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BASE PARACOMPACTNESS IN BITOPOLOGICAL SPACES

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ABSTRACT. This paper defines base paracompactness in bitopological spaces and also studied the properties of pairwise base paracompact spaces and their relations with other bitopological spaces. In this study, many well known theorems are generalized concerning base paracompactness.

1. INTRODUCTION

The concept of bitopological spaces can be represented as a space of the form (S, μ_1, μ_2) where μ_1,μ_2 are two topologies on S. this is related to previous study that has been done on bitopological spaces whereby each of the topologies are set of points that has nearby points related to satisfy a set of axioms.Pairwise hausdorff (P-hausdorff), Pairwise regular (P-regular), Pairwise normal (P-normal) spaces were explain by Kelly (1963) [2] with common several standard result referred to as Tietze extintion. Further work in the field of bitopological spaces was carried out by Kim (1968) [3] and Patty (1967) [4]. A pairwise open cover of S and a cover $\tilde{U}$ of (S, μ_1, μ_2) known a pairwise open cover of (S, μ_1, μ_2) if $\tilde{U} \subseteq \mu_1 \cup \mu_2$, $\tilde{U} \cap \mu_i \neq \emptyset, i = 1, 2$, was explained by Fletcher et al (1969) [1]. Bitopological space $(S.\mu_1, \mu_2)$ is pairwise paracompact if it

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has locally finite parallel subtle distinction for every pairwise open cover$\tilde{U}$ of (S, μ_1, μ_2) and totally paracompact if it has a locally limited subcover for every open base of it (Ford, 1963, [7]). The relationship between minimum and maximum inductive magnitude in metric spaces was shown and conculusion was drawn that locally compact spaces can be totally paracompact. Base paracompact is a common to the concept of totally paracompactness, Porter (2003) [5] investigated many properties of base paracompact spaces. In this study, the concept of pairwise base paracompact spaces will be introduced. First we will define pairwise bases of the bitopologicl spaces and the concept of pairwise base paracompact, and also introduce some of their properties. In addition we will state a well known definition which will be used to state some theorems and results. In this study (S, μ) denotes a topological space and (S,μ_1, μ_2) denotes the bitopological spaces. Let $\mathbb{R}$, $\mathbb{Z}$, $\mathbb{N}$ and $\mathbb{Q}$ represent the set of all real, integer, natural and rational number respectively. Furthermore $\mu_{coc}, \mu_{dis}, \mu_s, \mu_u, \mu_l, \mu_r, \mu_{cof}$ represent cocountable, discrete, sorgenfrey, usual, left ray, right ray and cofinite topologies on a non empty set S. The smallest closed set which contains the given set A is represented by clA, w(S) denotes the weight of the space S, therefore, w(S)=$\mid \tilde{B} \mid$ where $\tilde{B}$ has the smallest cardinal number among all other bases of S.

2. Base paracompactness in bitopological spaces

The concept, properties and relationship among base paracompactness in bitopological spaces and other spaces will be introduced and disscused here. However let us recall two important defintions which will be need in sequance.

Definition 2.1. *If every μ_i-free variable set of $\tilde{V}$ is included in some μ_i-free variable set of $\tilde{V}$, i=1,2. Then a pairwise free variable cover $\tilde{V}$ of S is said to be parellel subtil distinction of a pairwise free variable $\tilde{V}$.*

Definition 2.2. *If for each s $\in$ S and E $\subseteq$ S, then we would refer to E as μ_{us} open and if E is $\mu_1$open (resp μ_2 open) set of $\tilde{U}$.*

Definition 2.3. *If for each s $\in$ S, there exist a μ_{us} neighborhood of x having at least one element in common with a limited number of members of $\tilde{V}$. A subtle distiction $\tilde{V}$ of a pairwise free variable cover $\tilde{U}$ of S is said to be locally limited.*

Definition 2.4. *[8] Lets represent a bitopological space as (S, μ_1, μ_2):*

- *A chosen set K is said to be pairwise open if K is equally μ_1-open and μ_2-open in S.*
- *A chosen set K is said to be pairwise closed if K is equally μ_1-closed and μ_2-closed in S.*
- *If the elements of a cover of a bitopological space (S, μ_1, μ_2) are members of μ_1 and μ_2, the cover is called pairwise open and it includes at minimum one occupied member of each μ_1 and μ_2.*

