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SOLVING BI-OBJECTIVE INTERVAL ASSIGNMENT PROBLEM USING GENETIC APPROACH

T. K. Buvaneshwari[1] and D. Anuradha

ABSTRACT. This paper develops a new algorithm for obtaining a set of all efficient/ non-efficient solutions for bi-objective interval assignment problem using genetic algorithm (GA) approach. The working theory of the proposed model is performed by a numerical example.

1. INTRODUCTION

The assignment problem (AP) has been widely frequented in many real-life situations including human resource planning. The classical AP may be a popular combinatorial optimization problem that involves one-to-one matching items including two finite sets to get minimum cost or maximum profit. Hungarian method is the most classical apporach to unravel AP presented by Kuhn [1]. Many other researchers developed different methodologies for solving the APs [2–4]. Our target is to solve multiple objectives simultaneously. Most of the studies of AP models are disscussed about with one target. There is a few research paper available in the multi-objective assignment problem (MOAP). The entries in the cost matrix are certainly not consistently crisp. These limits are

[1]*corresponding author*

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ambiguous in many applications and the interval signifies these uncertain parameters. Sobana and Anuradha [5] solved the bi-objective interval AP by using the minor minimum method. Biswas and Pramanik [6] discussed the MOAP with fuzzy costs for the case of military affairs. Salehi [7] proposed an approach for solving MOAP with interval parameters. Sargam Majumdar [8] implemented a new method called the Hungarian interval method in linear assignment interval problems. Kai et al. [9] were to suggest an interval parameter MOAP, to the best of our insight. They used arithmetic intervals to transform their model into crisp form. In recent decades, genetic algorithms (GA) have been successfully implemented based on natural genetics and selection mechanics in a wide variety of standardized search and optimization algorithms. It was first imagined by John Holland(1970) and later developed by various researchers. Each potential solution is encoded as a string, creating a string population that is further processed by three operators: selection, crossover, and mutation. Initialization is a process where individual strings are copied according to their fitness function. Crossover is the process of swapping a possibility for the content of two strings at some point(s). The mutation is eventually the method of flipping the value in a string with a very low probability at a specific location. A detailed GA analysis can be found in [10]. Chu't' and Beasley [11] have solved the AP using the GA. Dörterler [12] recommended a new GA which is based on agent crossover for a generalized AP. Toroslu and Arslanoglu [13] have introduced a new method to solve the GA for the personnel AP with multiple objectives. Na et al. [14] suggested a hybrid GA for cloud hospital online patient assignment issues. Dhodiya and Tailor [15] solved the fuzzy MOAP using exponential membership function by GA based hybrid approach. Majumdar and Bhunia [16] proposed an elitist GA to solve the generalized AP with imprecise cost/time. Bhunia et al. [17] studied a GA approach for unbalanced APs in an interval environment. Karthy and Ganesan [18] proposed a multi-objective transportation problem by the GA approach. Anuradha and Bhavani [19] found all efficient solutions to the bi-criteria traveling salesman problem in multi-perspective metrics.

In Section 2, the mathematical model of bi-objective interval assignment problem (BOIAP) is presented. In Section 3, our genetic algorithm approach procedure, which is implemented in the BOIAP to achieve efficient/ non-efficient solutions is proposed. In Section 4, experimental study of our method is performed. This work is concluded in Section 5.

2. Mathematical Model of Bi-Objective Interval Assignment Problem (BOIAP)

We consider n donors in a tissue bank and n hospitals to process the tissue for their patients. One hospital must be associated with one donor only. A penalty c_{ij}^L and c_{ij}^U is the cost of transport and the total time to reach the hospital, which is incurred when a hospital $j(j = 1, 2, ..., n)$ is processed by the donors $i(i = 1, 2, ..., n)$. Let x_{ij} denote the assignment of j^{th} hospital to i^{th} donors. Our aim is to determine the assignment of donors to hospitals at minimum assignment cost and time to reach the hospital.

Now, the mathematical model of the above BOIAP is given as follows.

(F) Minimize $[Z_1, Z_2] = \sum_{i=1}^{n}\sum_{j=1}^{n}[c_{ij}^L, c_{ij}^U]x_{ij}$

Minimize $[Z_3, Z_4] = \sum_{i=1}^{n}\sum_{j=1}^{n}[t_{ij}^L, t_{ij}^U]x_{ij}$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1, 2, ..., n \text{ and } j \neq i$$

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, ..., n \text{ and } i \neq j$$

$$x_{ij} = \begin{cases} 1; & \text{donors assign to hospital } i \text{ to } j \\ 0; & \text{otherwise} \end{cases}$$

The basic definitions of the arithmetic operations, partial ordering of closed bounded intervals, optimal solutions of the interval, efficient/non-efficient solutions of the interval and best compromise solution can be obtained in [5, 20].

