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ON RNδP-OPEN SETS

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ABSTRACT. The aim of this paper is to introduce a new class of sets called rnδp-open sets in nano topological spaces. we characterize these sets and study some of their fundamental properties. Also, we introduce and study the notion of rnδp-continuity.

1. INTRODUCTION

M.L. Thivagar and C. Richard [6] initiated the study of nano topological spaces with respect to a subset Y of a universe which is defined in terms of lower approximation, upper approximation, and boundary regions. They have also defined nano-closed sets, nano-interior and nano-closure. Recently, S.Parimala et.al., introduced the class of sets, namely nano δ-open,nano δ-preopen, nano δ-semiopen, nano $\delta\alpha$-open sets in nano topological spaces. In this paper, we introduce and study a new class of sets called rnδp-open sets. Also,some properties of rnδp-continuous functions are obtained.

Throughout this paper, (U,τ_R(X),(V,$\tau_R{}^*$(Y))(or simply U,V) represent nano topological spaces on which no separation axioms are assumed unless explicitly stated and f:(U,τ_R(X)$\rightarrow$(V,$\tau_R{}^*$(Y)) denotes a function f of a NTS space U

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into a NTS V. Let M $\subseteq$ U, then Ncl(M) = $\cap${F: M $\subseteq$ F and $F^c \in \tau_R$(X) } is the nano closure of M and Nint(M) = $\cup${O: O $\subseteq$ M and O $\in \tau_R$(X) } is the nano interior of M.

2. Preliminaries

Definition 2.1. *[5] Let U be a non-empty finite set of objects, called the universe, and R be an equivalence relation on U named as the indiscernibility relation. The pair (U,R) is said to be the approximation space. Let Y$\subseteq$U.*

- (i) *The lower approximation of Y with respect to R is L_R(Y)=$\cup_{y\in U}$\{ R(y):R(y) $\subseteq$ Y \} where R(y) denotes the equivalence class determined by y$\in$U.*
- (ii) *The upper approximation of Y with respect to R is HR(Y)=$\cup_{y\in U}$\{ R(y):R(y) $\cap$ Y $\neq \phi$\}.*
- (iii) *The boundary region of Y with respect to R is B_R(Y))=H_R(Y))-L_R(Y)).*

Definition 2.2. *[6] Let U be the universe and R be an equivalence relation on U.Let Y $\subseteq$ U. Let τ_R(Y)=N^T=\{U,ϕ,L_R(Y)),H_R(Y)),B_R(Y))\} is called the nano topology on U.The pair (U,N^T) is called nano topological space(briefly,NTS).*

Elements of the nano topology N^T are known as the nano open sets and the relative complements of nano open sets are called nano closed sets.

Definition 2.3. *[1, 3, 6] Let (U,N^T) be a NTS and $M_1 \subseteq$ U, then M_1 is said to be:*

- (i) *nano regular open if M_1 = Nint(Ncl(M_1));*
- (ii) *nano δ-open if M_1 = Nint$_\delta$(M_1) where Nint$_\delta$(M_1) = $\cup${B:B is a nano regular open and B $\subseteq M_1$};*
- (iii) *nano δ-preopen if $M_1 \subseteq$ Nint(Ncl$_\delta$(M_1));*
- (iv) *nano δ-semiopen if $M_1\subseteq$ Nint(Ncl$_\delta$(M_1));*
- (v) *nano $\delta\alpha$-open if $M_1 \subseteq$ Nint(Ncl(Nint$_\delta$(M_1)));*
- (vi) *nano e-open if $M_1 \subseteq$ Ncl(Nint$_\delta$(M_1))$\cup$Nint(Ncl$_\delta$(M_1)).*

The complements of the above respective open sets are their respective closed sets.

The class of nano δ-preopen(resp., nano δ-semiopen, nano e-open, nano δ-open) sets of (U,N^T) is denoted by NδPO(U) (resp., NδSO(U), NEO(U) and NδO(U)) and the class of nano δ-preclosed (resp., nano δ-semiclosed, nano e-closed, nano δ-closed) sets of (U,N^T) is denoted by NδPC(U) (resp., NδSC(U), NEC(U) and NδC(U)).

