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SEQUENCE OF SOFT POINTS IN SOFT Δ-METRIC SPACES

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ABSTRACT. In this paper, we have constructed a sequence of soft points in one soft set with respect to a fixed soft point of another soft set. The convergence and boundedness of these sequences in soft Δ-metric spaces are defined and their properties are established. Further, the complete soft Δ-metric spaces are introduced by defining soft Δ-Cauchy sequences.

1. INTRODUCTION

The concept of soft sets initiated by Molodtsov [4] brought a rapid contribution to the area of mathematical modeling. Molodtsov gave the definition of soft set as, [4] a structure $(\mathcal{A}, \Theta)$ is a soft set over an initial universe $\mathcal{U}$ if $\mathcal{A}$ is a mapping from Θ, a set of parameters, to the power set of $\mathcal{U}$. Maji et al. [3] continued the study on soft sets and defined null soft set, absolute soft set and complement of a soft set. Further, the ideas of subsets, supersets, equality, union and intersection of soft sets are initiated in [3]. In 2012, the notions of soft elements, soft real sets and soft real numbers were proposed by Samanta et al. [1]. The soft real set $(\mathcal{F}, \Theta)$ is a mapping from a set of parameters Θ into $\mathbb{B}(\mathbb{R})$, the set of all non-empty bounded subsets of real numbers $\mathbb{R}$, that is $\mathcal{F} : \Theta \to \mathbb{B}(\mathbb{R})$

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and a soft real number is a singleton set identified with the corresponding soft element, denoted by $\widetilde{a}$ [1]. Also, in [1], the authors have depicted the ideas of positive, negative, non-negative and non-positive soft real sets and defined some arithmetic operations on soft real sets. The relations such as '$<$', '$>$', '$\leq$' and '$\geq$' on soft real numbers were defined by Samanta et al. [2]. A soft set $(\mathcal{P}, \Theta)$ over $\mathcal{U}$ is said to be a soft point if there is exactly one $e \in \Theta$ such that $\mathcal{P}(e) = \{a\}$ for some $a \in \mathcal{U}$ and for all $\theta \in \Theta \setminus \{e\}$, $\mathcal{P}(\theta) = \emptyset$ and is denoted by $\mathcal{P}_e^a$ [2]. Let $SP(\mathcal{U}, \Theta)$ denote the set of all soft points of $(\mathcal{U}, \Theta)$.

Patil et al. [5] introduced a new type of soft metric on two soft sets, namely, soft Δ-metric and is defined as, a mapping $\Delta : SP(\mathcal{U}_1, \Theta) \times SP(\mathcal{U}_2, \Theta) \to \mathbb{R}(\Theta)^+$, the set of all positive soft real numbers, is said to be a soft Δ-metric on soft sets $(\mathcal{U}_1, \Theta)$ and $(\mathcal{U}_2, \Theta)$ with $(\mathcal{U}_1, \Theta) \cap (\mathcal{U}_2, \Theta) = \widetilde{\emptyset}$, if for any $\mathcal{P}_e^x$, $\mathcal{P}_f^z \in SP(\mathcal{U}_1, \Theta)$ and $\mathcal{Q}_\sigma^y \in SP(\mathcal{U}_2, \Theta)$, Δ satisfies the following conditions:

(i) $\Delta(\mathcal{P}_e^x, \mathcal{Q}_\sigma^y) > \widetilde{0}$,
(ii) $\Delta(\mathcal{P}_e^x, \mathcal{Q}_\sigma^y) = \Delta(\mathcal{Q}_\sigma^y, \mathcal{P}_e^x)$,
(iii) $d(\mathcal{P}_e^x, \mathcal{P}_f^z) \leq \Delta(\mathcal{P}_e^x, \mathcal{Q}_\sigma^y) + \Delta(\mathcal{P}_f^z, \mathcal{Q}_\sigma^y)$.

Then, $(\mathcal{U}_1, \mathcal{U}_2, \Delta, \Theta)$ is called a soft Δ-metric space. Further, the notions of soft Δ-open balls, soft Δ-closed balls, soft Δ-open sets and soft Δ-closed sets are introduced with their properties in [5].

The main aim of this work is to introduce the sequence of soft points in one soft set with respect to a fixed soft point of another soft set and hence to study the convergence property in soft Δ-metric spaces. Also, soft Δ-bounded and soft Δ-Cauchy sequences are defined and their properties are scrutinized.

