Advances in Mathematics: Scientific Journal **10** (2021), no.2, 787–797
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.2.10

THE SECOND HYPER ZAGREB INDEX OF SOME GRAPH OPERATIONS

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ABSTRACT. In this paper, some exact expressions for the second hyper Zagreb index of graph operations such as Join, Composition, Cartesian, Corona and Tensor products of graphs will be presented.

1. INTRODUCTION

All graphs considered here are simple, connected and finite. Let $V(G)$, $E(G)$, $d_G(v)$ and $d_G(u,v)$ denote the vertex set, the edge set, the degree of a vertex and the distance between the vertices u and v of a graph G respectively. A graph with n vertices and m edges is called a (n,m) graph. First we present the definitions and notations which are required throughout this paper.

A topological index of a graph G is a real number which is invariant under automorphism of G and doesnot depend on the labeling or pictorial representation of a graph. The Wiener index [10] is the first and most studied topological indices and it is defined as

$$W(G) = \frac{1}{2} \sum_{\{u,v\} \subseteq V(G)} d_G(u,v).$$

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2020 *Mathematics Subject Classification.* 05C07, 05C12.

Key words and phrases. topological indices, join, composition, cartesian product, corona product and tensor product.

Submitted: 15.01.2021; *Accepted:* 30.01.2021; *Published:* 11.02.2021.

Gutman et.al., [4] introduced the first and second Zagreb indices of a graph G as follows:

$$M_1(G) = \sum_{uv\in E(G)} (d_G(u)+d_G(v)) = \sum_{u\in V(G)} d_G^2(u) \text{ and } M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v).$$

Shirdel et.al [9] defined the hyper-Zagreb index as

$$HM(G) = \sum_{uv\in E(G)} [d_G(u) + d_G(v)]^2$$

and presented the exact formulae for the Hyper-Zagreb index of some well-known graphs.

Farahani et.al [2] defined the second hyper-Zagreb index as

$$HM_2(G) = \sum_{uv\in E(G)} d_G^2(u)d_G^2(v).$$

A forgotten topological index F-index [3] is defined for a graph G as

$$F(G) = \sum_{v\in V(G)} d_G^3(v) = \sum_{uv\in E(G)} [d_G^2(u) + d_G^2(v)].$$

In 2018, Nilanjan.De [7] introduced a new index to calculate the F-index and co-index of some derived graphs. In 2020, Abdu Alameri et.al. [1] named that index as Y-index which is defined as

$$Y(G) = \sum_{v\in V(G)} d_G^4(v).$$

V.R. Kulli [6] introduced the first and second Gourava indices and defined as

$$GO_1(G) = \sum_{uv\in E(G)} (d_G(u) + d_G(v)) + (d_G(u)d_G(v))$$

and

$$GO_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v)(d_G(u) + d_G(v)).$$

The Join $G_1 + G_2$ of graphs G_1 and G_2 is a graph $G_1 + G_2$ with vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1) \text{ and } v \in V(G_2)\}$

$$|E(G_1 + G_2)| = |E(G_1)| + |E(G_2)| + |V(G_1)|\,|V(G_2)|\,.$$

The composition $G_1[G_2]$ of graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$ is the graph with vertex set $V(G_1) \times$

$V(G_2)$ and $u = (u_1, v_1)$ is adjacent to $v = (u_2, v_2)$ whenever u_1 is adjacent to u_2 or $u_1 = u_2$ and v_1 is adjacent to v_2.

The Cartesian product of the graphs G_1 and G_2 denoted by $G_1 \Box G_2$ is the graph with vertex set $V(G_1) \times V(G_2)$ and two vertices (u_1, u_2) and (v_1, v_2) are adjacent whenever (i) $u_1 = v_1$ and $u_2 v_2 \in E(G_2)$ or (ii) $u_2 = v_2$ and $u_1 v_1 \in E(G_1)$.

The corona product of the graphs G_1 and G_2 denoted by $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and $|V(G_1)|$ disjoint copies of G_2, and then joining the i^{th} vertex of G_1 to every vertex in i^{th} copy of G_2.

