b$ *:* $(b \cdot (b, a^6b) \cdot b) = (1, a^6) \cdot b = (b, a^6b)$ *;*
if $g = ab$ *:* $(ab \cdot (b, a^6b) \cdot ab) = (a, a^7) \cdot ab = (a^2b, a^8b)$ *;*
if $g = a^2b$ *:* $(a^2b \cdot (b, a^6b) \cdot a^2b) = (a^2, a^8) \cdot a^2b = (a^4b, a^{10}b)$ *;*

if $g = a^3b$ *:* $(a^3b \cdot (b, a^6b) \cdot a^3b) = (a^3, a^9) \cdot a^3b = (a^6b, b)$ *;*
if $g = a^4b$ *:* $(a^4b \cdot (b, a^6b) \cdot a^4b) = (a^4, a^{10}) \cdot a^4b = (a^8b, a^2b)$ *;*
if $g = a^5b$ *:* $(a^5b \cdot (b, a^6b) \cdot a^5b) = (a^5, a^{11}) \cdot a^5b = (a^{10}b, a^4b)$ *;*
if $g = a^6b$ *:* $(a^6b \cdot (b, a^6b) \cdot a^6b) = (a^6, 1) \cdot a^6b = (b, a^6b)$ *;*
if $g = a^7b$ *:* $(a^7b \cdot (b, a^6b) \cdot a^7b) = (a^7, a) \cdot a^7b = (a^2b, a^8b)$ *;*
if $g = a^8b$ *:* $(a^8b \cdot (b, a^6b) \cdot a^8b) = (a^8, a^2) \cdot a^8b = (a^4b, a^{10}b)$ *;*
if $g = a^9b$ *:* $(a^9b \cdot (b, a^6b) \cdot a^9b) = (a^9, a^3) \cdot a^9b = (a^6b, b)$ *;*
if $g = a^{10}b$ *:* $(a^{10}b \cdot (b, a^6b) \cdot a^{10}b) = (a^{10}, a^4) \cdot a^{10}b = (a^8b, a^2b)$ *; and*
if $g = a^{11}b$ *:* $(a^{11}b \cdot (b, a^6b) \cdot a^{11}b) = (a^{11}, a^5) \cdot a^{11}b = (a^{10}b, a^4b)\cdot$

Therefore, we have found the following orbit:

$$11)\ O(b, a^6b) = \{(b, a^6b), (a^2b, a^8b), (a^4b, a^{10}b), (a^6b, b), (a^8b, a^2b), (a^{10}b, a^4b)\}$$
$$= O(a^2b, a^8b) = O(a^4b, a^{10}b) = O(a^6b, b) = O(a^8b, a^2b) = O(a^{10}b, a^4b).$$