Definition 2.4. *[1, 3] If (U,N^T) is a NTS and $M \subseteq U$, then the nano δ-pre (resp., nano δ-semi, nano δ-, nano e-) interior of M is the union of all NδPO(U) (resp., NδSO(U), NδO(U) and NEO(U)) sets contained in M and denoted by $Nint_{\delta p}(M)$ (resp., $Nint_{\delta s}(M)$, $Nint_{\delta}(M)$ and $Nint_e(M)$) and the nano δ-pre(resp., nano δ-semi, nano δ-, nano e-) closure of M is the intersection of all NδPC(U) (resp., NδSC(U), NδC(U) and NEC(U)) sets containing M and denoted by $Ncl_{\delta p}(M)$ (resp., $Ncl_{\delta s}(M)$, $Ncl_{\delta}(M)$ and $Ncl_e(M)$).*

Lemma 2.1. *[1]Let M be a subset of a NTS (U, N^T), then:*

- (i) *$Ncl_{\delta p}(M)=M \cup Ncl(Nint_{\delta}(M))$ and $Nint_{\delta p}(M) = M \cup Nint(Ncl_{\delta}(M))$.*
- (ii) *$Ncl_{\delta s}(M)=M \cup Nint(Ncl_{\delta}(M))$ and $Nint_{\delta s}(M) = M \cup Ncl(Nint_{\delta}(M))$.*
- (iii) *$Nint_{\delta p}(Ncl_{\delta p}(M))= Ncl_{\delta p}(M)\cap Nint(Ncl_{\delta}(M)$.*

Lemma 2.2. *Let M be a subset of a NTS (U, N^T), then $Ncl_e(M)=Ncl_{\delta p}(M)\cap Ncl_{\delta s}(M)$.*

Definition 2.5. *[4] In a NTS (U,N^T), let $M \subseteq U$. Then M is said to be a nano δ-dense set if $Ncl_{\delta}(M)=U$.*

Definition 2.6. *[4] A NTS (U,N^T) is said to be nano δ-submaximal if every nano δ-dense subset of U is nano δ-open.*

Definition 2.7. *[2] A function $f:(U,\tau_R(X)\to(V,\tau_R{}^*(Y))$ is called nano δ-precontinuous if $f^{-1}(K)$ is nano δ-preopen in $(U,\tau_R(X)$ for every $K\in \tau_R{}^*(Y)$.*

3. Regular nano δ-preopen sets

Definition 3.1. *Let (U,N^T) be a NTS. Then a set $H \subseteq U$ is said to be:*

(1) *regular nano δ-preopen(briefly,rnδp-open) if $H = Nint_{\delta p}(Ncl_{\delta p}(H))$;*
(2) *regular nano δ-preclosed(briefly,rnδp-closed) if $H = Ncl_{\delta p}(Nint_{\delta p}(H))$.*

The class of rnδp-open (resp.,rnδp-closed) sets of (U,N^T) will be denoted by $RN_{\delta p}O(U)$ (resp,$RN_{\delta p}C(U)$).

Theorem 3.1. *In a NTS (U,N^T),let $M_1,M_2 \subseteq U$. If $M_1 \subseteq M_2$ then $Nint_{\delta p}(Ncl_{\delta p}(M_1)$ $\subseteq Nint_{\delta p}(Ncl_{\delta p}(M_2))$.*

Proof. Obvious. □

Theorem 3.2. *For a subset M_1 of a NTS (U,N^T), the following properties hold:*

(i) *$M_1 \subseteq Nint_{\delta p}$($Ncl_{\delta p}$($M_1$) if M_1 is nano δ-preopen.*

(ii) *$Ncl_{\delta p}$($Nint_{\delta p}$(M_1)) $\subseteq M_1$ if M_1 is nano δ-preclosed.*

(iii) *$Ncl_{\delta p}$(M_1) is rnδp-closed if M_1 is nano δ-preopen.*

(iv) *$Nint_{\delta p}$(M_1) is rnδp-open if M_1 is nano δ-preclosed.*

Proof.