The tensor product of the graphs G_1 and G_2 denoted by $G_1 \times G_2$ is the graph with vertex set $V(G_1) \times V(G_2)$ and edge set is defined as

$$E(G_1 \times G_2) = \{(u_1, v_1)(u_2, v_2) : u_1 u_2 \in E(G_1) \text{ and } v_1 v_2 \in E(G_2)\}.$$

In this paper, we calculate the second hyper Zagreb index for Join, Composition, Cartesian, Corona product and tensor products of graphs. We begin this paper with the following Lemmas.

Lemma 1.1. *[5, 8]*

(a) If a is a vertex of $G_1 + G_2$ then we've

$$d_{G_1+G_2}(a) = \begin{cases} d_{G_1}(a) + |V(G_2)|\,, & \textit{if } a \in V(G_1) \\ d_{G_2}(a) + |V(G_1)|\,, & \textit{if } a \in V(G_2) \end{cases} .$$

(b) If (a, b) is a vertex of $G_1[G_2]$ then $d_{G_1[G_2]}((a, b)) = |V(G_2)| d_{G_1}(a) + d_{G_2}(b)$.

(c) $d_{G_1 \Box G_2}((u_i, v_j)) = d_{G_1}(u_i) + d_{G_2}(v_j)$, where $(u_i, v_j) \in V(G_1 \Box G_2)$.

(d)

$$d_{G_1 \odot G_2}(u) = \begin{cases} d_{G_1}(u) + n_2 & if\ u \in V(G_1) \\ d_{G_1}(u) + n_2 & if\ u \in V(G_{2,i})\ for\ some\ 0 \le i \le n_1 - 1, \end{cases}$$

where $u \in V(G_1 \odot G_2)$ $G_{2,i}$ is the ith copy of the graph G_2 in $G_1 \odot G_2$.

(e) $d_{G_1 \times G_2}(u_i, v_j) = d_{G_1}(u_i) d_{G_2}(v_j)$, where $(u_i, v_j) \in V(G_1 \times G_2)$.

2. The Second Hyper Zagreb Index of Join of Graphs

In this section, we find the exact value of the second hyper Zagreb index of Join of graphs.

Theorem 2.1. *Let $G_i, i = 1, 2$ be a (n_i, m_i) graph. Then*

$$\begin{aligned}
HM_2(G_1+G_2) &= HM_2(G_1) + n_2^2 HM(G_1) + n_2^4 m_1 + 2n_2 GO_2(G_1) \\
&+ 2n_2^2 M_2(G_1) + 2n_2^3 M_1(G_1) + HM_2(G_2) + n_1^2 HM(G_2) \\
&+ n_1^4 m_2 + 2n_1 GO_2(G_2) + 2n_1^2 M_2(G_2) + 2n_1^3 M_1(G_2) \\
&+ M_1(G_1)M_1(G_2) + n_1^2 n_2 M_1(G_1) + n_1 n_2^2 M_1(G_2) + n_1^3 n_2^3 \\
&+ 4n_1 m_2 M_1(G_1) + 4m_1 n_2 M_1(G_2) + 16 m_1 m_2 n_1 n_2 + 4m_1 n_2^2 n_1^2 \\
&+ 4n_1^2 n_2^2 m_2
\end{aligned}$$

Proof.

$$\begin{aligned}
HM_2(G_1+G_2) &= \sum_{xy\in E(G_1+G_2)} d^2_{G_1+G_2}(x) d^2_{G_1+G_2}(y) \\
&= \sum_{xy\in E(G_1)} d^2_{G_1+G_2}(x) d^2_{G_1+G_2}(y) + \sum_{xy\in E(G_2)} d^2_{G_1+G_2}(x) d^2_{G_1+G_2}(y) \\
&+ \sum_{xy\in\{xy:\ x\in V(G_1), y\in V(G_2)\}} d^2_{G_1+G_2}(x) d^2_{G_1+G_2}(y) \\
&= J_1 + J_2 + J_3,
\end{aligned}$$

where J_1, J_2 and J_3 are the terms of the above sums taken in order which are calculated as follows.

