Advances in Mathematics: Scientific Journal **10** (2021), no.2, 959–970
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.2.25

ULTIMATE BOUNDEDNESS RESULT FOR A CERTAIN SYSTEM OF THIRD ORDER NONLINEAR DIFFERENTIAL EQUATIONS

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ABSTRACT. The paper studies the problem of ultimate boundedness behaviour of solutions for a certain system of third order nonlinear differential equations. Using the Lyapunov's second (or direct) method, we obtain sufficient conditions for the ultimate boundedness of solutions for the non-homogeneous nonlinear differential equations. Our results complement and extend some well known results on the third order differential equations in the literature.

1. INTRODUCTION

In the relevant literature, many works have been done on the qualitative properties of solutions of third order nonlinear differential equations. The specific property we shall be interested in is the ultimate boundedness property of solutions because it plays vital role in characterizing the behaviour of solutions of nonlinear differential equations and critical parameters describing dynamical systems. The number of results related to the ultimate boundedness of solutions of third order differential equations are few, see [3, 5, 10–13] and the literature cited therein. Although all these works were done with the aid of Lyapunov

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2020 *Mathematics Subject Classification.* 34D40,34D20, 34C25.
Key words and phrases. Ultimate Boundedness, complete Lyapunov function, nonlinear third order system.
Submitted: 23.01.2021; *Accepted:* 07.02.2021; *Published:* 25.02.2021.

function but the Lyapunov function used in most of these works are either incomplete or contain signum functions. see [4]. This development continues until in a sequence of results [1,2,8,9] made use of a complete Lyapunov function to obtain their results. In particular, [9] used a suitable complete Lyapunov function to obtain ultimate boundedness result for a more general equation. It must be noted here that finding an appropriate Lyapunov function remains a general problem. see [6].

This paper is concerned with the problem of ultimate boundedness of solutions of the nonlinear differential equations of the form

$$x''' + \psi(x, x', x'')x'' + \phi(x, x', x'')x' = p(t, x, x', x''), \tag{1.1}$$

where $\psi \in C(\mathbb{R}\times\mathbb{R}\times\mathbb{R}, \mathbb{R})$, $\phi \in C(\mathbb{R}\times\mathbb{R}\times\mathbb{R}, \mathbb{R})$ and $p \in C([0, \infty)\times\mathbb{R}\times\mathbb{R}\times\mathbb{R}, \mathbb{R})$, $\mathbb{R}$ the real line $-\infty < t < \infty$. We consider the case in (1.1) specialized for the case in [9] where $\phi(x, x', x'')x' = g(x, x')+h(x, x', x'')$ is much more complicated. The difficulty increases depending on the assumptions made on ϕ and the requirement for a complete Lyapunov function. Our motivation comes from [9] and the papers mentioned above. Consequently, we shall give an example to illustrate the effectiveness of the result obtained and provide geometric arguments on the behaviour of solutions of the nonlinear third order system. The results obtained here extend and complement the results of [9] some others mentioned in the literature.

Our main result is the following theorem.

2. Main Result

Theorem 2.1. *In addition to the basic assumptions imposed on functions ψ, ϕ and p appearing in* (1.1)*, we further suppose that the functions $\psi(0, 0, 0) = 0$, $\phi(0, 0, 0) = 0$ and ψ'_x, ϕ'_y are continuous for all x, y, z. We assume that there exist positive constants a, b, c and δ_o, $ab - c > 0$, $c < 1$ such that the following conditions hold:*

(i) $\psi(x, y, z) \geq a$, *for all x, y, z;*
(ii) $\phi(x, y, z) \geq b$, *for all x, y, z;*
(iii) $\phi(x, 0, 0) \geq \delta_o$, *for all x;*
(iv) $0 < \phi'_x(x, 0, 0) \leq c$, $\phi'_z(x, 0, 0) \leq 0$ *for all x,* $\psi'_z(x, y, \theta_1 z) \geq 0$*;*

(v) $y\int_0^y \phi'_x(x,\sigma,0)\sigma d\sigma \leq 0$, $z\int_0^y \phi'_x(x,\sigma,0)\sigma d\sigma \leq 0$ *and* $y\int_0^y \psi'_x(x,\sigma,0)\sigma d\sigma \leq 0$*;*

(vi) $|p(t,x,y,z)| \leq D_o$.