(i) As $M_1 \subseteq Ncl_{\delta p}(M_1)$ and M_1 is nano δ-preopen, then $M_1 \subseteq Nint_{\delta p}(Ncl_{\delta p}(M_1))$.

(ii) Since $Nint_{\delta p}(M_1) \subseteq M_1$ and M_1 is nano δ-preclosed, then $Ncl_{\delta p}(Nint_{\delta p}(M_1)$ $\subseteq M_1$.

(iii) Suppose that M_1 is nano δ-preopen.

By (i), we have $Ncl_{\delta p}(Nint_{\delta p}(Ncl_{\delta p}\ (M_1)) \supseteq Ncl_{\delta p}(M_1)$. On the other hand, $Ncl_{\delta p}(M_1) \supseteq Nint_{\delta p}(Ncl_{\delta p}(M_1)$. So that $Ncl_{\delta p}(M_1) \supseteq Ncl_{\delta p}(Nint_{\delta p}(Ncl_{\delta p}(M_1))$. Therefore $Ncl_{\delta p}(Nint_{\delta p}(Nclt_{\delta p}(M_1)) = Ncl_{\delta p}(M_1)$.

(iv) Suppose that M_1 is nano δ-preclosed.

By (ii), we have $Nint_{\delta p}(Ncl_{\delta p}(Nint_{\delta p}(M_1))\subseteq Nint_{\delta p}(M_1)$. On the other hand, we have $Nint_{\delta p}(M_1) \subseteq Ncl_{\delta p}(Nint_{\delta p}(M_1)$. So that $Nint_{\delta p}(M_1) \subseteq Nint_{\delta p}(Ncl_{\delta p}$ $(Nint_{\delta p}(M_1))$. Therefore $Nint_{\delta p}(Ncl_{\delta p}(Nint_{\delta p}(M_1))=Nint_{\delta p}(M_1)$. Thus $Nint_{\delta p}(M_1)$ is rnδp-open. □

Theorem 3.3. *In a NTS (U,N^T) and let $M \subseteq U$. Then:*

(i) *$Nint_{\delta p}$($Ncl_{\delta p}$(M)) is rnδp-open.*

(ii) *$Ncl_{\delta p}$($Nint_{\delta p}$(M)) is rnδp-closed.*

Proof. (i) We have $Nint_{\delta p}(Ncl_{\delta p}(Nint_{\delta p}(Ncl_{\delta p}(M)) \subseteq Nint_{\delta p}(Ncl_{\delta p}(Ncl_{\delta p}(M)) = Nint_{\delta p}(Ncl_{\delta p}(M))$ and $Nint_{\delta p}(Ncl_{\delta p}(Nint_{\delta p}(Ncl_{\delta p}(M)) \supseteq Nint_{\delta p}(Nint_{\delta p}(Ncl_{\delta p}(M))$ $= Nint_{\delta p}(Ncl_{\delta p}(M))$. Therefore $Nint_{\delta p}(Ncl_{\delta p}(Nint_{\delta p}(Ncl_{\delta p}(M)) = Nint_{\delta p}(Ncl_{\delta p}(M))$. □

Theorem 3.4. *Let (U,N^T) be a NTS,then the following properties hold:*

(i) *Every rnδp-open set is nano δ-preopen.*

(ii) *Every rnδp-open set is nano e-open.*

(iii) *Every rnδp-open set is nano e-closed.*

Proof.

(i) Let M_1 be rnδp-open, then

$$M_1 = Nint_{\delta p}(Ncl_{\delta p}(M_1) = Ncl_{\delta p}(M_1) \cap Nint(Ncl_{\delta}(M_1)) \subseteq Nint(Ncl_{\delta}(M_1)).$$

Hence M_1 is nano δ-preopen.

(ii) Let M_1 be rnδp-open, then

$$M_1 = Nint_{\delta p}(Ncl_{\delta p}(M_1) = Ncl_{\delta p}(M_1) \cap Nint(Ncl_\delta(M_1)) \subseteq Nint(Ncl_\delta(M_1)) \subseteq Nint(Ncl_\delta(M_1)) \cup Ncl(Nint_\delta(M_1)).$$

Hence M_1 is nano e-open.

