ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **10** (2021), no.2, 1125–1130
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.2.38

e AND r ENERGIES OF SOME DISEMIGRAPHS

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ABSTRACT. Semigraph is a generalization of graph, introduced by E. Sampathkumar in 2000. In this paper the energies of some fundamental disemigraphs are obtained by defining two types of adjacency matrix to a disemigraph. They are e-adjacency and r-adjacency matrices. In this paper energy of Linear disemigraphs, Disemigraph Cycles and cartiecian product of two linear disemigraphs are discussed.

1. INTRODUCTION

The concept of Semigraph was first introduced by E. Sampathkumar [1], which has been enriched by variety of graph properties related not only to the vertices but also to the edges. It was originated in 2000 and so far many papers were published related to the matrices and spectrum of semigraphs.

Associated with semigraph, incidence matrix and adjacency matrix A are introduced by Charuseela Despande [5], Laplacian matrix L=D-A is introduced by Ambika K. Biradar [6] of a semigraph. Using Laplacian matrix many graph properties can be identified and is used in Machine Learning applications. Also its spectrum is used to find clusters in data mining. Because of the peculiarity of

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2020 *Mathematics Subject Classification.* 05C50, 05C20.

Key words and phrases. disemigraph, linear disemigraph, disemigraph cycle, e-adjacency spectrum, r-adjacency spectrum,energy.

Submitted: 06.11.2020; *Accepted:* 21.11.2020; *Published:* 04.03.2021.

the structure of a semigraph, different types of graph parameters such as vertex degree, adjacency, isomorphism can be defined. [2, 3] discuss e and r adjacency matrix of linear semigraphs. In this paper two different adjacency matrix of a disemigraph is introduced and their spectra and energies are analysed. For the study we included Linear Disemigraph, Disemigraph Cycles and Cartiecian product of two linear disemigraphs.

We use the following definitions.

Definition 1.1. *[1] Semigraph G is a pair (V, X) where V is a non empty set whose elements are called vertices of G and X is a set of n-tuples caiied edges of G of distinct vertices for various n≥2 satisfying the following conditions.*

SG1-Any two edges have at most one vertex in common SG2-Two edges $(u_1, u_2, \ldots, u_n)$ *and* $(v_1, v_2, \ldots, v_m)$ *are considered to be equal iff* (i) $m = n$ *and* (ii) *either* $u_i = v_i$ *for 1≤i≤n or* $u_i = v_{n-i+1}$ *for 1≤i≤n.*

Definition 1.2. *[1] A linear semigraph is a semigraph with a single edge and n vertices.It is represented by* L_n*.*

Definition 1.3. *[1] A disemigraph or a directed semigraph D is G=(V, X) where X is the set of ordered n-tuples of distinct vertices of G for various n≥2.*

There are different types of adjacencies for two vertices in a semigraph.Some of them are listed here [1].

Two vertices u and v in a semigraph S are said to be

i. adjacent if they belong to the same edge.
ii. e-adjacent if they are the end vertices of an edge.
iii. r-adjacent if they are incident with r edges.

Similar adjacencies can be defined for disemigraph also.

Suppose a=$(u_1, u_2, \ldots, u_n)$ is an arc in a disemigraph. we say that for $1 \leq i < j \leq n$, u_i is adjacent to u_j and u_j is adjacent to u_i.

Definition 1.4. *[4] Cartesian product of Path disemigraphs: Let* $D_1 = (V_1, X_1)$ *and* $D_2 = (V_2, X_2)$ *be two disemigraphs.The cartesian product of* D_1 *and* D_2*, denoted by* D_1XD_2 *is defined as follows:*
The vertex set of D_1XD_2 *is* V_1XV_2*. For any vertex u in*V_1 *and any arc E=*$(v_1, v_2, \ldots, v_r)$ *in* X_2*,* $((u, v_1), (u, v_2), \ldots, (u, v_r))$ *is an arc in* D_1XD_2*. Also for any arc E=* $(u_1, u_2, \ldots, u_s)$ *in* X_1 *and for any vertex v in* V_2*,* $((u_1, v), (u_2, v), \ldots, (u_s, v))$ *is an arc in* D_1XD_2*.*

2. Main Results

Now we define e-adjacency and r-adjacency matrix for a disemigraph and will derive characteristic polynomial for linear disemigraph and will find spectrum and energy of some disemigraphs.