Then, there exist a finite constant $D_1 = D_1(a,b,c,d_o,D_o)$ *such that every solution* $x(t)$ *of* (1.1) *satisfies*

$$|x(t)| \leq D_1, \quad |x'(t)| \leq D_1, \quad |x''(t)| \leq D_1 \quad \textit{for all } t \geq 0.$$

Lemma 2.1. *Clearly* $V(0,0,0) = 0$ *and there exists finite constants* $k_1 > 0$ *and* $k_2 > 0$ *such that*

$$k_1(x^2+y^2+z^2) \leq V(x,y,z) \leq k_2(x^2+y^2+z^2). \tag{2.1}$$

Our proof of Theorem 2.1 rests entirely on Lemma 2.1 and the scalar function $V = V(x,y,z)$ defined by

$$\begin{aligned}
2V = {} & \left[2\int_0^x \phi(\nu,0,0)\nu d\nu - \frac{\mu_2}{b}\phi^2(x,0,0)\right] + \mu_2 b\left[y + \frac{\phi(x,0,0)}{b}\right]^2 \\
& + \left[2\int_0^y \psi(x,\sigma,0)\sigma d\sigma - \mu_2^{-1} - \mu_1\right]y^2 + \mu_2(z+\mu_2^{-1}y)^2 \\
& + \mu_2\left[2\int_0^y \phi(x,\sigma,0)\sigma d\sigma - by^2\right] + \mu_1(b-\mu_1)x^2 \\
& + a\left[2\int_0^x \phi(\nu,0,0)\nu d\nu - \phi^2(x,0,0)\right] + [a^{-\frac{1}{2}}y + a^{\frac{1}{2}}\phi(x,0,0)]^2 \\
& + \left[2\int_0^y \phi(x,\sigma,0)\sigma d\sigma - a^{-1}y^2\right] + [2\int_0^y \psi(x,\sigma,0)\sigma d\sigma - ay^2] \\
& + (\mu_1 x + ay + z)^2,
\end{aligned} \tag{2.2}$$

where

$$\frac{1}{a} < \mu_2 < \frac{b}{c},$$

and

$$\mu_1 < \min\left\{b, (1+a), \frac{(ab-c) - 2^{-1}(1+\mu_2)}{[a+2^{-1}(\phi(x,y,z)-b)^2]}, \frac{2(a\mu_2 - 1)}{[\psi(x,y,z)-a]^2}\right\}, \tag{2.3}$$

and μ_1, μ_2 are some positive constants.

By the hypothesis of the Theorem 2.1, we have that this term in (2.2),

$$
\begin{aligned}
&2\int_0^x \phi(\nu,0,0)\nu d\nu - \frac{\mu_2}{b}\phi^2(x,0,0)\\
&= 2\left[\int_0^x \phi(\nu,0,0)\nu d\nu - \frac{\mu_1}{b}\int_0^x \phi(\nu,0,0)\frac{d\phi(\nu,0,0)}{d\nu}d\nu - \frac{\mu_2}{b}\phi^2(0,0)\right]\\
&= 2\left[\int_0^x \phi(\nu,0,0)\nu d\nu - \frac{\mu_2}{b}\int_0^x \phi(\nu,0,0)\phi'_\nu(\nu,0,0)\nu d\nu\right]\\
&= 2\int_0^x (1 - \frac{\mu_2}{b}\phi'_\nu(\nu,0,0))\phi(\nu,0,0)\nu d\nu\\
&= 2\left[1 - \frac{\mu_2}{b}\phi'_\nu(\nu,0,0)\right]\phi(\nu,0,0)\int_0^x \nu d\nu \geq (1 - \frac{\mu_2}{b}c)\delta_o x^2.
\end{aligned}
$$