When $\omega = (ab, a^7b)$*, then* $O(ab, a^7b) = \{g \cdot (ab, a^7b) \cdot g^{-1} \mid g \in D_{24}\}$*. Substituting* g *with all elements of* D_{24}*, the following conditions are obtained:*
if $g = 1$ *:* $(1 \cdot (ab, a^7b) \cdot 1) = (ab, a^7b) \cdot 1 = (ab, a^7b)$ *;*
if $g = a$ *:* $(a \cdot (ab, a^7b) \cdot a^{11}) = (a^2b, a^8b) \cdot a^{11} = (a^3b, a^9b)$ *;*
if $g = a^2$ *:* $(a^2 \cdot (ab, a^7b) \cdot a^{10}) = (a^3b, a^9b) \cdot a^{10} = (a^5b, a^{11}b)$ *;*
if $g = a^3$ *:* $(a^3 \cdot (ab, a^7b) \cdot a^9) = (a^4b, a^{10}b) \cdot a^9 = (a^7b, ab)$ *;*
if $g = a^4$ *:* $(a^4 \cdot (ab, a^6b) \cdot a^8) = (a^5b, a^{11}b) \cdot a^8 = (a^9b, a^3b)$ *;*
if $g = a^5$ *:* $(a^5 \cdot (ab, a^7b) \cdot a^7) = (a^6b, b) \cdot a^7 = (a^{11}b, a^5b)$ *;*
if $g = a^6$ *:* $(a^6 \cdot (ab, a^7b) \cdot a^6) = (a^7b, ab) \cdot a^6 = (ab, a^7b)$ *;*
if $g = a^7$ *:* $(a^7 \cdot (ab, a^7b) \cdot a^5) = (a^8b, a^2b) \cdot a^5 = (a^3b, a^9b)$ *;*
if $g = a^8$ *:* $(a^8 \cdot (ab, a^7b) \cdot a^4) = (a^9b, a^3b) \cdot a^4 = (a^5b, a^{11}b)$ *;*
if $g = a^9$ *:* $(a^9 \cdot (ab, a^7b) \cdot a^3) = (a^{10}b, a^4b) \cdot a^3 = (a^7b, ab)$ *;*
if $g = a^{10}$ *:* $(a^{10} \cdot (ab, a^7b) \cdot a^2) = (a^{11}b, a^5b) \cdot a^2 = (a^9b, a^3b)$ *;*
if $g = a^{11}$ *:* $(a^{11} \cdot (ab, a^7b) \cdot a) = (b, a^6b) \cdot a = (a^{11}b, a^5b)$
if $g = b$ *:* $(b \cdot (ab, a^7b) \cdot b) = (a^{11}, a^5) \cdot b = (a^{11}b, a^5b)$ *;*
if $g = ab$ *:* $(ab \cdot (ab, a^7b) \cdot ab) = (1, a^6) \cdot ab = (ab, a^7b)$ *;*
if $g = a^2b$ *:* $(a^2b \cdot (ab, a^7b) \cdot a^2b) = (a, a^7) \cdot a^2b = (a^3b, a^9b)$ *;*
if $g = a^3b$ *:* $(a^3b \cdot (ab, a^7b) \cdot a^3b) = (a^2, a^8) \cdot a^3b = (a^5b, a^{11}b)$ *;*
if $g = a^4b$ *:* $(a^4b \cdot (ab, a^7b) \cdot a^4b) = (a^3, a^9) \cdot a^4b = (a^7b, ab)$ *;*
if $g = a^5b$ *:* $(a^5b \cdot (ab, a^7b) \cdot a^5b) = (a^4, a^{10}) \cdot a^5b = (a^9b, a^3b)$ *;*
if $g = a^6b$ *:* $(a^6b \cdot (ab, a^7b) \cdot a^6b) = (a^5, a^{11}) \cdot a^6b = (a^{11}b, a^5b)$ *;*

if $g = a^7b$ *:* $(a^7b \cdot (ab, a^7b) \cdot a^7b) = (a^6, 1) \cdot a^7b = (ab, a^7b)$ *;*
if $g = a^8b$ *:* $(a^8b \cdot (ab, a^7b) \cdot a^8b) = (a^7, a) \cdot a^8b = (a^3b, a^9b)$ *;*
if $g = a^9b$ *:* $(a^9b \cdot (ab, a^7b) \cdot a^9b) = (a^8, a^2) \cdot a^9b = (a^5b, a^{11}b)$ *;*
if $g = a^{10}b$ *:* $(a^{10}b \cdot (ab, a^7b) \cdot a^{10}b) = (a^9, a^3) \cdot a^{10}b = (a^7b, ab)$ *; and*
if $g = a^{11}b$ *:* $(a^{11}b \cdot (ab, a^7b) \cdot a^{11}b) = (a^{10}, a^4) \cdot a^{11}b = (a^9b, a^3b)\cdot$