(iii) Suppose that M_1 is rnδp-open, then

$$\begin{aligned} M_1 &= Nint_{\delta p}(Ncl_{\delta p}(M_1) \\ &= Ncl_{\delta p}(M_1) \cap Nint(Ncl_\delta(M_1)) \\ &= [M_1 \cup Ncl(Nint_\delta(M_1)] \cap Nint(Ncl_\delta(M_1) \\ &= [M_1 \cap Nint(Ncl_\delta(M_1)] \cup [Ncl(Nint_\delta(M_1) \cap Nint(Ncl(M_1)] \end{aligned}$$

As M_1 is nano δ-preopen, then

$$M_1 = M_1 \cup [Ncl(Nint_\delta(M_1) \cap Nint(Ncl_\delta(M_1)] = Ncl_e(M_1).$$

Hence M_1 is nano e-closed.

The converse inclusions in Theorem 3.4 may not hold as shown by the following example. □

Example 1. *Let $U = \{b_1, b_2, b_3, b_4, b_5\}$ with $U \backslash R = \{\{b_1, b_3\}, \{b_2\}, \{b_4\}, \{b_5\}\}$ and let $X = \{b_1, b_4, b_5\}$, $N^T = \{U, \phi, \{b_4, b_5\}, \{b_1, b_3, b_4, b_5\}, \{b_1, b_3\}\}$. Then $\{b_1 b_3, b_4, b_5\}$ is nano δ-preopen(hence nano e-open) but not rnδp-open and $\{b_2\}$ is nano e-closed but not rnδp-open.*

Lemma 3.1. *In a NTS (U,N^T) and let $M \subseteq U$ be nano regular open. Then $M = Nint(Ncl(M)) = Nint(Ncl_\delta(M)$.*

Theorem 3.5. *In a NTS (U,N^T), every nano regular open set is rnδp-open.*

Proof. Let M_1 be a nano regular open set, then by Lemmas 2.1 and 2.2, we have $Nint_{\delta p}(Ncl_{\delta p}(M_1) = Ncl_{\delta p}(M_1) \cap Nint(Ncl_\delta(M_1)) = Ncl_{\delta p}(M_1) \cap M_1 = M_1$. Hence M_1 is rnδp-open.

Converse of the above theorem need not be true as shown by the following example. □

Example 2. *In Example 1,the set $\{b_1\}$ is rnδp-open but it is not nano regular open.*

Theorem 3.6. *Let M_1 be a nano δ-preclosed subset of a NTS (U,N^T). Then following statements are equivalent:*

(1) *M_1 is nano δ-preopen;*
(2) *M_1 is rnδp-open.*

Proof.

(1)→(2): By (1), $M_1 = Nint_{\delta p}(M_1)$ and, by hypothesis, $M = Ncl_{\delta p}(M_1)$. Therefore, $Nint_{\delta p}(Ncl_{\delta p}(M_1))=Nint_{\delta p}(M_1)= M_1$.

(2)→(1): Obvious from Theorem 3.4(i). □

Remark 3.1. *The class of rnδp-open sets is not closed under finite union as well as finite intersection. It will be shown in the following example.*

Example 3. *In Example 1,* $\{b_1, b_3\}$ *and* $\{b_4, b_5\}$ *are rnδp-open sets but* $\{b_1, b_3\} \cup \{b_4, b_5\} = \{b_1, b_3, b_4, b_5\} \notin RN\delta P(U)$*. Moreover,* $\{b_1, b_2, b_4\}$ *and* $\{b_1, b_2, b_5\}$*are rnδp-open sets but* $\{b_1, b_2, b_4\} \cap \{b_1, b_2, b_5\} = \{b_1, b_2\} \notin RN\delta P(U)$*.*

Theorem 3.7. *In a nano δ-partition space* (U,N^T)*, let* $M\subseteq U$*. Then the following are equivalent:*