$$\begin{aligned}
J_1 &= \sum_{xy\in E(G_1)} d^2_{G_1+G_2}(x) d^2_{G_1+G_2}(y) \\
&= \sum_{xy\in E(G_1)} \left[\left(d_{G_1}(x) + n_2\right)\left(d_{G_1}(y) + n_2\right)\right]^2 \\
&= \sum_{xy\in E(G_1)} \left[d_{G_1}(x) d_{G_1}(y) + n_2\left(d_{G_1}(x) + d_{G_1}(y)\right) + n_2^2\right]^2 \\
&= HM_2(G_1) + n_2^2 HM(G_1) + n_2^4 m_1 + 2n_2 GO_2(G_1) + 2n_2^2 M_2(G_1) + 2n_2^3 M_1(G_1)
\end{aligned}$$

$J_2 = \sum_{xy\in E(G_2)} d^2_{G_1+G_2}(x) d^2_{G_1+G_2}(y)$

$$= \sum_{xy \in E(G_2)} \left[\left(d_{G_2}(x) + n_1 \right) \left(d_{G_2}(y) + n_1 \right) \right]^2$$

$$= \sum_{xy \in E(G_2)} \left[d_{G_2}(x) d_{G_2}(y) + n_1 \left(d_{G_2}(x) + d_{G_2}(y) \right) + n_1^2 \right]^2$$

$$= HM_2(G_2) + n_1^2 HM(G_2) + n_1^4 m_2 + 2n_1^2 M_2(G_2) + 2n_1^3 M_1(G_2) + 2n_1 GO_2(G_2)$$

$$J_3 = \sum_{xy \in \{xy:\, x \in V(G_1),\, y \in V(G_2)\}} \left(d_{G_1+G_2}(x) d_{G_1+G_2}(y) \right)^2$$

$$= \sum_{\substack{x \in V(G_1) \\ y \in V(G_2)}} \left[\left(d_{G_1}(x) + n_2 \right) \left(d_{G_2}(y) + n_1 \right) \right]^2$$

$$= \sum_{\substack{x \in V(G_1) \\ y \in V(G_2)}} \left[d_{G_1}(x) d_{G_2}(y) + n_1 d_{G_1}(x) + n_2 d_{G_2}(y) + n_2 n_1 \right]^2$$

$$= M_1(G_1) M_1(G_2) + n_1^2 n_2 M_1(G_1) + n_1 n_2^2 M_1(G_2) + n_1^3 n_2^3 + 4n_1 m_2 M_1(G_1)$$
$$+ 4m_1 n_2 M_1(G_2) + 16 m_1 m_2 n_1 n_2 + 4m_1 n_2^2 n_1^2 + 4n_1^2 n_2^2 m_2$$

Adding J_1, J_2, J_3 we get the desired result. □

3. The Second Hyper Zagreb Index of the composition of two graphs

In this section we find the exact value of the composition of two graphs.

Theorem 3.1. *Let $G_i, i = 1, 2$ be a (n_i, m_i) graph. Then*

$$\begin{aligned} HM_2(G_1[G_2]) = {} & n_2^6 HM_2(G_1) + n_2^3 M_1(G_2) F(G_1) + m_1 (M_1(G_2))^2 \\ & + 4m_2 n_2^4 GO_2(G_1) + 4m_2 n_2 M_1(G_1) M_1(G_2) + n_2^4 m_2 Y(G_1) \\ & + n_2^2 M_1(G_1) HM(G_2) + 16 m_2^2 n_2^2 M_2(G_1) + 2n_2^2 M_1(G_1) M_2(G_2) \\ & + 4m_1 n_2 GO_2(G_2) + n_1 HM_2(G_2) + 2n_2^3 F(G_1) M_1(G_2). \end{aligned}$$

Proof.