Therefore, we have found another orbit:

$$12)\ O(ab, a^7b) = \{(ab, a^7b), (a^3b, a^9b), (a^5b, a^{11}b), (a^7b, ab), (a^9b, a^3b), (a^{11}b, a^5b)\}$$
$$= O(a^3b, a^9b) = O(a^5b, a^{11}b) = O(a^7b, ab) = O(a^9b, a^3b) = O(a^{11}b, a^5b).$$

Based on above calculation we list all the orbits of the set Ω *for* D_{22} *and relabel them with* O_i *where* $i = 1, 2, 3 \ldots, K(\Omega)$ *for convenience as follows:*

$$O_1 = O(1, a^6) = \{(1, a^6)\},$$
$$O_2 = O(a^6, 1) = \{(a^6, 1)\},$$
$$O_3 = O(1, b) = \{(1, b), (1, a^2b), (1, a^4b), (1, a^6b), (1, a^8b), (1, a^{10}b)\},$$
$$O_4 = O(b, 1) = \{(b, 1), (a^2b, 1), (a^4b, 1), (a^6b, 1), (a^8b, 1), (a^{10}b, 1)\},$$
$$O_5 = O(1, ab) = \{(1, ab), (1, a^3b), (1, a^5b), (1, a^7b), (1, a^9b), (1, a^{11}b)\},$$
$$O_6 = O(ab, 1) = \{(ab, 1), (a^3b, 1), (a^5b, 1), (a^7b, 1), (a^9b, 1), (a^{11}b, 1)\},$$
$$O_7 = O(b, a^6) = \{(b, a^6), (a^2b, a^6), (a^4b, a^6), (a^6b, a^6), (a^8b, a^6), (a^{10}b, a^6)\},$$
$$O_8 = O(a^6, b) = \{(a^6, b), (a^6, a^2b), (a^6, a^4b), (a^6, a^6b), (a^6, a^8b), (a^6, a^{10}b)\},$$
$$O_9 = O(ab, a^6) = \{(ab, a^6), (a^3b, a^6), (a^5b, a^6), (a^7b, a^6), (a^9b, a^6), (a^{11}b, a^6)\},$$
$$O_{10} = O(a^6, ab) = \{(a^6, ab), (a^3b, a^6), (a^5b, a^6), (a^7b, a^6), (a^9b, a^6), (a^{11}b, a^6)\},$$
$$O_{11} = O(b, a^6b) = \{(b, a^6b), (a^2b, a^8b), (a^4b, a^{10}b), (a^6b, b), (a^8b, a^2b), (a^{10}b, a^4b)\},$$
$$O_{12} = O(ab, a^7b) = \{(ab, a^7b), (a^3b, a^9b), (a^5b, a^{11}b), (a^7b, ab), (a^9b, a^3b), (a^{11}b, a^5b)\}.$$

According to Definition 2.2, the number of orbits in D_{24} *is* $K(\Omega) = 12$*.*

3.2. **Constructing Generalized Conjugacy Class Graphs.**

We have found that with respect to conjugation action, the number of orbits of D_{14}, D_{18} and D_{22} is $K(\Omega) = 2$. According to Definition 2.3, we need to exclude central orbits for each D_{2n} when $n = 7, 9$ and 11. However, in this case, O_1 and

O_2 are the only orbits that can be found. Both of them are non central orbits. Consequently, the vertices for $\Gamma^{\Omega_c}_{D_{14}}, \Gamma^{\Omega_c}_{D_{18}}$ and $\Gamma^{\Omega_c}_{D_{22}}$ are $|V(\Gamma^{\Omega_c}_{D_{2n}})| = 2 - 0 = 2$. Observing, the cardinalities of each orbit, we can see that $|O_1| = |O_2| \neq 1$ and are not set-wise relatively prime. All the vertices (non central orbits) are connected by an edge with each others and giving us a complete graph on two vertices, K_2. The graph is illustrated in Figure 1.