2. Main results

Definition 2.1. *A sequence of soft points $\{\mathcal{Q}_{\sigma,n}^y\}_n$ of $(\mathcal{U}_2, \Theta)$ in a soft Δ-metric space $(\mathcal{U}_1, \mathcal{U}_2, \Delta, \Theta)$ called a soft Δ-sequence and is said to be soft Δ-convergent, converges to a soft point $\mathcal{R}_\lambda^x$ of $(\mathcal{U}_2, \Theta)$ with respect to a soft point $\mathcal{P}_e^a$ of $(\mathcal{U}_1, \Theta)$, where $\Delta(\mathcal{P}_e^a, \mathcal{R}_\lambda^x) = \widetilde{s}$, $\widetilde{s} \in \mathbb{R}(\Theta)^+$ if $\Delta(\mathcal{P}_e^a, \{\mathcal{Q}_{\sigma,n}^y\}_n) \to \widetilde{s}$ (or equivalently $\Delta(\mathcal{P}_e^a, \mathcal{R}_\lambda^x) - \Delta(\mathcal{P}_e^a, \{\mathcal{Q}_{\sigma,n}^y\}_n) \to \widetilde{0}$) as $n \to \infty$.*

That is, for any $\widetilde{\epsilon} > \widetilde{0}$, there exists $m \in \mathbb{N}$ such that $\{\mathcal{Q}_{\sigma,n}^y\}_n \subseteq \Delta B(\mathcal{P}_e^a, \widetilde{\epsilon})$ and $\mathcal{R}_\lambda^x \in \Delta B(\mathcal{P}_e^a, \widetilde{\epsilon})$, whenever $n \geq m$ and is denoted by $\Delta \lim_{n\to\infty} \mathcal{Q}_{\sigma,n}^y = \mathcal{R}_\lambda^x$.

Here, $\mathcal{R}^x_\lambda$ is said to be a soft Δ-limit point of $\{\mathcal{Q}^y_{\sigma,n}\}_n$ with respect to $\mathcal{P}^a_e$ in $(\mathcal{U}_1, \mathcal{U}_2, \Delta, \Theta)$.

Theorem 2.1. *If exist, soft Δ-limit point of a soft Δ-sequence in a soft Δ-metric space is unique.*

Proof. Let $\{\mathcal{Q}^y_{\sigma,n}\}_n$ be a soft Δ-convergent sequence in $(\mathcal{U}_1, \mathcal{U}_2, \Delta, \Theta)$. Suppose, $\{\mathcal{Q}^y_{\sigma,n}\}_n$ converges to two different soft Δ-limits $\mathcal{R}^x_\lambda$ and $\mathcal{S}^z_\delta$ with respect to $\mathcal{P}^a_e$. Then, for any $\widetilde{\epsilon_1}$, $\widetilde{\epsilon_2} > \widetilde{0}$ there exist m_1, $m_2 \in \mathbb{N}$ such that $\{\mathcal{Q}^y_{\sigma,n}\}_n \subseteq \Delta B(\mathcal{P}^a_e, \widetilde{\epsilon_1})$, $\mathcal{R}^x_\lambda \in \Delta B(\mathcal{P}^a_e, \widetilde{\epsilon_1})$ whenever $n \geq m_1$ and $\{\mathcal{Q}^y_{\sigma,n}\}_n \subseteq \Delta B(\mathcal{P}^a_e, \widetilde{\epsilon_2})$, $\mathcal{S}^z_\delta \in \Delta B(\mathcal{P}^a_e, \widetilde{\epsilon_2})$ whenever $n \geq m_2$. Let $\widetilde{\epsilon} = min\{\widetilde{\epsilon_1}, \widetilde{\epsilon_2}\}$ and $m = max\{m_1, m_2\}$. Now, $\{\mathcal{Q}^y_{\sigma,n}\}_n \subseteq \Delta B(\mathcal{P}^a_e, \widetilde{\epsilon})$ and $\mathcal{R}^x_\lambda$, $\mathcal{S}^z_\delta \in \Delta B(\mathcal{P}^a_e, \widetilde{\epsilon})$ whenever $n \geq m$, which contradicts the soft Δ-Hausdorff property of soft Δ-metric spaces [5]. Therefore, $\mathcal{R}^x_\lambda = \mathcal{S}^z_\delta$. □

