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CERTAIN SUBCLASS OF MULTIVALENT FUNCTIONS WITH FINITELY MANY FIXED COEFFICIENTS BY USING SALAGEAN DIFFERENTIAL OPERATOR

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ABSTRACT. We investigate and study a subclass of analytic multivalent functions with finitely many fixed coefficients defined by using Salagean differential operator. Our main objective is to find coefficient estimates, closure theorems, integral operators and extreme points are applied to in this class.

1. INTRODUCTION

Denote $A_p(n)$ be the class of functions f of the form

$$f(z) = z^p + \sum_{k=n}^{\infty} a_{p+k} z^{p+k}, \quad (a_{p+k} \geq 0; p, n \in \mathbb{N}), \tag{1.1}$$

which are analytic and multivalent in the open unit disc $E = \{z : z \in \mathbb{C}, |z| < 1\}$. We write $A_{(1,1)} = A$.

Let $S_p(n)$ denote the new subclass of $A_p(n)$, which also multivalent in E. T. Rosy [13] introduced the class $SD(\alpha)$, which was recently studied by S. Sunil Varma and T. Rosy [12], R. Ezhilarasi et al. [4], M. Shanthi and C. Selvaraj [11],

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that the functions consists of the form (1.1) satisfying the following analytic criteria

$$Re\left\{\frac{f(z)}{z}\right\} \geq \alpha\left|f^{'}(z) - \frac{f(z)}{z}\right|, \quad \alpha \geq 0.$$

In earlier many authors like K.K. Dixit and I.B. Misra [2], S. Owa and H.M. Srivastava [9], Murugusundramoorthy and Magesh [7] considered a subclass of $S_p(n)$ by fixing finitely many coefficients.

Now, let $T_p(n)$ be the subclass of $S_p(n)$ consisting of functions f of the form

$$f(z) = z^p - \sum_{k=n}^{\infty} a_{p+k} z^{p+k}, \quad (a_{p+k} \geq 0; p, n \in \mathbb{N}). \tag{1.2}$$

The subclasses of $A_p(n)$ were introduced by M. I. S. Robertson [10]. Further properties of subclass of multivalent analytic functions were studied by many authors like J. L. Lio [5] and [3, 8]. A function $f \in A_p(n)$ is said to be in the classes $N^*_{(p,n,\alpha)}(\alpha)$ and $T_p(n)(\alpha)$ of multivalent starlike of order α in E and multivalent convex of order α in E, if it satisfies the following inequalities.

$$Re\left\{\frac{zf^{'}(z)}{f(z)}\right\} > \alpha, \quad z \in E, (0 \leq \alpha < p, p \in \mathbb{N}),$$

and

$$Re\left\{1 + \frac{zf^{''}(z)}{f^{'}(z)}\right\} > \alpha, \quad z \in E, (0 \leq \alpha < p, p \in \mathbb{N}). \tag{1.3}$$

Let the functions $f(z) \in A_p(n)$, then we get,

$$D^m f(z) = D(D^{m-1} f(z)) = p^m z^p + \sum_{k=n}^{\infty} (p+k)^m a_{p+k} z^{p+k}.$$

By the work of Muhammet Kamali and Fatma Sagoz [6], we define $\rho : A_p(n) \to A_p(n)$,

$$\rho(m, \lambda, p) = \left(\frac{1}{p^m} - \lambda\right) D^m f(z) + \frac{\lambda z}{p}(D^m f(z))^{'},$$

$0 \leq \lambda \leq \frac{1}{p^m}$, $m \in \mathbb{N} \cup \{0\}$.

Now we consider a subclass $TSD(p, n, m, \alpha)$ with finitely fixed many coefficients defined by using Salagean differential operator. For convenience, $\rho(m, \lambda, p)$ is denoted by $G(z)$. Further we define,

$$TDSD(p, n, m, \alpha) = T_{(p,n)} \cap TSD(p, n, m, \alpha).$$

Definition 1.1. *A functions $f \in S_p(n)$ is in the $TSD(p,n,m,\alpha)$, if it satisfies the following analytic condition*

$$Re\left\{\frac{G(z)}{z}\right\} \geq \alpha\left|G^{'}(z)-\frac{G(z)}{z}\right|,$$

where $\alpha \geq 0, 0 \leq \lambda \leq \frac{1}{p^m}, m \in \mathbb{N} \cup (0)$.

Now, we prove a necessary and sufficient condition for the functions in $S_p(n)$ to be in $TSD(p,n,m,\alpha)$.