Meanwhile, the number of orbits with respect to conjugation action of D_{12}, D_{16}, D_{20}, D_{24} on Ω denoted by $K(\Omega) = 12$. By Definition 2.3, we need to exclude central orbits listed below for each D_{2n} when $n = 6, 8, 10$ and 12:

$D_{12} : O_1 = (1, a^3)$ and $0_2 = O(a^3, 1)$;
$D_{16} : O_1 = (1, a^4)$ and $0_2 = O(a^4, 1)$;
$D_{20} : O_1 = (1, a^5)$ and $0_2 = O(a^5, 1)$ and
$D_{24} : O_1 = (1, a^6)$ and $0_2 = O(a^6, 1)$.

FIGURE 1. $\Gamma^{\Omega_c}_{D_{14}}$, $\Gamma^{\Omega_c}_{D_{18}}$ and $\Gamma^{\Omega_c}_{D_{22}}$ as complete graph on two vertices, K_2

Consequently, the vertices for $\Gamma^{\Omega_c}_{D_{12}}, \Gamma^{\Omega_c}_{D_{16}}, \Gamma^{\Omega_c}_{D_{20}}$ and $\Gamma^{\Omega_c}_{D_{24}}$ are $V(\Gamma^{\Omega_c}_{D_{2n}}) = \{O_3, O_4, O_5, \ldots, O_{10}, O_{11}, O_{12}\}$ and hence the number of vertices $| V(\Gamma^{\Omega_c}_{D_{2n}}) | = 12 - 2 = 10$. Observing the cardinalities of each orbit for each group, we can see that

$$|O_3| = |O_4| = |O_5| = |O_6|, \ldots = |O_{12}| \neq 1$$

and are not set wise relatively prime. For instance, in $D_{12}, |O_3|, \ldots, = |O_{12}| = 3$ and gcd $(3, 3) \neq 1$. Hence, all the vertices (non central orbits) are connected by and edge with each others and giving us complete graph on ten vertices, K_{10}. The graph is illustrated in Figure 2.

3.3. **Relevant Matrices for Generalized Conjugacy Class Graph.**

The complete graphs K_2 and K_{10} can be represented as adjacency, incidence and Laplacian matrices.

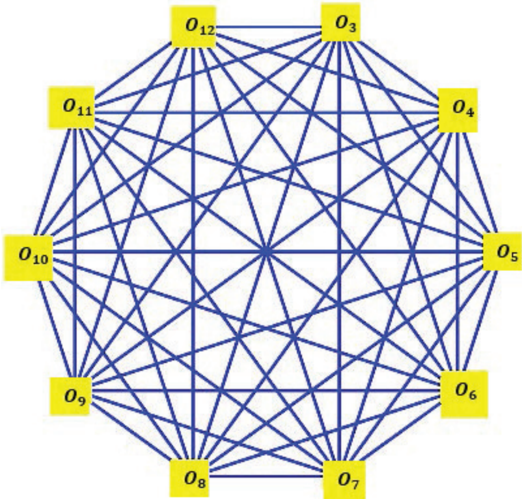

FIGURE 2. $\Gamma^{\Omega_c}_{D_{12}}$, $\Gamma^{\Omega_c}_{D_{16}}$, $\Gamma^{\Omega_c}_{D_{20}}$, and $\Gamma^{\Omega_c}_{D_{24}}$ as complete graph on ten vertices, K_{10}

By Definition 2.4, the following matrices are adjacency matrices for complete graphs K_2 and K_{10}

$$A(K_2) = \begin{array}{c} \\ v_1 \\ v_2 \end{array} \begin{array}{c} \begin{array}{cc} v_1 & v_2 \end{array} \\ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{array},$$

and

$$A(K_{10}) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \end{array} \begin{array}{c} \begin{array}{cccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}.$$

By Definition 2.5, the generalized conjugacy class graph can also be represented as incident matrices. The incidence matrix for complete graph, K_2 is:

$$I(K_2) = \begin{array}{c} \\ v_1 \\ v_2 \end{array} \begin{array}{c} e_1 \\ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{array}$$

and the incidence matrix for complete graph, K_{10} is provided in Appendix A due to its large dimension.