$$HM_2(G_1[G_2]) = \sum_{\substack{(u,x)(v,y)\in E(G_1[G_2])\\ uv\in E(G_1)}} \Big[d_{G_1[G_2]}(u,x).d_{G_1[G_2]}(v,y)\Big]^2 + \sum_{\substack{(u,x)(u,y)\in E(G_1[G_2])\\ xy\in E(G_2)}} \Big[d_{G_1[G_2]}(u,x).d_{G_1[G_2]}(u,y)\Big]^2 = K_1 + K_2$$

where K_1 and K_2 are the terms of the above sums taken in order which are calculated as follows:

$$\begin{aligned}K_1 &= \sum_{\substack{(u,x)(v,y)\in E(G_1[G_2])\\ uv\in E(G_1)}} \Big[d_{G_1[G_2]}(u,x).d_{G_1[G_2]}(v,y)\Big]^2\\ &= \sum_{x\in V(G_2)}\sum_{y\in V(G_2)}\sum_{uv\in E(G_1)} \Big[\Big(n_2 d_{G_1}(u) + d_{G_2}(x)\Big)\Big(n_2 d_{G_1}(v) + d_{G_2}(y)\Big)\Big]^2\\ &= n_2^6 HM_2(G_1) + n_2^3 M_1(G_2)F(G_1) + m_1(M_1(G_2))^2\\ &+ 4m_2 n_2^4 GO_2(G_1) + 16 m_2^2 n_2^2 M_2(G_1) + 4m_2 n_2 M_1(G_2)M_1(G_1)\end{aligned}$$

$$\begin{aligned}K_2 &= \sum_{\substack{(u,x)(u,y)\in E(G_1[G_2])\\ xy\in E(G_2)}} \Big[d_{G_1[G_2]}(u,x).d_{G_1[G_2]}(u,y)\Big]^2\\ &= \sum_{u\in V(G_1)}\sum_{xy\in E(G_2)} \Big[\Big(n_2 d_{G_1}(u) + d_{G_2}(x)\Big)\Big(n_2 d_{G_1}(u) + d_{G_2}(y)\Big)\Big]^2\\ &= n_2^4 m_2 Y(G_1) + n_2^2 M_1(G_1)HM(G_2) + n_1 HM_2(G_2) + 2n_2^3 F(G_1)M_1(G_2)\\ &+ 2n_2^2 M_1(G_1)M_2(G_2) + 4m_1 n_2 GO_2(G_2)\end{aligned}$$

It is easy to see that the summation of K_1 and K_2 completes the proof. □

4. The Second Hyper Zagreb Index of Cartesian Product of Graphs

In this section, we find the exact value of the second hyper Zagreb index of cartesian product of graphs.

Theorem 4.1. *Let $G_i, i = 1, 2$ be a (n_i, m_i) graph. Then*

$$\begin{aligned} HM_2(G_1\Box G_2) &= m_2Y(G_1) + 2F(G_1)M_1(G_2) + 4M_1(G_1)M_2(G_2) + M_1(G_1)F(G_2) \\ &+ n_1HM_2(G_2) + 4m_1GO_2(G_2) + m_1Y(G_2) + 2F(G_2)M_1(G_1) \\ &+ 4M_1(G_2)M_2(G_1) + M_1(G_2)F(G_1) + n_2HM_2(G_1) \\ &+ 4m_2GO_2(G_1) \end{aligned}$$

Proof.

$$\begin{aligned} HM_2(G_1\Box G_2) &= \sum_{(u,x)(v,y)\in E(G_1\Box G_2)} d^2_{G_1\Box G_2}(u,x)d^2_{G_1\Box G_2}(v,y) \\ &= \sum_{u\in V(G_1)}\sum_{xy\in E(G_2)} d^2_{G_1\Box G_2}(u,x)d^2_{G_1\Box G_2}(u,y) \\ &\quad + \sum_{x\in V(G_2)}\sum_{uv\in E(G_1)} d^2_{G_1\Box G_2}(u,x)d^2_{G_1\Box G_2}(v,x) \\ &= \sum_{u\in V(G_1)}\sum_{xy\in E(G_2)} [d_{G_1}(u)+d_{G_2}(x)]^2[d_{G_1}(u)+d_{G_2}(y)]^2 \\ &\quad + \sum_{x\in V(G_2)}\sum_{uv\in E(G_1)} [d_{G_1}(u)+d_{G_2}(x)]^2[d_{G_1}(v)+d_{G_2}(x)]^2 \\ &= S_1 + S_2, \end{aligned}$$