Theorem 1.1. *A function $f(z)$ of the form* (1.2) *is in the class $TDSD(p,n,m,\alpha)$, if, and only if,*

$$\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]\,|a_{p+k}| \leq 1+\alpha(p-1). \tag{1.4}$$

Proof. Let the function f of the form (1.2) satisfies (1.4). Then

$$\begin{aligned}
&Re\left\{\frac{G(z)}{z}\right\}-\alpha\left|G^{'}(z)-\frac{G(z)}{z}\right| \geq 1-\left|\frac{G(z)}{z}-1\right|-\alpha\left|G^{'}(z)-\frac{G(z)}{z}\right|\\
&=1-\left|z^{p-1}+\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right]a_{p+k}z^{p+k-1}-1\right|\\
&\quad-\alpha|pz^{p-1}+\sum_{k=n}^{\infty}(p+k)^{m+1}\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right]a_{p+k}z^{p+k-1}-z^{p-1}\\
&\quad-\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right]a_{p+k}z^{p+k-1}|\\
&=1-\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right]|a_{p+k}|\\
&\quad+\alpha\left[(p-1)-\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right](p+k-1)|a_{P+k}|\right]\\
&=1+\alpha(p-1)-\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]\,|a_{p+k}|\\
&\geq 0.
\end{aligned}$$

Conversely, let $Re\left\{\frac{G(z)}{z}\right\}-\alpha\left|G^{'}(z)-\frac{G(z)}{z}\right| > 0$, which implies,

$$Re\left\{z^{p-1}+\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right]a_{p+k}z^{p+k-1}\right\}$$
$$-\alpha|pz^{p-1}+\sum_{k=n}^{\infty}(p+k)^{m+1}\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right]a_{p+k}z^{p+k-1}-z^{p-1}$$
$$-\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right]a_{p+k}z^{p+k-1}|>0,$$
$$Re\left\{|z^{p-1}|+\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right]|a_{p+k}||z^{p+k-1}|\right\}$$
$$-\alpha|pz^{p-1}+\sum_{k=n}^{\infty}(p+k)^{m+1}\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right]a_{p+k}z^{p+k-1}-z^{p-1}$$
$$-\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right]a_{p+k}z^{p+k-1}|>0.$$

Letting z to take real values and as $|z|\to 1$, we get

$$1+\alpha(p-1)-\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]\,|a_{p+k}|\geq 0,$$

which implies,

$$\sum_{k=n}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]\,|a_{p+k}|\leq 1+\alpha(p-1).$$

□

Corollary 1.1. *Let $f\in TDSD(p,n,m,\alpha)$. Then*

$$a_{p+k}\leq\frac{1+\alpha(p-1)}{(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]}. \tag{1.5}$$

Now we introduce the subclass $TDSD(p,n,m,\alpha,q_{p+m})$ of $TDSD(p,n,m,\alpha)$ consisting of functions

$$f(z)=z^p-\sum_{i=n}^{m}\frac{1+\alpha(p-1)}{(p+i)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+i}{p}-1\right)\right][1+\alpha(p+i-1)]}-\sum_{k=m+1}^{\infty}a_{p+k}z^{p+k},$$

where $p,n\in\mathbb{N}, i=n,n+1,\ldots,m, 0\leq q_{p+i}\leq 1, 0\leq\sum_{i=n}^{m}q_{p+i}\leq 1.$

2. Coefficient Estimates

We now investigate and determine the coefficient estimate for functions in the class $TDSD(p, n, m, \alpha, q_{p+m})$.

Theorem 2.1. *A function of the form* (1.5) *belongs to the class* $TDSD(p, n, m, \alpha, q_{p+m})$, *if, and only if,*

$$\sum_{k=m+1}^{\infty} (p+k)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+k}{p} - 1\right)\right] [1 + \alpha(p+k-1)]\, a_{p+k}$$
$$\leq [1 + \alpha(p-1)][1 - \sum_{i=n}^{m} q_{p+i}],$$

where $0 \leq \alpha < p, 0 \leq q_{p+i} \leq 1, 0 \leq \sum_{i=n}^{m} q_{p+i} \leq 1.$

Proof. From (1.5), we have

$$a_{p+i} \leq \frac{[1 + \alpha(p-1)]q_{p+i}}{(p+i)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+i}{p} - 1\right)\right] [1 + \alpha(p+i-1)]},$$

where $p, n \in \mathbb{N}, i = n, n+1, \ldots, m, 0 \leq q_{p+i} \leq 1, 0 \leq \sum_{i=n}^{m} q_{p+i} \leq 1.$ By Theorem 1.1,

$$\sum_{k=n}^{\infty}(p+k)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+k}{p} - 1\right)\right] [1 + \alpha(p+k-1)]\, a_{p+k},$$

which implies

$$\sum_{i=n}^{m}(p+i)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+i}{p} - 1\right)\right] [1 + \alpha(p+i-1)]\, a_{p+i}$$
$$+ \sum_{k=m+1}^{\infty} (p+k)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+k}{p} - 1\right)\right] [1 + \alpha(p+k-1)]\, a_{p+k}$$
$$\leq 1 + \alpha(p-1)$$

$$\sum_{k=m+1}^{\infty} (p+k)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+k}{p} - 1\right)\right] [1 + \alpha(p+k-1)]\, a_{p+k}$$
$$\leq [1 + \alpha(p-1)][1 - \sum_{i=n}^{m} q_{p+i}].$$