Definition 2.2. *A soft Δ-sequence $\{\mathcal{Q}^y_{\sigma,n}\}_n \subseteq SP(\mathcal{U}_2, \Theta)$ in a soft Δ-metric space $(\mathcal{U}_1, \mathcal{U}_2, \Delta, \Theta)$ is said to be soft Δ-bounded if the set $\{\Delta(\mathcal{P}^a_e, \{\mathcal{Q}^y_{\sigma,n}\}_n) : \mathcal{P}^a_e \in SP(\mathcal{U}_1, \Theta), n \in \mathbb{N}\}$ of soft real numbers is bounded.*

That is, there exists $\widetilde{k} > \widetilde{0}$ such that $\Delta(\mathcal{P}^a_e, \{\mathcal{Q}^y_{\sigma,n}\}_n) \leq \widetilde{k}$ for all $n \in \mathbb{N}$.

Definition 2.3. *A soft Δ-sequence $\{\mathcal{Q}^y_{\sigma,n}\}_n \subseteq SP(\mathcal{U}_2, \Theta)$ in a soft Δ-metric space $(\mathcal{U}_1, \mathcal{U}_2, \Delta, \Theta)$ is said to be a soft Δ-Cauchy sequence if for any $i, j \in \mathbb{N}$ there exists $m \in \mathbb{N}$ such that $\Delta(\mathcal{P}^a_e, \{\mathcal{Q}^y_{\sigma,i}\}) - \Delta(\mathcal{P}^a_e, \{\mathcal{Q}^y_{\sigma,j}\}) \to \widetilde{0}$, whenever $i, j \geq m$.*

That is, for sufficiently large values of i and j (or equivalently as $i, j \to \infty$), we have, $\Delta(\mathcal{P}^a_e, \{\mathcal{Q}^y_{\sigma,i}\}) - \Delta(\mathcal{P}^a_e, \{\mathcal{Q}^y_{\sigma,j}\}) \to \widetilde{0}$ or $\Delta(\mathcal{P}^a_e, \{\mathcal{Q}^y_{\sigma,i}\}) \to \Delta(\mathcal{P}^a_e, \{\mathcal{Q}^y_{\sigma,j}\})$.

Theorem 2.2. *In a soft Δ-metric space $(\mathcal{U}_1, \mathcal{U}_2, \Delta, \Theta)$, a soft point $\mathcal{R}^x_\lambda$ of $(\mathcal{U}_2, \Theta)$ is a soft Δ-limit point of $(\mathcal{B}, \Theta) \subseteq (\mathcal{U}_2, \Theta)$ with respect to $\mathcal{P}^a_e \in SP(\mathcal{U}_1, \Theta)$ if and only if there exists a soft Δ-sequence in $(\mathcal{B}, \Theta)$ which converges to $\mathcal{R}^x_\lambda$ with respect to $\mathcal{P}^a_e$.*

Proof. Let $\mathcal{R}^x_\lambda$ be a soft Δ-limit point of $(\mathcal{B}, \Theta)$ with respect to $\mathcal{P}^a_e$. Then, we have the following cases: Case (i) If $\mathcal{R}^x_\lambda \in SP(\mathcal{B}, \Theta)$, then the constant soft Δ-sequence $\{\mathcal{R}^x_\lambda, \mathcal{R}^x_\lambda, ...\}$ in $(\mathcal{B}, \Theta)$ converges to $\mathcal{R}^x_\lambda$ with respect to $\mathcal{P}^a_e$. Case (ii) If $\mathcal{R}^x_\lambda \notin SP(\mathcal{B}, \Theta)$, then for every soft Δ-open ball $\Delta B(\mathcal{P}^a_e, \widetilde{r})$ containing $\mathcal{R}^x_\lambda$, we have, $[\Delta B(\mathcal{P}^a_e, \widetilde{r}) \cap SP(\mathcal{B}, \Theta)] \setminus \{\mathcal{R}^x_\lambda\} \neq \widetilde{\emptyset}$. Therefore, for some $\mathcal{S}^z_\delta \in SP(\mathcal{B}, \Theta)$, $\{\mathcal{S}^z_{\delta,n}\}_n$ is a soft Δ-sequence in $(\mathcal{B}, \Theta)$ that converges to $\mathcal{R}^x_\lambda$ with respect to $\mathcal{P}^a_e$.