where S_1 and S_2 are the terms of the above sums taken in order which are calculated as follows:

$$\begin{aligned} S_1 &= \sum_{u\in V(G_1)}\sum_{xy\in E(G_2)} [d_{G_1}(u)+d_{G_2}(x)]^2[d_{G_1}(u)+d_{G_2}(y)]^2 \\ &= \sum_{u\in V(G_1)}\sum_{xy\in E(G_2)} [d^2_{G_1}(u)+d^2_{G_2}(x)+2d_{G_1}(u)d_{G_2}(x)] \\ &\quad [d^2_{G_1}(u)+d^2_{G_2}(y)+2d_{G_1}(u)d_{G_2}(y)] \\ &= m_2Y(G_1) + 2F(G_1)M_1(G_2) + 4M_1(G_1)M_2(G_2) + M_1(G_1)F(G_2) \\ &\quad + n_1HM_2(G_2) + 4m_1GO_2(G_2) \end{aligned}$$

$$\begin{aligned}
S_2 &= \sum_{x\in V(G_2)} \sum_{uv\in E(G_1)} [d_{G_1}(u)+d_{G_2}(x)]^2[d_{G_1}(v)+d_{G_2}(x)]^2 \\
&= \sum_{x\in V(G_2)} \sum_{uv\in E(G_1)} [d^2_{G_1}(u)+d^2_{G_2}(x)+2d_{G_1}(u)d_{G_2}(x)] \\
&\quad [d^2_{G_1}(v)+d^2_{G_2}(x)+2d_{G_1}(v)d_{G_2}(x)] \\
&= m_1Y(G_2)+2F(G_2)M_1(G_1)+4M_1(G_2)M_2(G_1)+M_1(G_2)F(G_1) \\
&\quad + n_2HM_2(G_1)+4m_2GO_2(G_1)
\end{aligned}$$

Adding S_1 and S_2 we get the required result. □

5. The Second Hyper Zagreb Index of Corona Product of Graphs

In this section, we find the exact value of the second hyper Zagreb index of corona product of graphs.

Theorem 5.1. *Let $G_i, i=1,2$ be a (n_i,m_i) graph. Then*

$$\begin{aligned}
HM_2(G_1\odot G_2) &= n_2^4m_1+2n_2^3M_1(G_1)+4n_2^2M_2(G_1)+n_2^2F(G_1)+HM_2(G_1) \\
&\quad + 2n_2GO_2(G_1)+M_1(G_1)M_1(G_2)+4m_2M_1(G_1)+n_2M_1(G_1) \\
&\quad + 4m_1n_2M_1(G_2)+16m_1m_2n_2+4n_2^2m_1+n_1n_2M_1(G_2) \\
&\quad + n_1n_2^2(n_2+4m_2)+n_1[m_2+2M_1(G_2)+4M_2(G_2)] \\
&\quad + n_1[F(G_2)+HM_2(G_2)+2GO_2(G_2)]
\end{aligned}$$

Proof. The copy of G_2 in $G_1\odot G_2$ corresponding to the vertex u in G_1 is denoted by $G_{2,u}$. The edge set of $G_1\odot G_2$ can be partitioned into three subsets as follows:

$$\begin{aligned}
E_1 &= \{uv\in E(G_1\odot G_2) : u,v\in V(G_1)\} \\
E_2 &= \{uv\in E(G_1\odot G_2) : u\in V(G_1), v\in V(G_{2,u})\} \\
E_3 &= \{uv\in E(G_{2,u}) : u\in V(G_1)\}
\end{aligned}$$

$$
\begin{aligned}
HM_2(G_1 \odot G_2) = & \sum_{uv \in E(G_1)} (d_{G_1}(u) + n_2)^2 (d_{G_1}(v) + n_2)^2 \\
& + \sum_{u \in V(G_1)} \sum_{xy \in E(G_2)} (d_{G_2}(x) + 1)^2 (d_{G_2}(y) + 1)^2 \\
& + \sum_{u \in V(G_1)} \sum_{x \in V(G_2)} (d_{G_1}(u) + n_2)^2 (d_{G_2}(x) + 1)^2 \\
& = S_1 + S_2 + S_3,
\end{aligned}
$$