Similarly,

$$a\left[2\int_0^x \phi(\nu,0,0)\nu d\nu - \phi^2(x,0,0)\right] \geq (1-c)a\delta_o x^2,$$

$$[2\int_0^y \psi(x,\sigma,0)\sigma d\sigma - \mu_2^{-1} - \mu_1]y^2 \geq (a - \mu_2^{-1} - \mu_1)y^2,$$

$$\mu_2\left[2\int_0^y \phi(x,\sigma,0)\sigma d\sigma - by^2\right] \geq 0,$$

Similarly, the term

$$[2\int_0^y \psi(x,\sigma,0)\sigma d\sigma - ay^2] \geq 0,$$

and finally,

$$\left[2\int_0^y \phi(x,\sigma,0)\sigma d\sigma - a^{-1}y^2\right] \geq (b - a^{-1})y^2.$$

Combining these estimates, we have

$$
\begin{aligned}
V \geq &\left[(1 - \frac{\mu_2}{b}c)\delta_o + (1-c)a\delta_o + \mu_1(b-\mu_1)\right]x^2 + \left[(a - \mu_2^{-1} - \mu_1) + (b - a^{-1})\right]y^2\\
&+ [a^{-\frac{1}{2}}y + a^{\frac{1}{2}}\phi(x,0,0)]^2 + \mu_2 b\left[y + \frac{\phi(x,0,0)}{b}\right]^2 + \mu_2(z + \mu_2^{-1}y)^2 + (\mu_1 x + ay + z)^2.
\end{aligned}
$$

By the earlier assumptions on a, b, c, the constants $(1-\mu_2 b^{-1}c)\delta_o, \mu_1(b-\mu_1), a\delta_o(1-c), (a - \mu_2^{-1} - \mu_1)$ and $(b - a^{-1})$ are positive by the inequalities in (2.3). So,

$$V(x,y,z) \geq \zeta_1(x^2 + y^2) + \mu_2(z + \mu_2^{-1}y)^2 + (\mu_1 x + ay + z)^2,$$

where $\zeta_1 = \min\left\{(1-\mu_2 b^{-1}c)\delta_o+\mu_1(b-\mu_1)+a\delta_o(1-c), (a-\mu_2^{-1}-\mu_1)+(b-a^{-1})\right\}$. Thus, it is evident from the terms contained in the above inequality that there exists a constant $k_1 > 0$ small enough such that

$$V(x, y, z) \geq k_1(x^2 + y^2 + z^2).$$

To prove the right side of inequality (2.1), the hypotheses (i) - (iii) of Theorem 2.1 and using the fact that

$$2|x||y| \leq x^2 + y^2$$

yields from V, term by term

$$\begin{aligned}
2a\mu_1|xy| &\leq 2|x||y| \leq a\mu_1(x^2 + y^2)\\
2a|yz| &\leq 2a|y||z| \leq a(y^2 + z^2)\\
2|yz| &\leq 2|y||z| \leq (y^2 + z^2)\\
2\mu_1|xz| &\leq 2\mu_1|x||z| \leq \mu_1(x^2 + z^2)\\
2\int_0^x \phi(\nu, 0, 0)\nu d\nu &\leq \delta_o x^2\\
2\int_0^y \phi(x, \sigma, 0)\sigma d\sigma &\leq by^2\\
2\mu_2\int_0^y \phi(x, \sigma, 0)d\sigma &\leq \mu_2 by^2\\
2a\int_0^x \phi(\nu, 0, 0)\nu d\nu &\leq abx^2\\
2\int_0^y \psi(x, \sigma, 0)d\sigma &\leq ay^2\\
2a\int_0^y \psi(x, \sigma, 0)d\sigma &\leq a^2y^2\\
2\mu_2\phi(x, 0, 0) &\leq \mu_2\delta_o x^2\\
2y\phi(x, 0, 0) \leq \delta_o|x||y| &\leq \delta_o(x^2 + y^2).
\end{aligned}$$

It follows that

$$
\begin{aligned}
V(x,y,z) \leq & \Big(2\delta_o + ab + \mu_1 b + a\mu_1 + \mu_1 + \mu_2\delta_o\Big)x^2 \\
& + \Big(2a + a(1-\mu_1) + b + 1 + a^2 + \mu_2 b + \delta_o\Big)y^2 \\
& + \Big(\mu_1 + \mu_2 + 2 + a\Big)z^2, \\
\leq & \ \zeta_2(x^2 + y^2 + z^2),
\end{aligned}
$$

where $\zeta_2 = \max\Big\{2\delta_o + ab + \mu_1 b + a\mu_1 + \mu_1 + \mu_2\delta_o, 2a + a(1-\mu_1) + b + 1 + a^2 + \mu_2 b + \delta_o, \mu_1 + \mu_2 + 2 + a\Big\}$.

