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PERFECT DISTANCE DOMINATING SET

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ABSTRACT. In this paper, finding perfect distance dominating set using the distance dominating concepts applied on graphs and its minimal dominating graph. A minimal dominating set of a graph is called perfect distance dominating set if it is act as one of cut vertex, perfect dominating set and there exist a minimal dominating graph of graph.

1. INTRODUCTION

In the area of graph theory, in 1975, domination was extended to distance domination by Meir and Moon [1]. Some of the results based on distance dominating set were extended by P.J. Slater in his paper R-domination in graphs in 1976 [3]. The concept of perfect dominating set was introduced by Marilynn Livingston and Quentin F. Stout in 1990 [2]. in 1995, the concepts of minimal dominating graph was introduced by V.R. Kulli in his paper [4]. In this article we discuss the vertex set, dominating set, perfect dominating set and distance dominating set for given graph, we just looking on among the sets there is some connection or relation were seen. A set is appearing in all the discussion, such a set is called prefect distance dominating set. For, we consider a graph, and

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finding its cut vertex set which is also it is act as dominating set and sometime perfect dominating set, but this set having single vertex. Suppose such a set having more than one vertex, this is not easy to finding as a unique, there is visible more number of such a set. In this case we finding minimal dominating graph of given graph and then apply some process as following chapters to reduce to single set.

In this connection, we have discussed some basic definitions, example of exist a perfect distance dominating set, not perfect distance dominating set, some processer to find perfect distance dominating set. Finally every minimal dominating graph has perfect distance dominating set.

2. Basic definitions

Definition 2.1. *Let $G = (V, E)$, be a graph. A set $D \subseteq V$ is a dominating set of G if every vertex in V/D is adjacent to some vertex in D.*

Definition 2.2. *A dominating set D of a graph G is perfect dominating set if each vertex of G is dominated by exactly one vertex in D.*

Definition 2.3. *A graph $G = (V, E)$, a distance dominating set is a subset D of V such that every vertex u not in D, the distance*

$$d(u, v) \leq k$$

for some v in D. Sometimes it is called k-distance dominating set or kd-dominating set. Note that, a classical dominating set is a $1d$-dominating set.

Definition 2.4. *A dominating set D of G is minimal if for any vertex $v \in D$, $D - \{v\}$ is not a dominating set of G.*

Definition 2.5. *Let S be a finite set and let $F = \{S_1, S_2, \ldots, S_n\}$ be a partition of S. Then the intersection graph $\Omega(F)$ of F is the graph whose vertices are the subsets in F and which two vertices S_i and S_j are adjacent, if, and only if,*

$$S_i \cap S_j \neq \Phi.$$

Definition 2.6. *The minimal dominating graph of a graph G is the intersection graph defined on the family of all minimal dominating sets of vertices in G.*

Example 1. *Consider the graphs and its minimal dominating graphs*

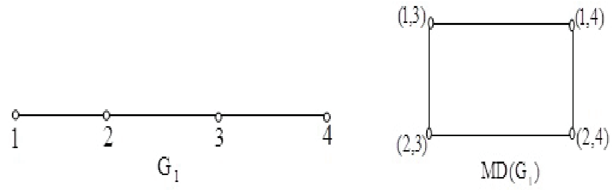

FIGURE 1.

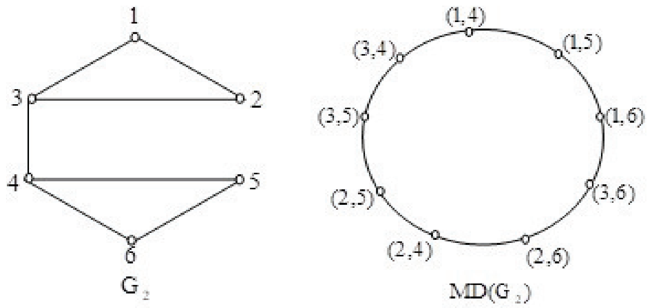

FIGURE 2.