(1) *M is nano δ-preopen;*

(2) *M is rnδp-open.*

Remark 3.2. *In a nano δ-partition space* (U,N^T)*, the class of rnδp-open sets is closed under arbitrary union.*

Lemma 3.2. *Let* M_1 *be a subset of a NTS* (U,N^T)*, then*

$$Nint_{\delta p}(Ncl_{\delta p}(M_1))=Nint_{\delta p}(Ncl_e(M_1)).$$

Proof. By Lemma 2.1, we have

$$\begin{aligned} Nint_{\delta p}(Ncl_{\delta p}(M_1)) &= Ncl_{\delta p}(M_1) \cap Nint(Ncl_{\delta}(M_1)) \\ &\subseteq Ncl_{\delta p}(M_1) \cap (M_1 \cup Nint(Ncl_{\delta}(M_1)) \\ &= Ncl_{\delta p}(M_1) \cap Ncl_{\delta s}(M_1) \\ &= Ncl_e(M_1), \text{ by Lemma 2.2.} \end{aligned}$$

Therefore, $Nint_{\delta p}(Ncl_{\delta p}(M_1)) \subseteq Nint_{\delta p}(Ncl_e(M_1))$.

The reverse inclusion is obvious. □

Theorem 3.8. *Let* (U,N^T) *be a NTS and* $M_1 \subseteq U$*,the following statements are equivalent:*

(1) M_1 *is rnδp-open;*

(2) M_1 *is nano e-closed and nano δ-preopen.*

Proof.

(1)→(2): Obvious from Theorem 3.4 [(i),(iii)].

(2)→(1): By (2),$M_1=Ncl_e(M_1)$ and $M_1= Nint_{\delta p}(M_1)$. By Lemma **??**,

$$\text{Nint}_{\delta p}(\text{Ncl}_{\delta p}(\text{M}_1)) = \text{Nint}_{\delta p}(\text{Ncl}_e(\text{M}_1)) = \text{Nint}_{\delta p}(\text{M}_1) = \text{M}_1.$$

Hence M_1 is rnδp-open. □

Lemma 3.3. *Let (U,N^T) be a NTS, then the following are equivalent:*

(1) *(U,N^T) nano δ-submaximal.*
(2) *Every nano δ-preopen set is nano δ-open.*

Theorem 3.9. *Let (U,N^T) be a nano δ-submaximal, then any finite intersection of nano δ-preopen sets is nano δ-preopen.*

Proof. Obvious since NδO(X) is closed under finite intersection. □

Theorem 3.10. *If a NTS (U,N^T) is nano δ-submaximal, then any finite intersection of rnδp sets is rnδp-open.*

Proof. Let $\{G_i$:i=1,2,...,n$\}$ be a finite family of rnδp-open sets. Since the space (U,N^T) is nano δ-submaximal, then by Theorem 3.9, we have $\bigcap_{i=n}^{n} G_i \in \delta$PO(U). By Theorem 3.2(i), $\bigcap_{i=n}^{n} G_i \subseteq \text{Nint}_{\delta p}(\text{Ncl}_{\delta p}(\bigcap_{i=n}^{n} G_i)$.

On the other hand,for each i, we have $\bigcap_{i=n}^{n} G_i \subseteq G_i$ and thus $\text{Nint}_{\delta p}(\text{Ncl}_{\delta p}(\bigcap_{i=n}^{n} G_i) \subseteq \text{Nint}_{\delta p}(\text{Ncl}_{\delta p}(G_i) = A_i$ as $\text{Nint}_{\delta p}(\text{Ncl}_{\delta p}(G_i) = G_i$. Therefore $\text{Nint}_{\delta p}(\text{Ncl}_{\delta p}(\bigcap_{i=n}^{n} G_i) \subseteq \bigcap_{i=n}^{n} G_i$. In consequence, $\bigcap_{i=n}^{n} G_i$ is rnδp-open in U. □

Definition 3.2. *In a NTS (U,N^T), let M $\subseteq$ U. Then M is called nano η-open if Nint(Ncl$_\delta$(M)) $\subseteq$ Ncl(Nint$_\delta$(M)).*

Theorem 3.11. *In a NTS (U,N^T),*

(i) *Every nano δ-semiopen set is nano η-set.*
(ii) *Every nano δ-semiclosed set is nano η-set.*

Proof.