Conversely,

$$Re\left\{\frac{G(z)}{z}\right\} - \alpha\left|G^{'}(z) - \frac{G(z)}{z}\right| > 0,$$

$$1+\alpha(p-1) - \sum_{i=n}^{m}(p+i)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+i}{p}-1\right)\right][1+\alpha(p+i-1)]\,a_{p+i} - \sum_{k=m+1}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]\,a_{p+k} \geq 0,$$

and

$$[1+\alpha(p-1)][1-\sum_{i=n}^{m} q_{p+i}] \geq \sum_{k=m+1}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]\,a_{p+k}.$$

Hence, $f \in TDSD(p, n, m, \alpha, q_{p+m})$.
The sharpness of the result follows by taking

$$f(z) = z^p - \sum_{i=n}^{m}\frac{[1+\alpha(p-1)]q_{p+i}}{(p+i)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+i}{p}-1\right)\right][1+\alpha(p+i-1)]}z^{p+i} - \frac{[1+\alpha(p-1)][1-\sum\limits_{i=n}^{m}q_{p+i}]}{(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]}z^{p+k},$$

where $k \geq n-1, p, n \in \mathbb{N}$. □

Corollary 2.1. *Let $f \in TDSD(p, n, m, \alpha, q_{p+m})$. Then, for $k \geq m+1$,*

$$a_{p+k} \leq \frac{[1+\alpha(p-1)][1-\sum\limits_{i=n}^{m}q_{p+i}]}{(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]}.$$

The result is sharp.

3. Closure theorems

Theorem 3.1. *The class* $f \in TDSD(p, n, m, \alpha, q_{p+m})$ *is convex.*

Proof. Let $f, g \in f \in TDSD(p, n, m, \alpha, q_{p+m})$. Then,

$$f(z) = z^p - \sum_{i=n}^{m} \frac{[1+\alpha(p-1)]q_{p+i}}{(p+k)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+k}{p} - 1\right)\right][1+\alpha(p+k-1)]} z^{p+i} - \sum_{k=m+1}^{\infty} a_{p+k} z^{p+k},$$

and

$$g(z) = z^p - \sum_{i=n}^{m} \frac{[1+\alpha(p-1)]q_{p+i}}{(p+k)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+k}{p} - 1\right)\right][1+\alpha(p+k-1)]} z^{p+i} - \sum_{k=m+1}^{\infty} b_{p+k} z^{p+k},$$

where $0 \le q_{p+i} \le 1, 0 \le \sum_{i=n}^{m} q_{p+i} \le 1$.

Let assume that $h(z) = \mu f(z) + (1-\mu) g(z)$. Then,

$$h(z) = z^p - \sum_{i=n}^{m} \frac{[1+\alpha(p-1)]q_{p+i}}{(p+i)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+i}{p} - 1\right)\right][1+\alpha(p+i-1)]} z^{p+i} - \sum_{k=m+1}^{\infty} [\mu a_{p+k} + (1-\mu) b_{p+k}] z^{p+k}.$$

Now,

$$\sum_{k=m+1}^{\infty} (p+k)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+k}{p} - 1\right)\right][1+\alpha(p+k-1)][\mu a_{p+k} + (1-\mu) b_{p+k}]$$
$$= \mu \sum_{k=m+1}^{\infty} (p+k)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+k}{p} - 1\right)\right][1+\alpha(p+k-1)] a_{p+k}$$
$$+ (1-\mu) \sum_{k=m+1}^{\infty} (p+k)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+k}{p} - 1\right)\right][1+\alpha(p+k-1)] b_{p+k}$$

$$\leq \mu[1+\alpha(p-1)][1-\sum_{i=n}^{m} q_{p+i}] + (1-\mu)[1+\alpha(p-1)][1-\sum_{i=n}^{m} q_{p+i}]$$
$$= [1+\alpha(p-1)][1-\sum_{i=n}^{m} q_{p+i}],$$

which implies $h(z) \in TDSD(p, n, m, \alpha, q_{p+m})$. □

Theorem 3.2. *Let*

$$f_m(z) = z^p - \sum_{i=n}^{m} \frac{[1+\alpha(p-1)]q_{p+i}}{(p+i)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+i}{p} - 1\right)\right][1+\alpha(p+i-1)]} z^{p+i} \tag{3.1}$$

and

$$\begin{aligned} f_{p+k}(z) = z^p &- \sum_{i=n}^{m} \frac{[1+\alpha(p-1)]q_{p+i}}{(p+i)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+i}{p} - 1\right)\right][1+\alpha(p+i-1)]} z^{p+i} \\ &- \frac{[1+\alpha(p-1)][1-\sum_{i=n}^{m} q_{p+i}]}{(p+k)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+k}{p} - 1\right)\right][1+\alpha(p+k-1)]} z^{p+k}, \end{aligned} \tag{3.2}$$

