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PAIR SUM MODULO LABELING OF GRAPH

P. Amudha[1] and J. Jayapriya

ABSTRACT. In this paper, we introduced a new labeling namely Pair sum modulo labeling and proven the existence of the same for certain type of graph, namely Petersen graph, bull graph, coconut tree graph, complete bipartite graph and bistar graph.

1. INTRODUCTION

Graph theory plays a vital role in the field of Science and Technology [1, 3]. In specific, it is broadly utilized within the areas like Sociology, Biology, Chemistry, biochemistry, communication networks and coding theory, algorithms and computation and operations research etc. Graph labeling is an important branch of graph theory which is exceptionally valuable in numerous areas. The assignment of numbers to the vertices or edges or both of a graph subject to certain conditions is called graph labeling. Rosa [5] introduced the idea of graph labeling in 1967. R.Ponraj [4,6] tried the pair sum labeling of some standard graphs. Gallian [2] regularly updates all the new labeling techniques in 'A dynamic survey of graph labeling'. So far more than 3000 papers are there in graph labeling

[1]*corresponding author*

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techniques. In this paper we introduced a new graph labeling for the vertices namely pair sum modulo labeling and examine the behaviour of coconut tree graph, bull graph, Petersen graph, bistar graphs etc. for this labeling technique.

2. Preliminaries

Definition 2.1. *A bistar graph is a graph obtained by joining the center(apex) vertices of two copies of* $K_{1,n}$ *by an edge. The vertex set of the bistar graph* $B_{n,n}$ *is* $V = \{u, v, u_i, v_i \ /1 \leq i \leq n\}$*, where* u, v *are apex vertices and* u_i, v_i *are pendent vertices.*

Definition 2.2. *A coconut tree CT(m,n) is the graph obtained from the path* P_n *by appending m new pendant edges at an end vertex of* P_n *.*

Definition 2.3. *A labeling of a graph G is an assignment of labels (represented by integers) to the vertices or edges or both subject to certain conditions.*

Definition 2.4. *An undirected simple graph G with p vertices and q edges is said to be a pair sum modulo graph if for an injective function* $f : V(G) \to \{\pm1, \pm2, \ldots, \pm p\}$ *there exists an induced edge labeling* $g : E(G) \to \{0, 1, 2, \ldots, q-1\}$ *such that* $g(uv) = (f(u) + f(v))(mod\ q)$ *is distinct for each edge uv.*

3. Main results

Theorem 3.1. *The Petersen graph admits pair sum modulo labeling.*

Proof. Consider the Petersen graph G(V,E) where $|V(G)|$ = 10 and $|E(G)|$ = 15. Let $f : V(G) \to \{\pm1, \pm2, \ldots, \pm10\}$ be a function such that for each edge e which is incident with vertices v_i, v_j has an unique label $(f(v_i) + f(v_j))(mod 15)$.

The labeling of the vertices of G are given below:

$f(v_i) = i$, for $1 \leq i \leq 6$;

$f(v_7) = 8,\ f(v_8) = 10,\ f(v_9) = 7,\ f(v_{10}) = -6.$

By using the pair sum modulo labeling, the induced edge labels becomes 4, 8, 7, 6, 5, 13, 11, 9, 12, 10, 1, 0, 14, 3, 2.

Figure 1 shows the pair sum modulo labeling of Petersen graph. Here the edge labels are distinct. Hence the pair sum modulo labeling exists for Petersen graph. □

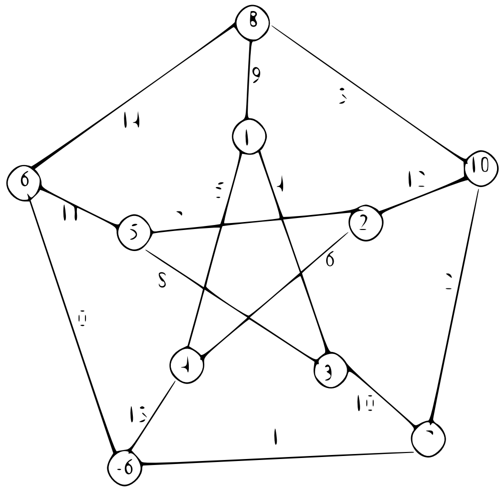

FIGURE 1. Pair sum modulo labeling of Petersen graph

Theorem 3.2. *The bull graph admits pair sum modulo labeling.*

Proof. Let G(V,E) be a bull graph with p vertices and q edges. Clearly p= $|V(G)|$ = 5 and q= $|E(G)|$ = 5. Also, $V(G)$ =$\{v_i, 1 \leq i \leq p\}$, $E(G)$ =$\{v_i v_{i+1}, 1 \leq i \leq p-1\} \cup \{v_{\frac{p-1}{2}} v_{\frac{p+3}{2}}\}$.