Conversely, let $\{\mathcal{S}^z_{\delta,n}\}_n$ be a soft Δ-sequence in $(\mathcal{B},\Theta)$ converges to $\mathcal{R}^x_\lambda$ with respect to $\mathcal{P}^a_e$. Then, for any $\widetilde{\epsilon} > \widetilde{0}$ there exists $m \in \mathbb{N}$ such that $\{\mathcal{S}^z_{\delta,n}\}_n \subseteq \Delta B(\mathcal{P}^a_e,\widetilde{\epsilon})$ and $\mathcal{R}^x_\lambda \in \Delta B(\mathcal{P}^a_e,\widetilde{\epsilon})$, whenever $n \geq m$. Therefore, $[\Delta B(\mathcal{P}^a_e,\widetilde{r}) \cap SP(\mathcal{B},\Theta)] \setminus \{\mathcal{R}^x_\lambda\} \neq \widetilde{\emptyset}$ and hence, $\mathcal{R}^x_\lambda$ is a soft Δ-limit point of $(\mathcal{B},\Theta)$ with respect to $\mathcal{P}^a_e$. □

Definition 2.4. *A soft Δ-subsequence of a soft Δ-sequence is a sequence of soft points of $(\mathcal{U}_2,\Theta)$ in a soft Δ-metric space $(\mathcal{U}_1,\mathcal{U}_2,\Delta,\Theta)$ obtained by deleting some or no soft points of soft Δ-sequence without altering the positions of remaining soft points.*

Theorem 2.3. *A soft Δ-sequence $\{\mathcal{Q}^y_{\sigma,n}\}_n$ in a soft Δ-metric space $(\mathcal{U}_1,\mathcal{U}_2,\Delta,\Theta)$ converges to $\mathcal{R}^x_\lambda \in SP(\mathcal{U}_2,\Theta)$ with respect to $\mathcal{P}^a_e \in SP(\mathcal{U}_1,\Theta)$ if and only if every soft Δ-subsequence $\{\mathcal{Q}^y_{\sigma,n_k}\}_{n_k}$ of $\{\mathcal{Q}^y_{\sigma,n}\}_n$ converges to $\mathcal{R}^x_\lambda$ with respect to $\mathcal{P}^a_e$.*

Proof. Since $\{\mathcal{Q}^y_{\sigma,n}\}_n$ converges to $\mathcal{R}^x_\lambda$ with respect to $\mathcal{P}^a_e$, there exists $m \in \mathbb{N}$ for any $\widetilde{\epsilon} > \widetilde{0}$ such that $\{\mathcal{Q}^y_{\sigma,n}\}_n \subseteq \Delta B(\mathcal{P}^a_e,\widetilde{\epsilon})$ and $\mathcal{R}^x_\lambda \in \Delta B(\mathcal{P}^a_e,\widetilde{\epsilon})$, whenever $n \geq m$. Now, for an arbitrary soft Δ-subsequence $\{\mathcal{Q}^y_{\sigma,n_k}\}_{n_k}$ of $\{\mathcal{Q}^y_{\sigma,n}\}_n$, we have, $\{\mathcal{Q}^y_{\sigma,n_k}\}_{n_k} \subseteq \Delta B(\mathcal{P}^a_e,\widetilde{\epsilon})$ for all $k \geq m$. Thus, $\{\mathcal{Q}^y_{\sigma,n_k}\}_{n_k}$ converges to $\mathcal{R}^x_\lambda$ with respect to $\mathcal{P}^a_e$. The converse part is obvious because, $\{\mathcal{Q}^y_{\sigma,n}\}_n$ itself is a soft Δ-subsequence of $\{\mathcal{Q}^y_{\sigma,n}\}_n$. □