(i) Let M be nano δ-semiopen, then M $\subseteq$ Ncl(Nint$_\delta$(M)) which implies,

$$\text{Nint}(\text{Ncl}_\delta(\text{M})) \subseteq \text{Ncl}(\text{Nint}_\delta(\text{M})).$$

Hence M is nano η-set.

(ii) Let K be nano δ-semiclosed, then Nint(Ncl$_\delta$(K)) $\subseteq$ K. Therefore

$$\text{Nint}(\text{Ncl}_\delta(\text{K})) \subseteq \text{Ncl}(\text{Nint}_\delta(\text{K})).$$

Hence K is nano η-set.

Converse of the above Theorem need not be true as shown by the following example. □

Example 4. *In Example 1,the set* $\{b_2\}$ *is* η*-set but it is not nano* δ*-semiopen. and the set* $\{b_1, b_3, b_4, b_5\}$ *is nano* η*-set but it is not nano* δ*-semiclosed.*

Remark 3.3. *DIAGRAM*

$$\begin{array}{ccccccc} \textit{nano regular open} & \rightarrow & \textit{nano } \delta\alpha\textit{-open} & \rightarrow & \textit{nano } \delta\textit{-semiopen} & \rightarrow & \textit{nano } \eta\textit{-set} \\ \downarrow & & \downarrow & & \downarrow & & \\ \textit{rn}\delta\textit{p-open} & \longrightarrow & \textit{nano } \delta\textit{-preopen} & \longrightarrow & \textit{nano e-open} & & \end{array}$$

Remark 3.4. *The notions of nano* η*-sets and rn*δ*p-open(hence nano* δ*-preopen, nano e-open)sets are independent of each other.*

Example 5. *Let (U,*N^T*) be a space as in Example 1. Then* $\{b_2\}$ *is nano* η*-set but not a nano e-open set and the set* $\{b_1, b_2, b_4, b_5\}$ *is rn*δ*p-open but it is not a* η*-set.*

Theorem 3.12. *In a NTS (U,*N^T*), let M* $\subseteq$ *U,the following are equivalent:*

(i) *M is nano* δ*-semiopen;*

(ii) *M is both nano e-open and nano* η*-set.*

Proof.

(i) $\rightarrow$(ii): Obvious.

(ii)$\rightarrow$(i): Let M be both nano e-open and nano η-set. Then, Nint(Ncl$_\delta$(M))$\cap$ Ncl(Nint$_\delta$(M)) $\subseteq$ M and Nint(Ncl$_\delta$(M)) $\subseteq$ Ncl(Nint$_\delta$(M)), and Nint(Ncl$_\delta$(M)) $\subseteq$ M. Hence M is nano δ-semiopen. □

Theorem 3.13. *For a subset M of a NTS (U,*N^T*), the following are equivalent:*

(i) *M is nano regular open;*

(ii) *M is rn*δ*p-open and* η*-set.*

Proof.

(i)$\rightarrow$(ii): It follows from Remark 3.3.

(ii)$\rightarrow$(i): Let M be rnδp-open and η-set. Then, by Lemma 2.1, we obtain

$$\begin{aligned} \text{M} &= \text{Nint}_{\delta p}(\text{Ncl}_{\delta p}(\text{M})) \\ &= (\text{M} \cup \text{Ncl}(\text{Nint}_\delta(\text{M}))\cap\text{Nint}(\text{Ncl}_\delta(\text{M})) \end{aligned}$$

$=(M\cap Nint(Ncl_{\delta}(M))\cup(Ncl(Nint_{\delta}(M))\cap Nint(Ncl_{\delta}(M)))$
$=(M\cap Nint(Ncl_{\delta}(M))\cup Nint(Ncl_{\delta}(M))$
$=Nint(Ncl_{\delta}(M))$
Therefore M = Nint(Ncl$_{\delta}$(M)) = Nint(Ncl(M)). Hence M is nano regular open. □