If we choose a positive constant k_2, then we have

$$V(x,y,z) \leq k_2(x^2 + y^2 + z^2).$$

Thus, (2.1) of lemma 2.1 is established where k_1, k_2 are finite constants.

Now, the proof of Theorem 2.1 follows.

Proof. It is convenient to consider equation (1.1) in equivalent system form

$$
\begin{aligned}
x' &= y, \\
y' &= z, \\
z' &= -\psi(x,y,z)z - \phi(x,y,z)y + p(t,x,y,z).
\end{aligned}
\tag{2.4}
$$

Thus, it suffices to prove Theorem 2.1, if we can show that the function V defined in (2.2) satisfies, for any solution $(x(t), y(t), z(t))$ of (2.4) such that

$$\frac{d}{dt}V(x,y,z) \leq -k(x^2 + y^2 + z^2), \quad \text{for some constant } k > 0.$$

Now, differentiating (2.2) along the system (2.4) and after simplification we get

$$\begin{aligned}
&\frac{d}{dt}V(x,y,z)\\
&= -a[\psi(x,y,z)-\psi(x,y,0)]yz+\mu_1 bxy+(1+a)\phi(x,0,0)xy\\
&+(1+\mu_2)y\int_0^y \phi'_x(x,\sigma,0)\sigma d\sigma+(1+\mu_2)y^2\phi_x(x,0,0)\\
&+(1+\mu_2)z\int_0^y \phi'_y(x,\sigma,0)\sigma d\sigma+(1+a)z^2+(1+\mu_2)z\phi(x,0,0)\\
&-(1+\mu_2)\psi(x,y,z)z^2-(1+\mu_2)\phi(x,y,z)yz-[\psi(x,y,z)-a]xz\\
&+(1+a)\int_0^y \psi'_x(x,\sigma,0)\sigma d\sigma-(1+a)y^2\phi(x,y,z)+\mu_1 ay^2-\mu_1 xy\phi(x,y,z)\\
&+[ax+(1+a)y+(1+\mu_2)z]p(t,x,y,z).
\end{aligned}$$

By (iv) and (v) of Theorem 2.1, we have that

$$\begin{aligned}
\frac{a[\psi(x,y,z)-\psi(x,y,0)]}{z^2}yz^2 = a\psi'_z(x,y,\theta_1 z)yz^2 &\geq 0,\\
y\int_0^y \phi_x(x,\sigma,0)\sigma d\sigma &\leq 0,\\
z\int_0^y \phi_y(x,\sigma,0)\sigma d\sigma &\leq 0,\\
\int_0^y \psi_x(x,\sigma,0)\sigma d\sigma &\leq 0
\end{aligned}$$

and the term

$$(1+\mu_2)z\phi(x,0,0)=(1+\mu_2)z^2\phi'_z(x,0,0)\leq 0.$$

Thus, we have

$$\begin{aligned}
\frac{d}{dt}V(x,y,z) \leq& -\frac{1}{2}[(1+a)-\mu_1]x^2-(b-\mu_2 c)y^2\\
&-[(ab-c)-2^{-1}(1+\mu_2)-\mu_1(a+2^{-1}(\phi(x,y,z)-b)^2)]y^2\\
&-[(a\mu_2-1)-\frac{\mu_1}{2}[\psi(x,y,z)-a]^2]z^2-\frac{1+\mu_2}{2}a^2z^2\\
&-\frac{1}{2}\Big\{\mu_1[x+(\phi(x,y,z)-b)y]^2+(1+a)[x+\phi(x,0,0)y]^2\Big\}\\
&-\frac{(1+\mu_2)}{2}[y+\phi(x,y,z)z]^2-\frac{\mu_1}{2}[x+(\psi(x,y,z)-a)z]^2\\
&+[ax+(1+a)y+(1+\mu_2)z]p(t,x,y,z).
\end{aligned}$$