3. Genetic algorithm approach

Step1: Consider a complete bipartite graph, F(S; D; S X D), with weights w(S, D) assigned to every edge (S, D). Construct subgraph F_1 and F_2 from the given graph F.

Step2: From the subgraph F_1, construct the subgraph F_{1L} and F_{1U} and obtain an optimal solution to the F_{1L} and F_{1U} by the Hungarian algorithm (HA).

Step3: Construct the subgraph F_{2L} and F_{2U} from the given subgraph F_2 and obtain an optimal solution to the F_{2L} and F_{2U} by the HA.

Step4: Take the optimal solution of F_{1L} and F_{1U} as a feasible solution of F_{2L}

andF_{2U} which is an efficient/non-efficient solution to F.

Step5: Select the chromosomes, which are used for the creation of a new generation. The method of selection can be chosen from the existing ones. Here, we choose a different combination of parents from the population and obtain the child.

Step6: Select parents from the subgraphs F_{2L} and F_{2U}. Choose at random a pair of parents for mating and apply single-point crossover to obtain the child. Repeat this procedure to obtain the efficient/non-efficient solutions to all combinations of parents to F_{2L} and F_{2U}.

Step7: Select a vertex randomly as a parent for the mutation for subgraphs F_{2L} and F_{2U} and apply swap mutation to obtain the child. Repeat this procedure to obtain the efficient/non-efficient solutions to all combinations of parents to F_{2L} and F_{2U}.

Step8: Now, we start with an optimal solution of F_{2L} and F_{2U} as a feasible solution of F_{1L} and F_{1U} which is an efficient/non-efficient solution to F.

Step9: Repeat step 5 to step 7 for the F_{1L} and F_{1U}.

Step 10: combine all the solutions of the F obtained by using the optimal solution of F_1 and F_2. From this, it is possible to achieve a set of efficient/non-efficient solutions to the F.

The GA approach for solving a BOIAP is shown below using an illustration.

4. Numerical illustration

A tissue bank has to sort out the assignment of three separate donors to three different hospital patients in different locations. Assume that two goals are taken into consideration:

(1) Assess the allocation that minimizes the overall cost of transport of donors to hospitals.
(2) Minimize the total time (in hrs) to reach the hospital.

As the allocation schedule has been pre-planned, we are usually unable to obtain this knowledge exactly. For this condition, the normal way to obtain interval data is through the assessment of the experience. The corresponding interval data are shown in the Figure 1.

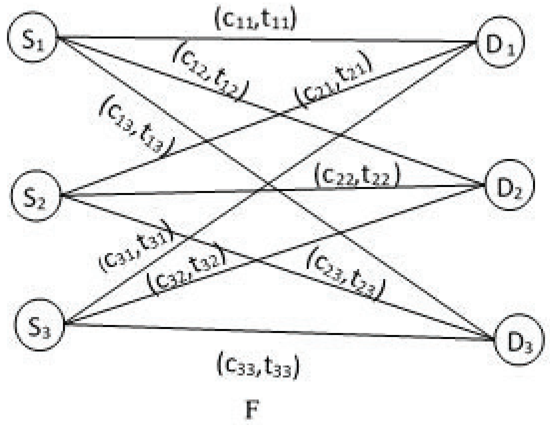

FIGURE 1. Illustration for BOIAP.

Now, using Step 1 the weighted subgraphs F_1 and F_2 to the given graph F are shown in the Figure 2.

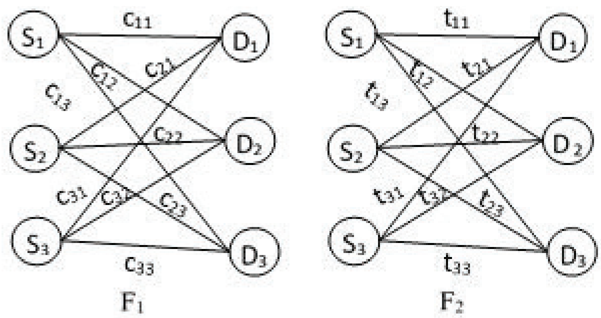

FIGURE 2. Subgraph F_1 and F_2 is the first and second objective of BOIAP.

Now, using Step 2 the weighted subgraphs F_{1L} and F_{1U} to the given graph F_1 are shown in the Figure 3.

Now, the optimal weight of F_{1L} and F_{1U} by HA is S1→D1, S2→D3, S3→D2, and optimal assignment weights are 7 and 13. Therefore, the optimal assignment weight of F_1 is [7, 13].

Now, using Step 3 the weighted subgraphs F_{2L} and F_{2U} to the given graph F_2 are shown in the Figure 4.