By Definition 2.6, the Laplacian matrix $L(G)$ of G is defined by $L(G) = D(G) - A(G)$, where $D(G)$ is the diagonal matrix of vertex degrees and $A(G)$ is the adjacency matrix of G. Therefore, Laplacian matrix for complete graph, K_2 and K_{10} can be constructed as follows:

$$L(K_2) = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix},$$

and

$$L(K_{10}) = \begin{bmatrix} 9 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 9 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 9 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 9 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 9 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & 9 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 9 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 9 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 9 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 9 \end{bmatrix}.$$

3.4. **Energy and Laplacian Energy.**

The results obtained can be used to find the energy and Laplacian energy. By using Definition 2.7 and Definition 2.8, the energy and Laplacian energy for complete graphs, K_2 and K_{10} can be determined. Based on the adjacency matrix of generalized conjugacy class graph A_{K_2}, the eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = -1$

with multiplicity 1. By Definition 2.7,

$$\varepsilon(\Gamma_{D_{2n}}^{\Omega}) = \sum_{i=1}^{2} \mid \lambda_i \mid = \mid 1 \mid + \mid -1 \mid = 2.$$

On the other hand, Laplacian energy for complete graph, K_2 also are calculated. Based on the Laplacian matrix of $L(K_2)$, the eigenvalues of Laplacian matrix of generalized conjugacy class graph $L(K_2)$ are $\mu_1 = 0$ and $\mu_2 = 2$ with multiplicity 1. By Definition 2.8,

$$\begin{aligned} LE(\Gamma_{D_{2n}}^{\Omega}) &= \sum_{i=1}^{2} \mid \mu_i - \frac{2m}{n} \mid = \mid 0 - \frac{2(1)}{2} \mid + \mid 2 - \frac{2(1)}{2} \mid \\ &= \mid -1 \mid + \mid 1 \mid = 2. \end{aligned}$$

Next, the energy and Laplacian energy for complete graph, K_{10} also can be calculated. Based on the adjacency matrix of $A_{K_{10}}$, the eigenvalues of adjacency matrix of conjugacy class graph $A_{K_{10}}$ are $\lambda_1 = 9$ (with multiplicity 1) and $\lambda_2, \lambda_3, \ldots, \lambda_{10} = -1$ with multiplicity 9. By Definition 2.7,

$$\varepsilon(\Gamma_{D_{2n}}^{\Omega}) = \sum_{i=1}^{10} \mid \lambda_i \mid = \mid 9 \mid +9 \mid -1 \mid = 18.$$

Lastly, based on the Laplacian matrix of $L(K_{10})$, the eigenvalues of Laplacian matrix of generalized conjugacy class graph $L(K_{10})$ are $\mu_1 = 0$ (multiplicity 1) and $\mu_2 = 10$ (multiplicity 9). By the definition of the Laplacian energy of graph in Definition 2.8,

$$\begin{aligned} LE(\Gamma_{D_{2n}}^{\Omega}) &= \sum_{i=1}^{10} \mid \mu_i - \frac{2m}{n} \mid = \mid 0 - \frac{2(45)}{10} \mid +9 \mid 10 - \frac{2(45)}{10} \mid \\ &= \mid -9 \mid +9 \mid 1 \mid = 18. \end{aligned}$$

As a result, for $n = 7, 9$ and 11, the energy and Laplacian energy is equal to two. Meanwhile, for $n = 6, 8, 10$ and 12, the energy and Laplacian energy is equal to 18.