2.1. **e-adjacency matrix of a disemigraph.** Let D=(V, X) be a disemigraph with vertices V= $(v_1, v_2, \ldots, v_n)$ and directed edge set X=$(e_1, e_2, \ldots, e_m)$ where $e_j = (i_1, i_2, \ldots, i_{kj})$ ordered kj-tuple, j=1, 2, ..., m. e-adjacency matrix AD_n of D=(V, X) is defined as

$$AD_n = [a_{ij}] = \left\{ \begin{array}{ll} 1, & v_i \text{ is adjacent to } v_j \text{ and are end vertices of same arc} \\ -1, & v_i \text{ is adjacent from } v_j \text{ and are end vertices of same arc} \\ 0, & \text{otherwise} \end{array} \right\}$$

2.2. **e-adjacency matrix of a linear disemigraph with n vertices.**
Let LD_n denote the linear disemigraph with n vertices.
e-adjacency matrix ofLD_3 is$AD_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ and its characteristic polynomial is $\phi(LD_3, \lambda) = -\lambda^3 - \lambda$.

For LD_4, the characteristic polynomial is $\phi(LD_4, \lambda) = \lambda^4 + \lambda$.

In general, For $n \geq 2$,

$$\phi(LD_n, \lambda) = (-1)^n \lambda^n + (-1)^{n-2} \lambda^{n-2} = (-1)^{n-2} \lambda^{n-2} [\lambda^2 + 1]$$

and $\phi(LD_n, \lambda) = -\lambda \, \phi(LD_{n-1}, \lambda)$.

Theorem 2.1. *The characteristic polynomial* $\phi(LD_n, \lambda)$ *of linear disemigraph* LD_n *satisfy the following properties:*

(1) $\phi(LD_n, \lambda) = \alpha_0 \lambda^n + \alpha_1 \lambda^{n-1} + \cdots + \alpha_n$ *where* $\alpha_0 = (-1)^n, \alpha_2 = (-1)^{n-2}$ *and* $\alpha_1 = \alpha_3 = \alpha_4 = \cdots = \alpha_n = 0$*;*

(2) $\phi(LD_n, \lambda) = (-1)^{n-2} \lambda^{n-2} \phi(LD_2, \lambda)$*, for* $n \geq 2$ *with* $\phi(LD_2 = \lambda^2 + 1$.

2.3. **e-energy of a linear disemigraph.** LD_n be a linear disemigraph with n vertices.If AD_n is the e-adjacency matrix of LD_n , the eigen values of AD_n denoted by $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ are said to be eigen values of LD_n.The e-energy of LD_n is $E(LD_n) = \sum_{i=1}^{n} |\lambda i|$.The spectrum of LD_n is $\begin{pmatrix} i & -i & 0 \\ 1 & 1 & n-2 \end{pmatrix}$ and e-energy is $E(LD_n) = 2$.

Theorem 2.2. *The e-energy of linear disemigraph is the sum of the degrees of the end vertices.*

2.4. **r-adjacency matrix of a disemigraph.** In a disemigraph, if $a = (u_1, u_2, \dots, u_n)$ is an arc, then we say that u_1 is adjacent to $u_2, u_3, \dots, u_n$ and u_n is adjacent from $u_1, u_2, \dots, u_{n-1}$. Also we consider u_i is adjacent to u_i itself in a. Also u_i and u_j are adjacent if either u_i is adjacent to u_j or u_i is adjacent from u_j. Then for a disemigraph D, r-adjacency matrix.

$$AD_n = [a_{ij}] = \left\{ \begin{array}{ll} r, & \text{the number of edges in which } u_i \text{ and } u_j \text{ are adjacent} \\ 0, & \text{otherwise} \end{array} \right\}.$$

2.5. **r-adjacency matrix of a linear disemigraph.** r-adjacency matrix of LD_3 is $AD_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and the characteristic polynomial is $\phi(LD_3, \lambda) = -\lambda^3 + 3\lambda^2$. Similarly $\phi(LD_4, \lambda) = \lambda^4 - 4\lambda^2$. In general $\phi(LD_n, \lambda) = (-1)^n \lambda^n + n(-1)^{n-1}\lambda^{n-1} = (-1)^{n-1}\lambda^{n-1}(n - \lambda)$. Also we have $\phi(LD_{n-1}, \lambda) = (-1)^{n-2}\lambda^{n-2}(n-1-\lambda)$. Therefore $\phi(LD_n, \lambda) = -\lambda\phi(LD_{n-1}, \lambda) + (-\lambda)^{n-1}$.