Corollary 2.1. *A soft Δ-sequence $\{\mathcal{Q}^y_{\sigma,n}\}_n$ in a soft Δ-metric space $(\mathcal{U}_1,\mathcal{U}_2,\Delta,\Theta)$ converges to $\mathcal{R}^x_\lambda \in SP(\mathcal{U}_2,\Theta)$ with respect to $\mathcal{P}^a_e \in SP(\mathcal{U}_1,\Theta)$ if and only if every soft Δ-subsequence of $\{\mathcal{Q}^y_{\sigma,n}\}_n$ has a soft Δ-convergent subsequence that converges to $\mathcal{R}^x_\lambda$ with respect to $\mathcal{P}^a_e$.*

Proof. Suppose, $\{\mathcal{Q}^y_{\sigma,n}\}_n$ does not converge to $\mathcal{R}^x_\lambda$ with respect to $\mathcal{P}^a_e$. Then, for any $m \in \mathbb{N}$ there exists $\widetilde{\epsilon} > \widetilde{0}$ such that $\{\mathcal{Q}^y_{\sigma,n}\}_n$ is not contained in $\Delta B(\mathcal{P}^a_e,\widetilde{\epsilon})$ and $\mathcal{R}^x_\lambda \in \Delta B(\mathcal{P}^a_e,\widetilde{\epsilon})$ for any $n \geq m$. This implies, there should be a soft Δ-subsequence of $\{\mathcal{Q}^y_{\sigma,n}\}_n$ which has no soft Δ-subsequence that converges to $\mathcal{R}^x_\lambda$ with respect to $\mathcal{P}^a_e$, which is a contradiction. Hence, $\{\mathcal{Q}^y_{\sigma,n}\}_n$ converges to $\mathcal{R}^x_\lambda$. By the Theorem 2.3 and the fact that every soft Δ-subsequence of a soft Δ-subsequence of $\{\mathcal{Q}^y_{\sigma,n}\}_n$ is also a soft Δ-subsequence of $\{\mathcal{Q}^y_{\sigma,n}\}_n$, the converse follows. □

Theorem 2.4. *A soft Δ-sequence $\{\mathcal{Q}^y_{\sigma,n}\}_n$ of soft points in a soft Δ-metric space $(\mathcal{U}_1,\mathcal{U}_2,\Delta,\Theta)$ converges to $\mathcal{R}^x_\lambda \in SP(\mathcal{U}_2,\Theta)$ with respect to $\mathcal{P}^a_e \in SP(\mathcal{U}_1,\Theta)$ if and* *only if for every soft Δ-neighbourhood $N(\mathcal{R}^x_\lambda)$ of $\mathcal{R}^x_\lambda$, there exists $k \in \mathbb{N}$ such that $\{\mathcal{Q}^y_{\sigma,n}\}_n \subseteq N(\mathcal{R}^x_\lambda)$ for all $n \geq k$.*

Theorem 2.5. *A soft subset $(\mathcal{B},\Theta)$ of $(\mathcal{U}_2,\Theta)$ in a soft Δ-metric space $(\mathcal{U}_1,\mathcal{U}_2,\Delta,\Theta)$ is soft Δ-closed if and only if every soft Δ-convergent sequence of soft points in $(\mathcal{B},\Theta)$ does not converge to a soft point of $(\mathcal{B},\Theta)'$.*

Proof. Let $\{\mathcal{Q}^y_{\sigma,n}\}_n$ be a soft Δ-convergent sequence of soft points in $(\mathcal{B},\Theta)$ and converges to $\mathcal{R}^x_\lambda \in (\mathcal{B},\Theta)'$. By Theorem 2.2, $\mathcal{R}^x_\lambda$ is a soft Δ-limit point of $(\mathcal{B},\Theta)$, which is a contradiction to $(\mathcal{B},\Theta)$ is soft Δ-closed. Converse follows from the Theorem 2.2 and from [5]. □

Theorem 2.6. *Every soft Δ-convergent sequence in a soft Δ-metric space is a soft Δ-Cauchy sequence.*