In figure 1: *The cut vertex sets are* $\{(2),(3),(2,3)\}$ *of* G_1.
In figure 2: *The cut vertex sets are* $\{(3),(4),(3,4)\}$ *of* G_2.
In figure 1: *The dominating sets are* $\{(1,3),(1,4),(2,3),(2,4)\}$ *as same as* $1d$*-dominating set of* G_1 *and* $\{(1,4),(2,3)\}$ *are perfect dominating sets.*
In figure 2: *The dominating sets are* $\{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$ *as same as* $1d$*-dominating set of* G_2 *and* $\{(1,5),(1,6),(2,5),(2,6),(3,4)\}$ *are perfect dominating sets.*
In figure 1 and 2: *Any adjacent vertex sets (minimal dominating sets) in minimal dominating graph of* G_1 *and* G_2 *is satisfied the condition* $S_i \cap S_j \neq \Phi$
In figure 1 and 2: *The perfect dominating sets of* G_1 *and* G_2 *are more than one set available.*
In figure 1: *The distance dominating sets* $\{(2),(3),(1,4)\}$ *which are* $2d$*-dominating sets of* G_1 *and* $\{(2),(3)\}$ *are perfect dominating sets.*
In figure 2: *The distance dominating sets* $\{(3),(4),(4,6),(4,5),(3,1),(3,2)\}$ *which are* $2d$*-dominating sets of* G_2 *and* $\{(3),(4)\}$ *are perfect dominating sets.*
In figure 1 and 2: *The perfect dominating sets of* $MD(G_1)$ *and* $MD(G_2)$ *are more than one set available.*

In figure 1: *The distance dominating sets are* $\{(1,3),(1,4),(2,3),(2,4)\}$ *as same as* $2d$*-dominating set of* $MD(G_1)$.

In figure 2: *The distance dominating sets are* $\{[(1,4),(2,4)],[(1,4),(2,6)],[(1,5),(2,5)],[(1,5),(2,4)],[(1,6),(3,5)],[(1,6),(2,5)],[(3,6),(3,4)],[(3,6),(3,5)],[(2,6),(3,4)]\}$ *which are* $2d$*-dominating sets of* $MD(G_2)$.

Example 2. *Consider the graph*

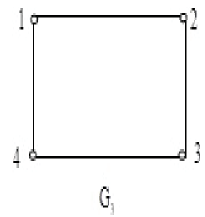

FIGURE 3.

In figure 3: *The cut vertex sets are* $\{(1,3),(2,4)\}$.
In figure 3: *The perfect dominating sets are* $\{(1,2),(2,3),(3,4),(4,1)\}$.
In figure 3: *The condition of minimal dominating graph is not satisfied, that is* $(1,3)\cap(2,4)=\Phi$.

Example 3. *Consider the graph:*

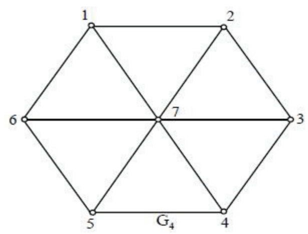

FIGURE 4.

In figure 4: *The cut vertex sets are* $\{(1,7,3),(1,7,4),(1,7,5),\ldots\}$.
In figure 4: *The perfect dominating sets is* $\{(7)\}$.
In figure 4: *The dominating sets are* $\{(1,4),(2,5),(3,6),(7)\}$.
In figure 4: *The condition of minimal dominating graph is not satisfied, that is*

$$(7)\cap \textit{(other dominating set)} = \Phi.$$

Example 4. *Consider the graph*

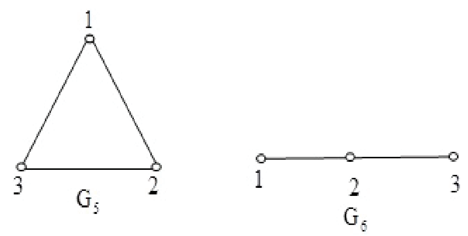

FIGURE 5.