Theorem 2.3. *The characteristic polynomial $\phi(LD_n, \lambda)$ of a linear disemigraph with respect to r-adjacency matrix satisfy the following properties: $\phi(LD_n, \lambda) = \alpha_0\lambda^n + \alpha_1\lambda^{n-1} + \cdots + \alpha_n$, where $\alpha_0 = (-1)^n$, $\alpha_1 = n(-1)^{n-1}$ and $\alpha_2 = \alpha_3 = \cdots = \alpha_n = 0$.*

2.6. **r-energy of a linear disemigraph.** $\phi(LD_n, \lambda) = (-1)^n\lambda^n + n(-1)^{n-1}\lambda^{n-1}$
The r-spectrum of LD_n is

$$\begin{pmatrix} 0 & n \\ n-1 & 1 \end{pmatrix}.$$

The r-energy is $E(LD_n) = n$.

Theorem 2.4. *The r-energy of linear disemigraph LD_n is the number of vertices in LD_n.*

2.7. **e-energy of a cycle semigraph and a cycle disemigraph.** For cycle C_3, e-adjacency matrix $e(C_3)$ is $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. For a directed cycle DC_3, e-adjacency matrix $e(DC_3)$ is the skewsymmetric matrix $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$.

Inserting a middle vertex in each edge of C_3, the resultant subdivision semigraph SC_3 posses e-adjacency matrix as a block matrix $e(SC_3) = \begin{bmatrix} [e(C_3)] & 0 \\ 0 & 0 \end{bmatrix}$ Similarly, Inserting a middle vertex in each directed edge of DC_3, the resultant subdivision disemigraph DSC_3 posses e-adjacency matrix as a block matrix $e(DSC_3) = \begin{bmatrix} [e(DC_3)] & 0 \\ 0 & 0 \end{bmatrix}$.

Theorem 2.5.

(i) *The e-adjacency spectrum and energy is invariant for a cycle C_n and a subdivision semigraph of C_n that is SC_n.*

(ii) *The e-adjacency spectrum and energy is invariant for a directed cycle DC_n and a subdivision disemigraph of DC_n that is DSC_n.*

(iii) *The r-adjacency spectrum and energy is variant for C_n and SC_n and also for DC_n and DSC_n.*

2.8. e and r-adjacency matrix of cartesian product of two linear disemigraphs.

Definition 2.1. *[4] Let $D_1 = (V_1, X_1)$ and $D_2 = (V_2, X_2)$ be two disemigraphs.Cartesian product $D = D_1XD_2$ is a disemigraph with vertex set $V = V_1XV_2$ and set X, where e is an arc between vertices (u_1, u_2) and (v_1, v_2) iff (i)$u_1 = v_1$ and there exist a partial arc or an arc (u_2, v_2) in X_2. or (ii)$u_2 = v_2$ and there exist a partial arc or arc $u_1 = v_1$ in X_1 and a vertex (u_1, u_2) in $D = D_1XD_2$ is a middle vertex if both u_1 and u_2 are middle vertices in $D - 1$ and D_2 respectively; otherwise the vertex is an end vertex in $D = D_1XD_2$. The direction of edges in $D = D_1XD_2$ is the same as the direction of edges in inducing graphs $D - 1$ and D_2.*

Results:

(i) The e-adjacency matrix of LD_3XLD_3 can be represented by a block matrix

$$\begin{pmatrix} A_1 & A_2 & I - A_2 \\ -A_2 & AD_3 & A_2 \\ A_2 - I & -A_2 & A_1 \end{pmatrix},$$

where

$$A_1 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \quad \text{and} \quad A_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

the e-adjacency of LD_3.

(ii) The r-adjacency matrix of LD_3XLD_3 can be represented by a block matrix

$$\begin{pmatrix} A_1 & I & A_2 \\ I & AD_3 & I \\ A_2 & I & A_1 \end{pmatrix},$$

where

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad \text{and} \quad A_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and AD_3 is the r-adjacency of LD_3.

3. CONCLUSION

We can further extend the concept of e and r-adjacency matrix, spectrum and energy to other disemigraphs.

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