Definition 2.5. *[9] A set $\tilde{B} \subseteq$ P(S) is said to be pairwise basis of the bitopological space (S, μ_1, μ_2) if and only if :*

- *If $\tilde{B} \subseteq \mu_1 \cap \mu_2$, i.e., $\tilde{B}$ is collection of pairwise open sets.*
- *For all $s \in S$ and for each open set U containig s, $\beta \in \tilde{B}$ exist s $\in \beta \subseteq \tilde{B}$.*

Definition 2.6. *A bitopological space (S, μ_1, μ_2) is called pairwise base paracompact if there is a pairwise base for (S, μ_1, μ_2) with w(S)=| $\tilde{B}$ |. Each pairwise open cover of S has pairwise locally finite subtle distinction by $\tilde{B}$ members.*

Remark 2.1. *A bitopological space (S, μ_1, μ_2) is called pairwise base paracompact if both (S, μ_1) and(S, μ_2) are base paracompact.*

Example 1. *Consider (S, μ_{cof}, μ_u) is pairwise base paracompact, since μ_{cof} and μ_u are both base paracompact spaces.*

Definition 2.7. *A subset M of space S is pairwise base paracompact relative to S, if $\exists$ pairwise open base $\tilde{B}$ of S, w(S)= | $\tilde{B}$ |, every pairwise open cover (in S) of M has pairwise locally limited (in S) partial refinement $B \subseteq \tilde{B}$, M $\subseteq \cup$ B.*

Lemma 2.1. *Each pairwise closed subset of pairwise base paracompact space S is pairwise base paracompact relative to S.*

Proof. Assume S to be pairwise base paracompact space and $\tilde{B}$ be a pairwise basis for S, let M be a pairwise closed of S, and assume $\tilde{U}$ to be a pairwise open cover in S of M. The open cover $\tilde{U}\cup$ (S $\setminus$ M) in S has a pairwise locally limited open subtile distinction B by $\tilde{B}$ members.Thus, $W = \{\beta \in B, \beta \cap M \neq \emptyset\}$ is pairwise locally limited (in S) partial subtle distinction of $\tilde{U}$. □

Theorem 2.1. *Assume S to be a pairwise base paracompact. If M is pairwise closed of w(S)=w(M), in that case M is pairwise base paracompact.*

Proof. By Lemma 2.1, M is pairwise base paracompact relative to S, because w(S)=w(M) and M is pairwise base paracompact. □

Definition 2.8. *A pairwse cover $\tilde{A}$ of a set S is pairwise star finite subtle distinction of pairwise cover $\tilde{B}$ if for each $\beta \in \tilde{B}$, ∃ A ∈ $\tilde{A}$, star $(\beta, \tilde{B})$=∪ $\{c \in \tilde{B}, c \cap \beta \neq \emptyset\}$ $\subseteq \tilde{A}$.*

Remark 2.2. *Each open cover of a paracompact space has an open star subtle distinction.*

Theorem 2.2. *S is pairwise base paracompact, If S is pairwise paracompact, and it is denumerable union of closed pairwise base paracompact sets relative to S.*