Proof. Let $\{\mathcal{Q}^y_{\sigma,n}\}_n \subseteq SP(\mathcal{U}_2,\Theta)$ be a soft Δ-convergent sequence in $(\mathcal{U}_1,\mathcal{U}_2,\Delta,\Theta)$ and converges to $\mathcal{R}^x_\lambda \in SP(\mathcal{U}_2,\Theta)$ with respect to $\mathcal{P}^a_e \in SP(\mathcal{U}_1,\Theta)$. Let $\Delta(\mathcal{P}^a_e,\mathcal{R}^x_\lambda) = \widetilde{s}$. Therefore, $\Delta(\mathcal{P}^a_e,\mathcal{Q}^y_{\sigma,i}) \to \widetilde{s}$ as $i \to \infty$ and $\Delta(\mathcal{P}^a_e,\mathcal{Q}^y_{\sigma,j}) \to \widetilde{s}$ as $j \to \infty$. Now, $\Delta(\mathcal{P}^a_e,\mathcal{Q}^y_{\sigma,i}) - \Delta(\mathcal{P}^a_e,\mathcal{Q}^y_{\sigma,j}) \to \widetilde{0}$ as $i,j \to \infty$. Hence, $\{\mathcal{Q}^y_{\sigma,n}\}_n$ is a soft Δ-Cauchy sequence. □

Theorem 2.7. *If a soft Δ-Cauchy sequence in a soft Δ-metric space has a soft Δ-convergent subsequence, then it is a soft Δ-convergent sequence.*

Proof. Let $\{\mathcal{Q}^y_{\sigma,n_k}\}_{n_k}$ be a soft Δ-convergent subsequence of a soft Δ-Cauchy sequence $\{\mathcal{Q}^y_{\sigma,n}\}_n$ and converges to $\mathcal{R}^x_\lambda \in SP(\mathcal{U}_2,\Theta)$ with respect to $\mathcal{P}^a_e \in SP(\mathcal{U}_1,\Theta)$. By Theorem 2.6, $\{\mathcal{Q}^y_{\sigma,n_k}\}_{n_k}$ is a soft Δ-Cauchy sequence. Therefore, $\Delta(\mathcal{P}^a_e,\{\mathcal{Q}^y_{\sigma,n_k}\}) - \Delta(\mathcal{P}^a_e,\{\mathcal{Q}^y_{\sigma,n}\}) \to \widetilde{0}$ as $k,n \to \infty$. Since n_k is an increasing sequence of positive integers and $n \to \infty$, we have, $\Delta(\mathcal{P}^a_e,\mathcal{R}^x_\lambda) - \Delta(\mathcal{P}^a_e,\{\mathcal{Q}^y_{\sigma,n}\}) \to \widetilde{0}$, implies $\{\mathcal{Q}^y_{\sigma,n}\}_n$ is a soft Δ-convergent sequence. □

Theorem 2.8. *Every soft Δ-Cauchy sequence in a soft Δ-metric space is soft Δ-bounded.*

Definition 2.5. *A soft Δ-metric space $(\mathcal{U}_1,\mathcal{U}_2,\Delta,\Theta)$ is said to be soft Δ-complete, if every soft Δ-Cauchy sequence in $(\mathcal{U}_1,\mathcal{U}_2,\Delta,\Theta)$ is soft Δ-convergent and converges to some soft point of $(\mathcal{U}_2,\Theta)$.*

Theorem 2.9. *A soft Δ-metric subspace $(\mathcal{A},\mathcal{B},\Delta_{sub},\Theta)$ of a soft Δ-metric space $(\mathcal{U}_1,\mathcal{U}_2,\Delta,\Theta)$ is soft Δ-complete if and only if $(\mathcal{B},\Theta)$ is soft Δ-closed in $(\mathcal{U}_1,\mathcal{U}_2,\Delta,\Theta)$.*

Proof. Let $(\mathcal{A}, \mathcal{B}, \Delta_{sub}, \Theta)$ be soft Δ-complete. Then, every soft Δ-Cauchy sequence in $(\mathcal{A}, \mathcal{B}, \Delta_{sub}, \Theta)$ is soft Δ-convergent and converges to some soft point of $(\mathcal{B}, \Theta)$. Also, every soft Δ-convergent sequence is soft Δ-Cauchy. Thus, every soft Δ-convergent sequence in $(\mathcal{A}, \mathcal{B}, \Delta_{sub}, \Theta)$ converges to a soft point of $(\mathcal{B}, \Theta)$ and hence $(\mathcal{B}, \Theta)$ is soft Δ-closed. Conversely, let $(\mathcal{B}, \Theta)$ be soft Δ-closed. Then, by Theorem 2.5 and Theorem 2.6, $(\mathcal{A}, \mathcal{B}, \Delta_{sub}, \Theta)$ is soft Δ-complete. □

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