Define a function $f : V(G) \to \{\pm 1, \pm 2, \dots, \pm 5\}$ by $f(v_i) = i, 1 \leq i \leq p$. By using the definition of pair sum modulo labeling the edge values are calculated and found to be distinct. Figure 2 shows the pair sum modulo labeling of bull graph.

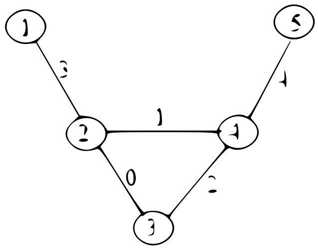

FIGURE 2. Pair sum modulo labeling of Bull graph

Hence the bull graph is a pair sum modulo graph. □

Theorem 3.3. *The coconut tree graph CT(2,n) admits pair sum modulo labeling.*

Proof. Let G(V,E) be a coconut tree graph CT(2,n). Clearly $|V(G)|$ = 2+n and $|E(G)|$ = 1+n. Let $V(G)$ =$\{v_i, 1 \leq i \leq n+2\}$, $E(G)$ =$\{v_1 v_2, v_2 v_i : 3 \leq i \leq n+2\}$.

Define a function $f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm(n+2)\}$ by

$f(v_i) = i - 1,\ 3 \le i \le n+2$;

$f(v_2) = 1;\ f(v_1) = 2 + n.$

By using the definition of pair sum modulo labeling the edge values are calculated and found to be distinct. Hence the coconut tree graph CT(2,n) is a pair sum modulo graph. □

Theorem 3.4. *The complete bipartite graph $K_{2,n}$ admits pair sum modulo labeling for all natural numbers n.*

Proof. Let G(V,E) be a complete bipartite graph $K_{2,n}$. Clearly $|V(G)|$ = 2+n and $|E(G)|$ = 2n.

Let $V(G)$ ={$v_i, 1 \le i \le n+2$}, $E(G)$ ={$v_1v_i, v_2v_i : 3 \le i \le n+2$}. Define a function $f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm(n+2)\}$ such that

$$f(v_1) = -1;\quad f(v_2) = -(n+1);\quad f(v_i) = i - 2,\ 3 \le i \le n+2.$$

By using the definition of pair sum modulo labeling the edge values are calculated and found to be distinct.

Hence the complete bipartite graph $K_{2,n}$ is a pair sum modulo graph. □

Theorem 3.5. *The bistar graph $B_{n,n}$ admits pair sum modulo labeling for all natural numbers n.*

Proof. Let G(V,E) be a bistar graph $B_{n,n}, n \in N$. Clearly $|V(G)|$ = 2n+2 and $|E(G)|$ = 2n+1.

Let $V(G)$ ={$u_i, v_i : 1 \le i \le n\,, n \in N\} \cup \{u, v\}$; $E(G) = \{uv, uu_i, vv_i : 1 \le i \le n\,, n \in N\}$, where u, v are apex vertices and $u_i,\ v_i$ are pendent vertices.

Case (i): When n=1

Let an injective function $f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm 4\}$ such that $f(u) = -1; f(v) = -2; f(u_1) = 2; f(v_1) = 1$. By using pair sum modulo labeling technique the distinct edge labels are obtained as $0, 1, 2$. Therefore $B_{1,1}$ admits pair sum modulo labeling.

Case (ii): When n>1

Let an injective function $f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm(2n+2)\}$ such that

$f(u) = -1;$

$f(v) = -2;$

$f(u_i) = i; 1 \le i \le n$;

$f(v_i) = 1 + n + i;\ 1 \leq i \leq n - 2;$
$f(v_i) = 2 + n + i;\ n - 1 \leq i \leq n$.

The induced edge labels are calculated by using the definition of pair sum modulo labeling and found to be distinct. Therefore $B_{n,n}$ admits pair sum modulo labeling for n>1. Hence the bistar graph $B_{n,n}$ is a pair sum modulo graph for all natural numbers n. □

4. CONCLUSION

In this paper the existence of pair sum modulo labeling of Peterson graph, Coconut tree graph CT(2,n), bistar graph, complete bipartite graphs $K_{2,n}$ and bull graph are shown.

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RESEARCH SCHOLAR, DEPARTMENT OF MATHEMATICS
SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY
CHENNAI-600119, INDIA
Email address: amutha106@gmail.com

DEPARTMENT OF MATHEMATICS
SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY
CHENNAI-600119, INDIA
Email address: priyanandam_1975@rediffmail.com