Proof. To show (S, μ_1, μ_2) is pairwise base paracompact, it is sufficient to show that both topological spaces (S, μ_1) and (S, μ_2) are base paracompact. Let $S = \cup_{i=1}^{\infty} F_i$ where every F_i is closed and base paracompact relative to S. Now ∀ i, ∃ a basis $\tilde{B}$ for S certifies base paracompactness relative to S for F_i. Assume that $\tilde{B} = \cup_{i=1}^{\infty} \beta_i$, be a ware that $\tilde{B}$ is a base for S and w(S)= $\mid \tilde{B} \mid$. In addition, $\tilde{B}$ certifies base paracompactness relative to S for each F_i. Assume $\tilde{U}$ to be an open cover for S. Now, there will be a locally limited subcollection A_o of $\tilde{B}$ that covers F_o and seperate $\tilde{U}$. By Remark 2.2, A_o has $(A_o)^*$ which is open star subtile distinction of the open cover A_o ∪ (S $\setminus$ F_o) of S. By induction, ∃ $A_n \subseteq \tilde{B}$ for i ≤ n which covers F_n and seperats $\tilde{U}$. Therefore, assume $(A_n)^*$ to be an star subtile distinction of the cover A_n ∪ (S $\setminus F_n$) of S and also every $(A_i)^*$ for each i ≤ n. For each j set $V_j = \{v \subseteq A_j$:vu for any u ∈ A_j and for every $i \leq j\}$. Let s ∈ S being given clearly, let $\cup_{i=1}^{\infty} A_i$ which covers S, assume j to be the minimum positive integer such s ∈ V for some V ∈ A_j. Assume V ⊆ U ∈ A_j for some i ≤ j. Then s ∈ U ∈ A_i, which disconfirm the choice of j. Hence v ∈ V_j and V covers S. Assume s ∈ S, then s ∈ F_i for some i. Then s ∈ W for some W ∈ $(A_i)^*$. Assume O be neighborhood of s with O ⊆ W, which meets limitedly many member of $\cup_{i=1}^{\infty} A_i$. Let V ∩ W ≠ ∅, for each v ∈ V_j with i ≤ j, because A_j seperates A_{j-1}, V ⊆ star (W, A_i) ⊆ star (W, $(A_i)^*$) ⊆ U ⊆ (A_i). This contradicts the choice of V_j. Hence V is locally finite refinement of U. Similarly, we show that(S, μ_2) is base paracompact, and hence (S, μ_1, μ_2) is pairwise base paracompact. □

Corollary 2.1. *Let S represents a pairwise base paracompact. M is base pairwise paracompact, if M ⊆ S is F_σ with w(S)=w(M).*

Proof. Because M is an F_σ set, then $M = \cup_{n=1}^{\omega} M_n$, where M_n is pairwise closed, by Lemma 2.1 every M_n is pairwise base paracompact relative to M because w(S)=w(M). Now with Theorem 2.2, M is pairwise base paracompact. □

Definition 2.9. *A pairwise open cover $\tilde{U} = \{u_\alpha, \alpha \in \Delta\}$ for a bitoplogical spaces (S,μ_1, μ_2) is called pairwise shrinkable $\iff$ $\exists$ a pairwise open cover $\tilde{V} = \{v_\alpha, \alpha \in \Delta\}$ such that $cl_1 v_\alpha$, $cl_2 v_\alpha \subseteq u_\alpha$.*

Definition 2.10. *A bitopological space (S,μ_1, μ_2) is caled pairwise locally base paracompact, if for each s$\in$ S, a pairwise open neighborhood O_s of s exists then $cl_1 O_s$, $cl_2 O_s$ are base paracompact spaces.*

Theorem 2.3. *A pairwise paracompact, pairwise locally base paracompact are base paracompact spaces.*

Proof. Let s $\in$ S; let O_s be a pairwise open neighborhood of s; $cl_1 O_s, cl_2 O_s$ are pairwise base paracompact; O=$\{O_s, s \in S\}$ be a pairwise open cover of (S, μ_1, μ_2). Since (S, μ_1, μ_2) is pairwise base paracompact, O has a pairwise locally limited open subtile distinction V, $\mid V \mid \leq$ w(S). Assume W shrinks O, that for each $\omega \in$ W there will be $O_\omega \in O$ such that $cl_1 W \subseteq O_\omega$ and $cl_2 W \subseteq O_\omega$. Hence, clW will be pairwise base paracompact for $\omega \in$ W. Assume B_x to be a pairwise abasis for O_ω which certifies base paracompactness relative to clW. Suppose $B = \{B_\omega, \omega \in W\}$ and assume $\tilde{U}$ to be a pairwise open cover of (S, μ_1, μ_2). Consider $\tilde{U}_\omega = \{U, U \cap clW \neq \emptyset\}$ there is a pairwise locally limited (in S) subtile distinction B'_ω of $\tilde{U}_\omega$, $\cup$ $B'_\omega \subseteq O_\omega$. It should be noted that B'_ω is pairwise locally limited in S. Therefore, $B' = \cup\{B'_\omega, \omega \in W\}$ is pairwise locally limited subtile distinction of $\tilde{U}$. □