4. Conclusion

The following Table 2 gathers all findings from this study - from the generalized conjugacy class graphs to the energy and Laplacian energy for each of D_{2n} considered. We can conclude that when n is odd $(n = 7, 9, 11)$, the generalized conjugacy class graphs are all isomorphic to a complete graph with two vertices, K_2. Meanwhile when n is even $(n = 6, 8, 10, 12)$, the generalized conjugacy class graphs are all isomorphic to a complete graph with ten vertices, K_{10}. Consequently, the dimension of adjacency and Laplacian matrices are similar according to even and odd cases. For n odd $(n = 7, 9, 11)$, the dimension of adjacency and Laplacian matrices is 2×2, whilst for n even $(n = 6, 8, 10, 12)$, the dimension is 10×10.

In addition to that, it is found that the energy and Laplacian energy have the same values conforming to even and odd cases as well. This will motivate us to investigate further if this situation has relation with the properties of certain conjugacy classes of this group.

TABLE 2. Generalized Conjugacy Class Graph and Its Graph Energies

Dihedral Group, D_{2n}	Generalized Conjugacy Class Graph, $\Gamma^{\Omega_c}_{D_{2n}}$	Energy, $\varepsilon(\Gamma_G)$	Laplacian Energy, $LE(\Gamma_G)$
D_{12}	K_{10}	18	18
D_{14}	K_2	2	2
D_{16}	K_{10}	18	18
D_{18}	K_2	2	2
D_{20}	K_{10}	18	18
D_{22}	K_2	2	2
D_{24}	K_{10}	18	18

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Appendix: Incidence Matrix of Complete Graph K_{10}

$I(K_{10}) =$

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
v_1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
v_2	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1
v_3	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
v_4	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
v_5	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0
v_6	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
v_7	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0
v_8	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
v_9	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
v_{10}	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0

	e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}
v_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v_2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
v_3	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0
v_4	0	0	1	0	0	0	0	0	0	1	1	1	1	1	1
v_5	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0
v_6	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
v_7	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0
v_8	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
v_9	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0
v_{10}	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1

	e_{31}	e_{32}	e_{33}	e_{34}	e_{35}	e_{36}	e_{37}	e_{38}	e_{39}	e_{40}	e_{41}	e_{42}	e_{43}	e_{44}	e_{45}
v_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v_5	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
v_6	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
v_7	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
v_8	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
v_9	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
v_{10}	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1

REFERENCES

[1] E. A. BERTRAM, M. HERZOG, A. MANN: *On a graph related to conjugacy classes of groups*, Bull. London Math. Soc., **22**(6) (1990), 569–575.

[2] F.M. GOODMAN: *Algebra : Abstract and Concrete : Stressing Symmetry*, 2nd ed., Prentice Hall, 2003.

[3] I. GUTMAN, B. ZHOU: *Laplacian energy of a graph*, Linear Algebra Appl., **414**(1) (2006), 29–37.

[4] R. B. BAPAT: *Graphs and Matrices*, 2nd ed., Springer-Verlag London, 2010.

[5] Y. TERANISHI: *Subgraphs and the laplacian spectrum of a graph*, Linear Algebra Appl., **435**(5) (2011), 1029–1033.

[6] G. CHARTRAND, P. ZHANG: *A First Course in Graph Theory*, 1st ed., Dover Publications, 2012.

[7] S. OMER, N.H. SARMIN, A. ERFANIAN, K. MORADIPOUR: *The probability that an element of a group fixes a set and its graph related to conjugacy classes*, J. Basic. Appl. Sci. Res., **3**(10) (2013), 369–380.

[8] S. OMER, N.H. SARMIN, A. ERFANIAN: *Generalized conjugacy class graph of some finite non-abelian groups*, AIP Conf. Proc., **1660**(2015), 1–5.

[9] J.J. ROTMAN: *Advance Modern Algebra*, 3rd ed., American Mathematical Society, 2017.

[10] N. ZAID, N.H. SARMIN, S.N.A. ZAMRI: *A variant of commutativity degree and its generalized conjugacy class graph*, Adv.Sci. Lett., **24**(6) (2018), 4429–4432.

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