If we choose

$$\mu_1 < \min\left\{b, (1+a), \frac{(ab-c) - 2^{-1}(1+\mu_2)}{[a + 2^{-1}(\phi(x,y,z) - b)^2]}, \frac{2(a\mu_2 - 1)}{[\psi(x,y,z) - a]^2}\right\}$$

we have

$$\frac{d}{dt}V(x,y,z) \leq -\frac{1}{2}[(1+a) - \mu_1]x^2 - (b - \mu_2 c)y^2 - \frac{1+\mu_2}{2}a^2 z^2 + [ax + (1+a)y + (1+\mu_2)z]|p(t,x,y,z)|.$$

It follows that

$$\frac{d}{dt}V(x,y,z) \leq -k_3(x^2+y^2+z^2) + k_4(|x|+|y|+|z|),$$

where

$$k_3 = \min\{(1+a) - \mu_1, b - \mu_2 c, (\frac{1+\mu_2}{2})a^2\}$$

and

$$k_4 = D_o max\{a, (1+a), (1+\mu_2)\}.$$

Hence,

$$\frac{d}{dt}V(x,y,z) \leq -k_3(x^2+y^2+z^2) + k_5(x^2+y^2+z^2)^{\frac{1}{2}}, \tag{2.5}$$

where $k_5 = 3^{\frac{1}{2}}k_4$.
If we choose $(x^2+y^2+z^2)^{\frac{1}{2}} \geq k_6 = k_5 k_3^{-1}$, inequality (2.5) implies that

$$\frac{d}{dt}V(x,y,z) \leq -k_3(x^2+y^2+z^2), \tag{2.6}$$

we immediately see that

$$\frac{d}{dt}V(x,y,z) \leq -k_7, \quad \text{provided,} \quad (x^2+y^2+z^2) \geq k_7 k_3^{-1}.$$

The remainder of the proof of Theorem 2.1 may now be obtained by the use of estimates (2.1) and (2.6) and an obvious adaptation of the Yoshizawa type reasoning in [7]. □

2.1. **Example.** Consider equation (1.1) in the form

$$\begin{aligned} &x''' + \left[\frac{x}{1+x^2} + (x')^3 + (2+x'')^2\right]x'' + \left[\frac{\ln(x+2)}{1+x^2} + (x')^2 + (x'')^2 + 1\right]x' \\ &= \frac{1}{1+t^2+x^2+x'^2+x''^2}, \end{aligned} \tag{2.7}$$

comparing it with equation (2.4), it is clear that

$$\psi(x,y,z) = \frac{x}{1+x^2} + y^3 + (2+z)^2,$$

$$\phi(x,y,z) = \frac{\ln(x+2)}{1+x^2} + y^2 + z^2 + 1$$

and

$$p(t,x,y,z) = \frac{1}{1+t^2+x^2+y^2+z^2}.$$

It is easy to check that the hypothesis in Theorem 2.1 are satisfied since

$$\psi(x,y,z) \geq 4 = a,$$

$$\phi(x,y,z) \geq 1.7 = b,$$

$$\phi(x,0,0) = \frac{\ln(x+2)}{1+x^2} + 1 \geq 0.7 = \delta_o,$$

$$\phi'_z(x,0,0) \leq 0,$$

$$\phi'_x(x,0,0) = \frac{1}{(1+x^2)}.\frac{1}{(x+2)} - \frac{2x\ln(x+2)}{(1+x^2)^2} \leq \frac{1}{2} = c < 1.$$

From inequality (2.3), we have

$$0.25 < \mu_2 < 3.4.$$

We pick $\mu_2 = 2$, so that

$$\mu_1 < \min\{1.7, 5, 0.72, 0.69\},$$

we choose $\mu_1 = 0.4$.

Further,

$$|p(t)| \leq \frac{1}{1+t^2} \leq 1.$$

Hence, this shows that all the conditions of Theorem 2.1 are satisfied. Thus, we conclude that all the solutions $(x(t), x'(t), x''(t))$ or $(x(t), y(t), z(t))$ equivalently of equation (2.7) are ultimately bounded.

2.2. **Ultimate boundedness analysis of solutions of nonlinear system** (2.7).

(1) In Figure 1, is a graph showing the solutions $x(t)$ in (blue), $y(t)$ in (green) and $z(t)$ in (red) of (2.7) satisfying all the conditions of Theorem 2.1 for $p \neq 0$ as $t \to \infty$.