Definition 2.11. *A pairwise bases $\tilde{B}$ for a topological space (S, μ_1, μ_2) is pairwise steady if for each point $s \in S$ and any pairwise neighborhood Uof S, a pairwise neighborhood $V \subseteq U$ of the point s exist, the set of all elements of $\tilde{B}$ meets finitly many of both V and S-U.*

Theorem 2.4. *The sets of greatest members from $\tilde{B}$, $(\tilde{B})^m = \{\beta \in \tilde{B}, if \beta \subseteq \beta' \in \tilde{B}, then \beta = \beta'\}$ is a pairwise locally finite cover of (S, μ_1, μ_2), if $\tilde{B}$ is pairwise steady bases for a space (S, μ_1, μ_2).*

Proof. We shall show $\cup(\tilde{B})^m = S$, for each s $\in$ S a pairwise U_0. Let s not be contained in any member of $(\tilde{B})^m$. Thus, we can define an infinite sequence

$U_0 \subseteq U_1 \subseteq U_2 \subseteq \ldots$ of $\tilde{B}$ elements, $U_i \neq U_{i+1}$ for i=1,2,..., i.e., an infinite subfamily of pairwise open sets $\{U_i, 1 \leq i \leq \infty\}$ of $\tilde{B}$ whose members contain s and meet $S - U_0$, which is impossible, hence s $\in (\tilde{B})^m$ and $\cup(\tilde{B})^m = S$. Now it can be seen if the pairwise base $\tilde{B}$ is pairwise regular, then the cover $(\tilde{B})^m$ is $(\tilde{B})^m$ which contains s, the set of all $\tilde{B}$ elements that meets both S-U and s and a pairwise neighborhood V of s. However, every $U' \in (\tilde{B})^m$ -V that contains s (meets V) also meets S-U, so that only finitly many members of $(\tilde{B})^m$ contains s (meets V) which shows that $(\tilde{B})^m$ is pairwise locally finite cover of S. ☐

Definition 2.12. *A pairwise basis $\tilde{B}$ for a topological space (S,μ_1, μ_2) is pairwise steady if for each point $s \in S$ and any pairwise neighborhood Uof S, a pairwise neighborhood $V \subseteq U$ of the point s exist, the set of all elements of $\tilde{B}$ meets finitly many of both V and S-U.*

Theorem 2.5. *The sets of greatest members from $\tilde{B}$, $(\tilde{B})^m$ =$\{\beta \in \tilde{B}$, if $\beta \subseteq \beta' \in \tilde{B}$,then$\beta = \beta'\}$ is a pairwise locally finite cover of (S,μ_1, μ_2), if $\tilde{B}$ is pairwise steady bases for a space (S,μ_1, μ_2).*

Proof. We shall show $\cup(\tilde{B})^m$=S, for each s$\in$ S a pairwise U_0. Let s not be contained in any member of $(\tilde{B})^m$. Thus, we can define an infinite sequence $U_0 \subseteq U_1 \subseteq U_2 \subseteq \ldots$ of $\tilde{B}$ elements, $U_i \neq U_{i+1}$ for $i = 1, 2, \ldots$, i.e., an infinite subfamily of pairwise open sets $\{U_i, 1 \leq i \leq \infty\}$ of $\tilde{B}$ whose members contain s and meet S-U_0, which is impossible, hence s $\in (\tilde{B})^m$ and $\cup(\tilde{B})^m$ =S.

Now it can be seen if the pairwise base $\tilde{B}$ is pairwise regular, then the cover $\tilde{B}^m$ is pairwise locally finite, take a point s$\in$ S, with a pairwise open set U $\in (\tilde{B})^m$ which contains s, the set of all $\tilde{B}$ elements that meets both S-U and s and a pairwise neighborhood V of s. However, every $U' \in \tilde{B}^m$-V that contains s (meets V) also meets S-U, so that only finely many members of $(\tilde{B})^m$ contains s (meets V) which shows that $(\tilde{B})^m$ is pairwise locally finite cover of S. ☐

Lemma 2.2. *A bitological spaces (S,μ_1, μ_2) is pairwise measurable if, and only if, it is a pairwise T_1-space and poses a pairwise steady base.*