Definition 3.3. *In (U,N^T), let M $\subseteq$ U.*

(1) *The rnδp-interior of M, denoted by $Nint^r_{\delta p}$(M) is defined as*

$$Nint^r_{\delta p}(M)=\bigcup\{K:K\subseteq M \text{ and } M\in RN\delta PO(U)\};$$

(2) *The rnδp-closure of M, denoted by $Ncl^r_{\delta p}$(M) is defined as*

$$Ncl^r_{\delta p}(M)=\bigcap\{F:M\subseteq F \text{ and } F\in RN\delta PC(U)\}.$$

Theorem 3.14. *In (U,N^T),let M $\subseteq$ U. Then the following hold:*

(a) *$Nint^r_{\delta p}(M)\subseteq M \subseteq Ncl^r_{\delta p}(M)$.*
(b) *If M is rnδp-open(rnδp-closed), then $Nint^r_{\delta p}(M)$ =M(resp,$Ncl^r_{\delta p}(M) = M$).*

Remark 3.5. *The converse of Theorem 3.14 (b) is true only when (U,N^T) is nano δ-partition.*

4. RNδP-CONTINUOUS FUNCTIONS:

Definition 4.1. *A function f:(U,τ_R(X)→(V,$\tau_R{}^*$(Y)) is said to be rnδp-continuous if f^{-1}(H) is rnδp-open in (U,τ_R(X)) for each H $\in \tau_R{}^*$(Y).*

Example 6. *Let U = $\{b_1, b_2, b_3, b_4, b_5\}$ with U\R= $\{\{b_1, b_3\}, \{b_2\}, \{b_4\}, \{b_5\}\}$ and let X = $\{b_1, b_4, b_5\}$, $\tau_R(X)$ =$\{U, \phi, \{b_4, b_5\}, \{b_1, b_3, b_4, b_5\}, \{b_1, b_3\}\}$. Let V = $\{a, b, c, d\}$ with V\R= $\{ \{a, c\}, \{b\}, \{d\}\}$ and let Y = $\{a, b\}$, $\tau_R{}^*(Y)$ =$\{V, \phi, \{b\}, \{a, b, c\}, \{a, c\}\}$. Define f:(U, τ_R(X)→(V, $\tau_R{}^*$(Y)) as $f(b_1) = a$, $f(b_2) = c = f(b_4)$ and $f(b_3) = b = f(b_5)$.Then $f^{-1}(\{b\})=\{b_3, b_5\}$ $f^{-1}(\{a, b, c\})=V$ and $f^{-1}(\{a, c\})= \{b_1, b_2, b_4\}$.*

That is, the inverse image of every nano open set is rnδp-open in U.Therefore, f is rnδp-continuous

Theorem 4.1. *A function f:(U,τ_R(X)→(V,$\tau_R{}^*$(Y)) is rnδp-continuous, if, and only if, f^{-1}(D) is rnδp-closed in (U,τ_R(X)) for every nano closed set D of (V,$\tau_R{}^*$(Y)).*

Proof. Let D be a nano closed set in V. Then V-D is a nano open set in V. Since f is rnδp-continuous, f^{-1}(V\D)=U\f^{-1}(D) is rnδp-open in U. Therefore, f^{-1}(D) is rnδp-closed.

Conversely, let F be a nano open set in (V,$\tau_R{}^*$(Y)),then V\F is a nano closed set in V. By assumtion, f^{-1}(V\F) = U\f^{-1}(F) is rnδp-closed in U. This implies f^{-1}(F) is rnδp-open in U. □

Theorem 4.2. *Every rnδp-contninuous function is nano δ-precontinuous.*

Example 7. *Consider (U,τ_R(X) and (V,$\tau_R{}^*$(Y)) as in Example 6. Define f:(U,τ_R(X) $\to$ (V,$\tau_R{}^*$(Y)) as f(b_1) = a, f(b_2) = b = f(b_5) and f(b_3) = c = f(b_4). Then f^{-1}($\{b\}$)=$\{b_2, b_5\}$ f^{-1}($\{a, b, c\}$)=V and f^{-1}($\{a, c\}$)= $\{b_1, b_3, b_4\}$. Therefore, f is nano δ-precontinuous but there exists $\{a, c\}$$\in$$\tau_R{}^*$(Y) such that f^{-1}($\{a, c\}$)= $\{b_1, b_3, b_4\}$ $\notin$ RNδPO(U). Hence f is not rnδp-continuous.*