$k \geq m+1$. *Then* $f \in TDSD(p, n, m, \alpha, q_{p+m})$*, if, and only if,* f *can be expressed in the form*

$$f(z) = \sum_{k=m}^{\infty} \mu_{p+k} f_{p+k}(z),$$

where $\mu_{p+k} \geq 0, k \geq m$ *and* $\sum_{k=m}^{\infty} \mu_{p+k} = 1$.

Proof. Let $f \in A_p(n)$ can be expressed in the form (3.1). Then

$$\begin{aligned} f(z) = z^p &- \sum_{i=n}^{m} \frac{[1+\alpha(p-1)]q_{p+i}}{(p+i)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+i}{p} - 1\right)\right][1+\alpha(p+i-1)]} z^{p+i} \\ &- \sum_{k=m+1}^{\infty} \frac{\mu_{p+k}[1+\alpha(p-1)][1-\sum_{i=n}^{m} q_{p+i}]}{(p+k)^m \left[\frac{1}{p^m} + \lambda\left(\frac{p+k}{p} - 1\right)\right][1+\alpha(p+k-1)]} z^{p+k}. \end{aligned}$$

Now,

$$\sum_{k=m+1}^{\infty} \frac{(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]\mu_{p+k}[1+\alpha(p-1)][1-\sum\limits_{i=n}^{m} q_{p+i}]}{(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]}$$

$$= \sum_{k=m+1}^{\infty} \mu_{p+k}[1+\alpha(p-1)][1-\sum_{i=n}^{m} q_{p+i}] = [1+\alpha(p-1)][1-\sum_{i=n}^{m} q_{p+i}] \sum_{k=m+1}^{\infty} \mu_{p+k}$$

$$= [1+\alpha(p-1)][1-\sum_{i=n}^{m} q_{p+i}](1-\mu_{p+m}) \leq [1+\alpha(p-1)][1-\sum_{i=n}^{m} q_{p+i}],$$

which implies $f \in TDSD(p,n,m,\alpha,q_{p+m})$.

Conversely, for $k \geq m+1$, Let

$$\mu_{p+k} = \frac{(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]\,a_{p+k}}{[1+\alpha(p-1)][1-\sum\limits_{i=n}^{m} q_{p+i}]}, k \geq m+1$$

and $\mu_{p+m} = 1 - \sum\limits_{k=m+1}^{\infty} \mu_{p+k}$. Thus f can be represented as $f = \sum\limits_{k=m}^{\infty} \mu_{p+k} f_{p+k}(z)$. □

Corollary 3.1. *The extreme points of the class $TDSD(p,n,m,\alpha,q_{p+m})$ are the functions $f_n, (n \geq k)$ given by* (3.1) *and* (3.2).

The Alexander operator [1] for the functions in the class $T_p(n)$ maps the class of starlike functions onto convex functions and is defined as

$$I(f) = \int_0^z \frac{f(t)}{t} dt.$$

4. Integral operator

Theorem 4.1. *Let f of the form* (1.5) *be in the class $TDSD(p,n,m,\alpha,q_{p+m})$. Then $I(f) \in TDSD(p,n,m,\alpha,q_{p+m})$, where $t_{p+m} = \frac{q_{p+m}}{(p+m)}$.*

Proof. We have

$$I(f) = z^p - \sum_{i=n}^{m} \frac{[1+\alpha(p-1)]t_{p+i}}{(p+i)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+i}{p}-1\right)\right][1+\alpha(p+i-1)]} z^{p+i}$$

$$- \sum_{k=m+1}^{\infty} \frac{a_{p+k}}{(p+k)} z^{p+k}.$$

Now,

$$\sum_{k=m+1}^{\infty}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]\frac{a_{p+k}}{(p+k)}$$
$$\leq\frac{1}{(p+m+1)}(p+k)^m\left[\frac{1}{p^m}+\lambda\left(\frac{p+k}{p}-1\right)\right][1+\alpha(p+k-1)]a_{p+k}$$
$$\leq\frac{1}{(p+m+1)}[1+\alpha(p-1)][1-\sum_{i=n}^{m}q_{p+i}],$$

which implies $I(f)\in TDSD(p,n,m,\alpha,q_{p+m})$. □

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