Proof. First we will check that every pairwise measurable space has a pairwise regular base (S, ρ, τ) be metric space for (S,μ_1, μ_2) and B_i a pairwise locally limited open refinement of the pairwise open cover $\{B(s, \frac{1}{4i}), s \in S\}$. Clearly $\tilde{B} = \{B_i, 1 \leq i \leq \infty\}$ is pairwise base for S. For each point s$\in$ S and for any pairwise neighborhood U of s, exist B(s,$\frac{1}{i}$) $\subseteq$ U. Let V_o= B(s,$\frac{1}{2i}$) and for $j =$

$1, 2, \ldots,$ let V_j be a pairwise neighborhood of s that meets only finitly many elements B_i, which is the set of all elements of $\tilde{B}$ that meets both V= $\cap\{V_j, 0 \leq j \leq i\}$ and S-U is limited. $\square$

Theorem 2.6. *Metric bitopological spaces are pairwise base paracompact spaces.*

Proof. Assume (S, μ_1, μ_2) be a pairwise measurable and suppose $\tilde{B}$ is pairwise steady basis for S and $\mid \tilde{B} \mid$=w(S). Suppose $\tilde{U}$ be a pairwise open cover of S, and suppose $C = \{\beta \in \tilde{B}, \beta \subseteq u, u \in \tilde{U}\}$. Note that C is pairwise steady bases for S, and hence C^m is pairwise locally limited subtile distinction of $\tilde{U}$ by $\tilde{B}$ elements. $\square$

Theorem 2.7. *Let us make $\tilde{B}$ a pairwise basis for a bitopological spaces S with $\mid \tilde{B} \mid$ =w(S). Thus, a pairwise bases $\tilde{B'}$ exists for (S, μ_1, μ_2) with $\mid \tilde{B'} \mid$=w(S) and $\tilde{B} \subseteq \tilde{B'}$, which is closed under finite unions and finite intersections, and complements of closure.*

Proof. Suppose $\tilde{B}_0 = \tilde{B}$ and suppose $\tilde{B}_1$ be all finite unions, finite intersections, and complements of clousers by elements of $\tilde{B}_0$. Be aware that $\mid \tilde{B}_1 \mid$ =w(S). Because $\mid \tilde{B} \mid$ =w(S), continue by induction, let $\mid \tilde{B}_{n-1} \mid$ =w(S), suppose $\tilde{B}_2$ be all finite unions, finite intersections and complements of clousers by members of $\tilde{B}_{n-1}$. Be aware that $\mid \tilde{B}_n \mid$=w(S). Because $\mid \tilde{B}_{n-1} \mid$ =w(S), then the bases $\tilde{B'}$= $\cup\{B_n, n \leq \omega\}$ is the needed bases. $\square$

Theorem 2.8. *A pairwise regular lindelof spaces are pairwise base paracompact.*

Proof. Suppose (S, μ_1, μ_2) be apairwise steady lindelof space, and suppose $\tilde{B'}$ be a pairwise bases for S with $\mid \tilde{B'} \mid$ =w(S). By Theorem 2.7, there is a pairwise base $\tilde{B}$ with $\tilde{B'} \subseteq \tilde{B}$ which is closed under finite unions and finite intersections and complements of clousers. Let $\tilde{U}$ be a pairwise open cover of S. Thus, for every s $\in$ S, there exist a pairwise open set V_s, $U_s \in \tilde{B}, s \in V_s \subseteq clV_s \subseteq U_s \subseteq$ U for some U $\subseteq \tilde{U}$. $\square$

Definition 2.13. *[6] A map f :$(S, \mu_1, \mu_2) \longrightarrow$ (T,ν_1, ν_2) is referred to as pairwise-continuous (pairwise open, pairwise closed, pairwise homemorphism, respectively) if the map f_1 :(S,μ_1) $\longrightarrow$ (T,ν_1) is continuous and f_2 :(S,μ_2) $\longrightarrow$ (T,ν_2) is continuous (open, closed, homemorphism, respectively).*

Definition 2.14. *A map f :$(S, \mu_1, \mu_2) \longrightarrow (T, \nu_1, \nu_2)$ is referred to as pairwise-perfect if the function f is pairwise continuous and pairwise closed and for all t $\in$ T, the set f^{-1} is pairwise compact.*

Theorem 2.9. *Let f :$(S, \mu_1, \mu_2) \longrightarrow (T, \nu_1, \nu_2)$ be a pairwise-perfect mapping, then S is pairwise base paracompact if T is so.*