(2) In Figure 2, Figure 3 and Figure 4, the ultimate boundedness behavior of solutions $x(t)$ in (blue) and $y(t)$ in (red) and $z(t)$ in (yellow) respectively in equation (2.7) for $p \neq 0$ where $x(t), y(t), z(t)$ are bounded for all $t \geq 0$.

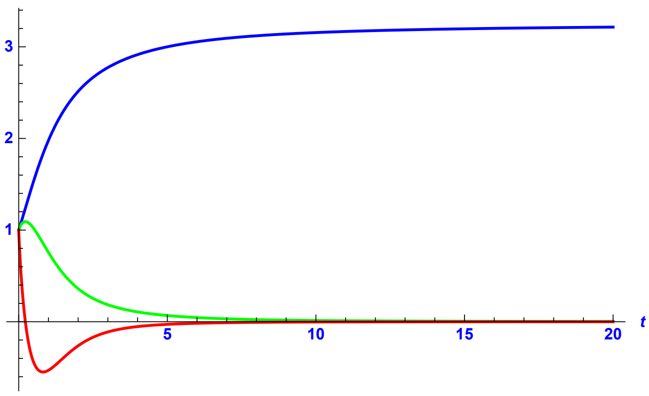

FIGURE 1. the solutions $x(t)$ in (blue), $y(t)$ in (green) and $z(t)$ in (red) of (2.7) satisfying all the conditions of Theorem 2.1 for $p \neq 0$ as $t \to \infty$.

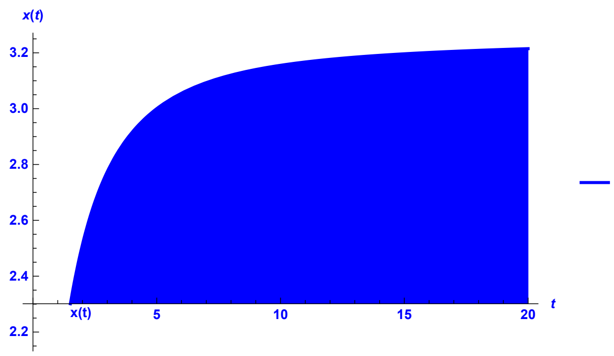

FIGURE 2. The graph of solution $x(t)$ of (2.7) satisfying the conditions of Theorem 2.1 for $p \neq 0$ is ultimately bounded as $t \to \infty$.

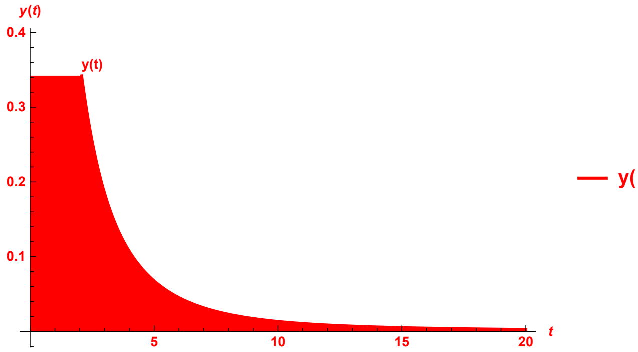

FIGURE 3. The graph of solution $y(t)$ of (2.7) satisfying the conditions of Theorem 2.1 for $p \neq 0$ is ultimately bounded as $t \to \infty$.

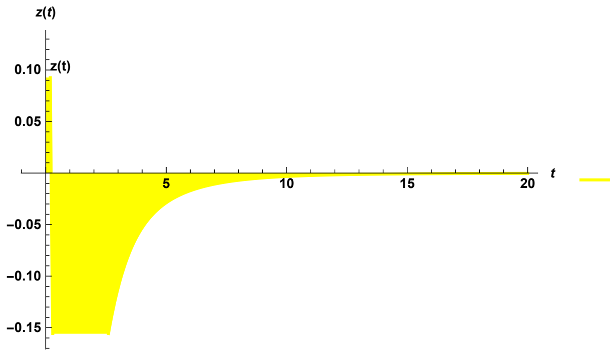

FIGURE 4. The graph of solution $z(t)$ of (2.7) satisfying the conditions of Theorem 2.1 for $p \neq 0$ is ultimately bounded as $t \to \infty$.

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