Now, the optimal weight of F_{2L} and F_{2U} by HA is S1→D1, S2→D3, S3 →D2, and optimal assignment weights are 7 and 12. Therefore, the optimal assignment weight of F_2 is $[7, 12]$.

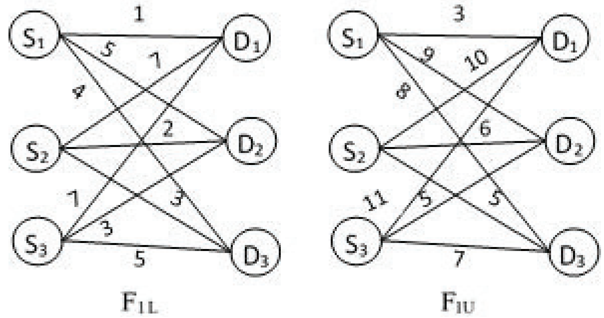

FIGURE 3. Subgraph F_{1L} and F_{1U} is the lower and upper of first objective AP.

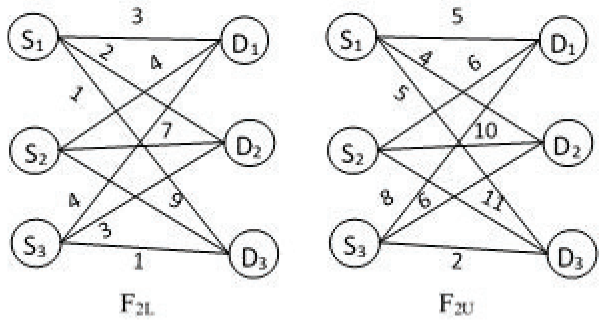

FIGURE 4. Subgraph F_{2L} and F_{2U} is the lower and upper of first objective AP.

Now, by using Step 4, consider the optimal solution F_{1L} and F_{1U} in the F_{2L} and F_{2U} as a feasible solution. Therefore, the assignment weight of F is $([7, 13], [15, 22])$ and the allotment is S1→D1, S2→D3, S3→D2.

Now, using Step 5 and Step 6, we choose the parents. In F_{2L}, we take edges of weights (3 2 1) as the parent 1 and (4 7 9) as the parent 2. In the F_{2U}, we take edges of weights (5 4 5) as the parent 1 and (6 10 11) as the parent 2. Then, we use the single point crossover, by making a cut point by selecting randomly between the two parents. After interchanging, the edges of weights transform as (3 7 9) and (4 2 1) in F_{2L}, (5 10 11) and (6 4 5) in F_{2U}. Subsequently, the resulting subgraph is the crossover, of F_{2L} and F_{2U}.Using the HA, the optimal allotment to the F_{2L} is S1→D1, S2→D2, S3 →D3, and the optimal assignment weight is 11 to F_{1L} is 8. The optimal allotment to the F_{2U} is S1→D1, S2→D2, S3→D3, and the optimal assignment weight is 17 to F_{1U} is 16. Therefore, the assignment weight of F is $([8, 16], [11, 17])$.

Repeat the above procedure to a remaining combination of parents for F_{2L} and F_{2U} to obtain the efficient/non-efficient solutions. Therefore, assignment weight of F is $([15, 25], [15, 23])$ and $([14, 23], [8, 17])$.

Therefore, the set of solutions in crossover from F_1 to F_2 is $\{([8, 16], [11, 17]), ([15, 25], [15, 23]), ([14, 23], [8, 17])\}$.

Now, using Step 5 and Step 7, we swap the randomly selected edges of a set of combinations of parents, for both the subgraph F_{2L} and F_{2U}. Here, swap the edge of the weight 2 and 3 in F_{2L} and 4 and 5 in F_{2U}. Subsequently, the resulting subgraph is the mutation of F_{2L} and F_{2U}. Using the HA, the optimal allotment to the F_{2L} is S1→D2, S2→D1, S3→D3, and the optimal assignment weight is 7 and F_{1L} is 17. The optimal allotment to the F_{2U} is S1→D2, S2→D1, S3→D3, and the optimal assignment weight is 12 and F_{1U} is 26. Therefore, the assignment weight of F is $([17, 26], [7, 12])$.

Repeat the above procedure to a remaining combination of parents for F_{2L} and F_{2U} to obtain the efficient/non-efficient solutions. Therefore, assignment weight of F is $([7, 16], [15, 17])$ and $([14, 23], [8, 17])$.

Therefore, the set of solutions in mutation from F_1 to F_2 is $\{([17, 26], [7, 12]), ([7, 16], [15, 17]), ([14, 23], [8, 17])\}$.