Proof. Suppose $\tilde{B}_t$ be a pairwise basis for T which certifies base paracompactness. Note w(S) $\geq$ w(T). Let $\tilde{B}_s$ be a pairwise basis for S with $\mid \tilde{B}_s \mid$ =w(S) and let $\tilde{B'}_x = \{\tilde{B}_s \cup f^{-1}(\tilde{B}), \tilde{B} \in \tilde{B}_t\} \cup \{\tilde{B} \cap f^{-1}(\tilde{B'}), \tilde{B} \in \tilde{B}_s, \tilde{B'} \in \tilde{B}_t\}$. Claim $\tilde{B'}_x$ is the pairwise base which witness base paracompactness of S. Now $\mid \tilde{B'}_x \mid$ =w(S). Suppose $\tilde{U} = \{U_p, p \in P\}$ be a pairwise open cover for S. For each t $\in$ T, choose a finite subset I(y) $\subseteq$ P, $f^{-1}(\tilde{B}) \subseteq \{U_p, p \in I\}$, Since f is pairwise closed, $\exists$ a pairwise set V_t of t, $f^{-1}(t) \subseteq f^{-1}(V_t) \subseteq \cup \{U_p, p \in P\}$. The pairwise cover $\{V_t, t \in T\}$ has a pairwise locally limited subtle distinction $\tilde{B'}_y \subseteq \tilde{B}_t$, Then $\{f^{-1}(\tilde{B}), \tilde{B} \in \tilde{B'}_y\}$, is pairwise locally limited and for each $\tilde{B}$ $\in \tilde{B'}_y$. $f^{-1}(\tilde{B}) \subseteq f^{-1}(V_t) \subseteq \cup \{U_p, p \in I_{y(\tilde{B})}\}$, for some t $\in$ T. Now $\tilde{B''}_x$ = $\{U_p \cap f^{-1}(\tilde{B}) : \tilde{B} \in \tilde{B'}_y, \text{and} p \in I_{t(\tilde{B})}\}$ is pairwise locally finite refinement of $\tilde{U}$ by members of $\tilde{B'}_x$. Hence S is pairwise base paracompact. □

Corollary 2.2. *Let S be a pairwise base paracompact, and suppose T be a pairwise compact space. Hence, S×T is pairwise base paracompact.*

Proof. The projection map f: S×T $\longrightarrow$ S is pairwise perfect mapping, and hence S×T is pairwise base paracompact. □

Theorem 2.10. *Assuming S is a pairwise base paracompact space while T is a pairwise σ-compact. Then S×T is pairwise base paracompact.*

Proof. Suppose T = $\cup \{C_i, i \leq \omega\}$, where every C_i is a pairwise compact subset of T. Let $\tilde{B}_s$ be a pairwise base of S which certify base paracompactness. Also assuming $\tilde{B}_t$ is a pairwise base for T with $\mid \tilde{B}_t \mid$ =w(T). Be aware that $\tilde{B}_s \times \tilde{B}_t$ is a pairwise basis for S $\times$ T with $\mid \tilde{B}_s \times \tilde{B}_t \mid$ =w(S $\times$ T), Theorem 2.3 satisfies to prove S $\times$ C_i is pairwise base paracompactrelative to $S \times T$. Hence we will prove that $\tilde{U}$ poses a pairwise locally limited subcover S $\times$ C_i by elements of $\tilde{B}_s$ $\times \tilde{B}_t$. Since s $\times$ C_i is pairwise paracompact, finitly many elements of $\tilde{U}$ exist, say $U_1 \times V_1 \cdots U_{ns} \times V_{ns}$ that cover s $\times$ C_i and define $V_s = \{V_1, \ldots, V_{ns}\}$ and W_s=$U_1 \cap \cdots \cap U_{ns}$ Be aware that W=$\{W_s, s \in S\}$ covers S. suppose W' be a pairwise locally limited refinement of W by $\tilde{B}_s$ elements. For every O $\in$ W', O $\subseteq$ W_{so}, for some $W_{so} \subseteq$ W. Then $\cup$ $\{O \times V, V \in V_{so}, O \in W'\}$ is apairwise locally limited distinction of $\tilde{U}$ covering S $\times$ C_i. □

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