where S_1, S_2 and S_3 are the terms of the above sums taken in order which are calculated as follows:

$$
\begin{aligned}
S_1 &= \sum_{uv \in E(G_1)} (d_{G_1}(u) + n_2)^2 (d_{G_1}(v) + n_2)^2 \\
&= \sum_{uv \in E(G_1)} [d_{G_1}^2(u) + n_2^2 + 2n_2 d_{G_1}(u)][d_{G_1}^2(v) + n_2^2 + 2n_2 d_{G_1}(v)] \\
&= n_2^4 m_1 + 2n_2^3 M_1(G_1) + 4n_2^2 M_2(G_1) + n_2^2 F(G_1) + HM_2(G_1) + 2n_2 GO_2(G_1) \\
S_2 &= \sum_{u \in V(G_1)} \sum_{xy \in E(G_2)} (d_{G_2}(x) + 1)^2 (d_{G_2}(y) + 1)^2 \\
&= \sum_{u \in V(G_1)} \sum_{xy \in E(G_2)} [d_{G_2}^2(x) + 2d_{G_2}(x) + 1][d_{G_2}^2(y) + 2d_{G_2}(y) + 1] \\
&= n_1[m_2 + 2M_1(G_2) + 4M_2(G_2) + F(G_2) + HM_2(G_2) + 2GO_2(G_2)] \\
S_3 &= \sum_{u \in V(G_1)} \sum_{x \in V(G_2)} (d_{G_1}(u) + n_2)^2 (d_{G_2}(x) + 1)^2 \\
&= \sum_{u \in V(G_1)} \sum_{x \in V(G_2)} [d_{G_1}^2(u) + 2n_2 d_{G_1}(u) + n_2^2][d_{G_2}^2(x) + 2d_{G_2}(x) + 1] \\
&= M_1(G_1)M_1(G_2) + 4m_2 M_1(G_1) + n_2 M_1(G_1) + 4m_1 n_2 M_1(G_2) + 4m_1 n_2^2 \\
&\quad + n_1 n_2 M_1(G_2) + 16 m_1 m_2 n_2 + 4n_1 n_2^2 m_2 + n_1 n_2^3.
\end{aligned}
$$

Adding S_1, S_2 and S_3 we get the required result. □

6. THE SECOND HYPER ZAGREB INDEX OF TENSOR PRODUCT OF GRAPHS

In this section, we find the exact value of the second hyper Zagreb index of tensor product of graphs.

Theorem 6.1. *Let* $G_i, i = 1, 2$ *be a* (n_i, m_i) *graph. Then* $HM_2(G_1 \times G_2) = 2HM_2(G_1)HM_2(G_2)$.

Proof.

$$\begin{aligned} HM_2(G_1 \times G_2) &= \sum_{(u,x)(v,y)\in E(G_1\times G_2)} d^2_{G_1\times G_2}(u,x)d^2_{G_1\times G_2}(v,y) \\ &\quad + \sum_{(u,y)(v,x)\in E(G_1\times G_2)} d^2_{G_1\times G_2}(u,y)d^2_{G_1\times G_2}(v,x) \\ &= \sum_{uv\in E(G_1)} \sum_{xy\in E(G_2)} d^2_{G_1}(u)d^2_{G_2}(x)d^2_{G_1}(v)d^2_{G_2}(y) \\ &\quad + \sum_{uv\in E(G_1)} \sum_{xy\in E(G_2)} d^2_{G_1}(u)d^2_{G_2}(y)d^2_{G_1}(v)d^2_{G_2}(x) \\ &= 2\sum_{uv\in E(G_1)} \sum_{xy\in E(G_2)} d^2_{G_1}(u)d^2_{G_1}(v)d^2_{G_2}(x)d^2_{G_2}(y) \\ &= 2HM_2(G_1)HM_2(G_2) \\ HM_2(G_1 \times G_2) &= 2HM_2(G_1)HM_2(G_2) \end{aligned}$$

□

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