Consequently, the set of all solutions $\mathbb{R}_1$ of the F found from F_1 to F_2 is $\{([17, 26], [7, 12]), ([7, 16], [15, 17]), ([14, 23], [8, 17]), ([8, 16], [11, 17]), ([15, 25],)\}$.

By using Step 8, consider the optimal solution F_{2L} and F_{2U} in the F_{1L} and F_{1U} as a feasible solution. Therefore, the assignment weight of F is $([17, 26], [7, 12])$ and the allotment is S1→D2, S2→D1, S3→D3.

Now, using Step 9, we obtain the set of all solutions $\mathbb{R}_2$ of F found from F_2 to F_1 is $\{([8, 16], [11, 17]), ([7, 13], [15, 22])\}$

Now using step 10, set of all solutions $\mathbb{R}$ of F found from F_1 to F_2 and from F_2 to F_1 is $\mathbb{R} = \mathbb{R}_1 U \mathbb{R}_2 = \{([17, 26], [7, 12]), ([7, 16], [15, 17]), ([14, 23], [8, 17]), ([8, 16], [11, 17]), ([15, 25], [15, 23]), ([7, 13], [15, 22])\}$.

The set of all solutions of F that we get through using GAA are Ideal Solution $([7, 13], [7, 12])^1$,Efficient solution $([17, 26], [7, 12])^2, ([7, 16], [15, 17])^3, ([14, 23],)^4, ([8, 16], [11, 17])^5, ([7, 13], [15, 22])^6$. Non-efficient solution $([15, 25], [15, 23])^7$ and the best Compromise solution $([8, 16], [11, 17])$.

By using [20], we obtain the mid-value of an interval is $(10, 9.5)^1$, $(21.5, 9.5)^2$, $(11.5, 16)^3$, $(18.5, 12.5)^4$, $(12, 14)^5$, $(10, 18.5)^6$ and $(18, 19)^7$ are extracted for plotting the solutions graphically.

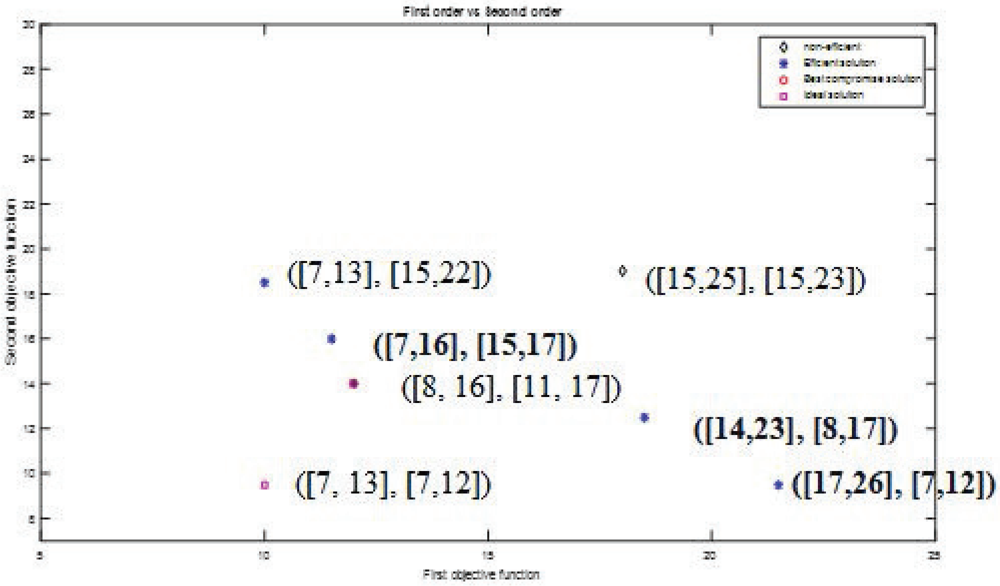

FIGURE 5. The solutions attained from the GAA.

From Figure 5, we see that the ideal solution and set of efficient / non-efficient solutions can be found by the proposed method.

5. CONCLUSION

In the present paper, a methodology to solve the bi-objective AP has been proposed and solved by GAA. It is found that the algorithm is very effective to find the best compromise solution. A great feature of this work is its simple calculation procedure compared to the other methods. As a whole, the proposed methodology doesn't require careful attention to the determinations of the weight among the resources. Moreover, it incorporates the priority of the resources in the decision-making process.

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DEPARTMENT OF MATHEMATICS
VELLORE INSITUTE OF TECHNOLOGY, VELLORE, INDIA
Email address: buvisparkle@gmail.com

DEPARTMENT OF MATHEMATICS
VELLORE INSITUTE OF TECHNOLOGY, VELLORE, INDIA
Email address: anuradhadhanapal1981@gmail.com