In figure 5: *The cut vertex* $\{2\}$ *of* G_6 *and there is no cut vertex in* G_5.
In figure 5: *The dominating sets are* $\{(1),(2),(3)\}$ *of* G_5 *and* $\{(2),(1,3)\}$ *of* G_6, *all are perfect dominating sets.*
In figure 5: *Does not exist minimal dominating sets fort the graphs* G_5 *and* G_6.

3. PERFECT DISTANCE DOMINATING SET

In this paper, we investigate the perfect distance dominating set, let us consider the a graph with vertices more than 3 (because, see example of the Figure 5), for that graph, we can find perfect dominating set at the same time it will be more than one sets are available which are called also perfect dominating sets, but we have investigated here which is best among those sets. So we can find the characteristics of those perfect dominating set.

Some of the graphs have single vertex act as perfect dominating set, see the example G_4 but the same time that vertex is not a cut vertex and cannot form the minimal dominating graph for that graph. In this case take as trivial.

Some of the graphs have collection of vertices set act as perfect dominating sets, see the example G_3 but the same time that vertices sets are cut vertex set. We observe that graph, suppose we can choose any one of among perfect dominating sets, we will say that why this set is best than other sets, in this stage we apply the concept of distance dominating because the distance is doing major role in graph theory and many of the application of graph theory. Some situation we cannot take decision for the best one and we have to check its minimal dominating graph, we can may or may not get best one and it is defending on its distance, but the example G_3 cannot form the minimal dominating graph for that graph. In this case take as not trivial.

The following process for find the perfect distance dominating set through minimal dominating graph,

Step 1: We can choose any minimal dominating set say S_i.
Step 2: Check the condition $S_i \cap C_k \neq \Phi$, where C_k is cut vertex set.
Step 3: Check the condition $P_i \cap C_k \neq \Phi$, where P_i is perfect dominating set.
The set P_i is called perfect distance dominating set.

Proposition 3.1. *The perfect distance dominating set of a graph G is cut vertex set of G, converse need not true. See the Figure 4.*

Proposition 3.2. *The perfect distance dominating set of a graph G is perfect dominating set of G, converse need not true. See the Figures 3 and 5.*

Proposition 3.3. *Every perfect dominating set is cut vertices set and converse need not true. See the Figures 3 and 5.*

Theorem 3.1. *Let G be a graph, if there exist a minimal dominating graph of G then G has perfect distance dominating set.*

Proof. Let G be a graph with $n \geq 4$ where n is number of vertices.
Then its minimal dominating graph $MD(G)$ as satisfied the following properties.
Let S be a finite set, Let $F = \{S_1, S_2, \ldots, S_n\}$ be a partition of S. Therefore

(i) The intersection graph $\Omega(F)$ of F is the graph whose vertices are the subsets in F.
(ii) Any two vertices S_i and S_j are adjacent if and only if $S_i \cap S_j \neq \Phi$.

Since, every $MD(G)$ has cut vertices sets and perfect dominating sets (see the Figure 1 and 2). So there also cut vertices set and perfect dominating set in G.

To collect the dominating set of G, say $S_1, S_2, \ldots, S_n$. Since $S_i \cap S_j \neq \Phi$ if S_i and S_j are adjacent. Since some of S_i are cut vertex set in G and some vertex $\{v_i\}$ is also in cut vertex in G.

Now, let A be the family of cut vertices sets. Let B be the family of perfect dominating sets. Then $A \cap B = C$. Where C is common vertices set between A and B. Thus C act as cut vertex set and perfect dominating set.

Therefore C is the perfect distance dominating set. Proof is now completed. □

4. Conclusion

In this article we investigated the cut vertex set, dominating set, perfect dominating set and distance dominating set for given graph, then there are more number of dominating sets available for a graph. We have to try to select one best set among all dominating sets, we have obtained result that every minimal dominating graph of graph has perfect distance dominating set, if a graph has no minimal dominating graph, then does not exit perfect distance dominating set as best one for connectivity of minimum distance. So, Perfect distance dominating set is more imparted to connectivity of electrical circuit which is reduce to distance and it can apply ant distance related concepts.

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