Theorem 4.3. *Let f:(U,τ_R(X)$\to$(V,$\tau_R{}^*$(Y)) be a function where (U,τ_R(X) is nano δ-partition.Then the following are equivalent:*

(1) *f is rnδp-continuous;*
(2) *For each subset B of V,Ncl$^r_{\delta p}$(f^{-1}(B)) $\subseteq$ f^{-1}(Ncl(B));*
(3) *For each subset A of U,f(Ncl$^r_{\delta p}$(A)) $\subseteq$ Ncl(f(A));*
(4) *For each subset B of V, f^{-1}(Nint(B)) $\subseteq$ Nint$^r_{\delta p}$(f^{-1}(B)).*

Proof.

(1)$\to$(2). Let f be rnδp-continuous and B $\subseteq$ V. Then f^{-1}(Ncl(B)) is rnδp-closed in U. Then cl$^N_{rp}$((f^{-1}(B)) $\subseteq$ Ncl$^r_{\delta p}$((f^{-1}(Ncl(B))=f^{-1}(Ncl(B).

(2)$\to$(1). Let B $\subseteq$ V be a nano closed set. Then by (2), Ncl$^r_{\delta p}$(f^{-1}(B)) $\subseteq$ f^{-1}(Ncl(B))=f^{-1}(B) $\Longrightarrow$ Ncl$^r_{\delta p}$(f^{-1}(B))=f^{-1}(B) since (U,τ_R(X) is nano δ-partition. Therefore, f^{-1}(B) is rnδp-closed in (U,τ_R(X)).

(2)$\to$(3). Let A $\subseteq$ U. Then f(A) $\subseteq$ V. By (2), we get

$$f^{-1}(\mathrm{cl}(\mathrm{f}(\mathrm{A}))) \supseteq \mathrm{cl}^N_{rp}(f^{-1}(\mathrm{f}(\mathrm{A}))) \supseteq \mathrm{Ncl}^r_{\delta p}(\mathrm{A}).$$

Therefore, f(Ncl$^r_{\delta p}$(A)) $\subseteq$ f(f^{-1}(Ncl(f(A))) $\subseteq$ Ncl(f(A).

(3)$\to$(2). Let B $\subseteq$ V and A = f^{-1}(B) $\subseteq$ U.Then by(3), f(Ncl$^r_{\delta p}$(f^{-1}(B))) $\subseteq$ Ncl(f(f^{-1}(B)) $\subseteq$ Ncl(B). $\Rightarrow$ Ncl$^r_{\delta p}$(f^{-1}(B)) $\subseteq$ f^{-1}(Ncl(B)).

(2)$\to$(4). Replace B by V \ B in (2), we get Ncl$^r_{\delta p}$(f^{-1}(V\B)) $\subseteq$ f^{-1}(Ncl(V\B)). $\Rightarrow$ Ncl$^r_{\delta p}$(X\f^{-1}(B)) $\subseteq$ f^{-1}(V\Nint(B)). Therefore, f^{-1}(Nint(B)) $\subseteq$ Nint$^r_{\delta p}$(f^{-1}(B)) for each B $\subseteq$ V.

(4)$\rightarrow$(1). Let H $\subseteq$ V be nano open. Then

$$f^{-1}(\mathrm{H}) = f^{-1}(\mathrm{Nint(H)}) \subseteq \mathrm{Nint}^{r}_{\delta p}(f^{-1}(\mathrm{H})$$

$\Rightarrow \mathrm{Nint}^{r}_{\delta p}(f^{-1}(\mathrm{H}) = f^{-1}(\mathrm{H})$ since(U,τ_R(X)) is nano δ-partition. Hence, f^{-1}(H) is rnδp-open in U. $\square$

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