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EFFICIENT DISJUNCTIVE DOMINATING SETS IN GRAPHS

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ABSTRACT. A vertex of a graph G = (V,E) dominates itself and all its neighbors. A vertex set D in G is an efficient dominating set for G if for every vertex $v \in V$, there is exactly one $u \in D$ dominating v. $D \subset V$ is a disjunctive dominating set if every vertex $v \in V$ is either dominated by vertices in D or has at least two vertices in D at a distance 2 from it. This paper introduces Efficient Disjunctive Dominating sets (EDD-sets) and Nearly efficient disjunctive dominating sets (NEDD-sets) in graphs. We examine the existence of EDD-sets in some graphs, characterize all paths,cycles, two dimensional grid graphs having EDD-sets and provide a proof to show the existence of an NEDD-set in an infinite two dimensional grid graph.

1. INTRODUCTION

Let $G = (V, E)$ be a simple graph. A vertex u is said to dominate a vertex v if $u = v$ or u is adjacent to v. A set of vertices $D \subset V$ is called a dominating set of G if every vertex of G is dominated by at least one member of D. The basic domination problem is to determine the minimum-size of a dominating

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set. Domination in graphs and many different variations of this concept have been widely studied by many researchers.

When each vertex of G is dominated by exactly one element of D, the set D is called an **Efficient Dominating Set**. This concept was originally defined by D.W. Bange, A.E. Barkauskas and P.J. Slater in [1].

W Goddard *et al.* [9] defined disjunctive domination in graphs. A subset D of vertices of G is called a disjunctive dominating set if every vertex in $V \setminus D$ is either adjacent to a vertex in D or has at least 2 vertices in D at a distance 2 from it. The disjunctive domination number of G, denoted by $\gamma_2^d(G)$, is the minimum cardinality of a disjunctive dominating set in G. From the definition of disjunctive domination, we note that if D is a disjunctive dominating set and $u \in V$ is not dominated by D, then there exists at least two vertices $v_1, v_2 \in D$ such that $d(u, v_1) = d(u, v_2) = 2$. In this case we say that $u \in V$ is disjunctively dominated or D-dominated by D. Some properties of disjunctive domination are studied in [6].

In this paper we make an attempt to study efficient disjunctive dominating sets in some graphs. We focus mainly on two dimensional grid graphs, i.e, the Cartesian product of two paths. Grid graphs have importance in computer architecture as they model parallel processor networks and they have applications in various fields like sensor networks, coding theory and robotics. Hence the study of graph theoretic properties of these graphs is a significant problem. Classical domination number of these graphs are investigated intensively by many researchers, for example, in [2–4, 7].

For all standard terminology and notation we follow [5]. The terms related to the theory of domination in graphs are used as in the sense of Haynes et al. [8]. $P_n \Box P_m$ denotes the cartesian product of the path graphs P_n and P_m which is known as the $n \times m$ complete grid graph. The graphs considered in this paper are simple, connected and nontrivial, unless otherwise specified.

2. Efficient Disjunctive Dominating Set

Definition 2.1. *Let $D \subset V$. Define a function $f_D : V \to R$ by*

$$f_D(u) = |N[u] \cap D| + \frac{1}{2}|N_2(u) \cap D|.$$

D is called an ***Efficient Disjunctive Dominating set*** *or* ***EDD-set*** *if $f_D(u) = 1$ for all $u \in V$. In other words, D is an efficient disjunctive dominating set if each vertex of V is either dominated by exactly one vertex in D or disjunctively dominated by exactly two vertices in D.*

An EDD-set is a disjunctive dominating set for which the total amount of domination and disjunctive domination done by it is minimum. Hence cardinality of an EDD-set is $\gamma_2^d(G)$.

Most graphs do not have an efficient disjunctive dominating set. If a graph G has an efficient disjunctive dominating set, we say that G is efficiently disjunctive dominatable graph or EDD-graph. Some examples of EDD-graphs are given in Figure 1.

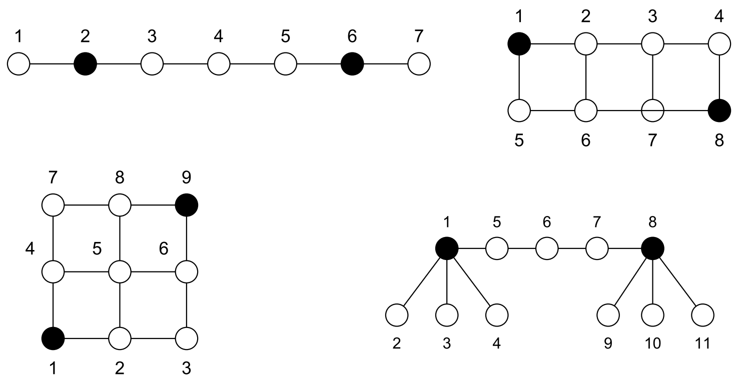

FIGURE 1. EDD-graphs

Example 1. *Petersen graph is not an EDD-graph. Disjunctive domination number of this graph is 2, as realized by any two vertices. But any pair of vertices of this graph lie on a C_5 shows that it has no EDD-set.*

Observation. All graphs having a universal vertex, i.e., a vertex which is adjacent to all other vertices of the graph, are EDD-graphs. In particular complete graphs, star and wheel graphs are EDD-graphs.

Lemma 2.1. *If D is an EDD-set, $d(u, v) \geq 4$ for every pair of vertices $u, v \in D$.*

Proof. Let $u, v \in D$ and $d(u, v) < 4$. Then there exist at least one vertex w on the uv-path such that $f_D(w) > 1$ which is not possible by definition of EDD-sets. □

Theorem 2.1. *For $n \geq 3$, C_n is an EDD-graph if and only if $n = 3$ or $n \equiv 0\ (mod\ 4)$, but $n \neq 4$.*

Proof. W. Goddard *et al.* [9] proved that, if $n \geq 3$,

$$\gamma_2^d(C_n) = \begin{cases} 2, & \text{if n=4} \\ \lceil \frac{n}{4} \rceil, & \text{otherwise} \end{cases}.$$

It is obvious that C_3 is an EDD-graph, but C_4 is not an EDD-graph. Let $n \geq 5$. An EDD-set in C_n, if it exists, has cardinality $\lceil \frac{n}{4} \rceil$. Let $\{1, 2, 3, \ldots, n\}$ be the vertices of C_n.

Case (i): $n = 4k,\ k \neq 1$. In this case $D = \{1, 5, 9, \ldots, 4k-3\}$ of cardinality k is an EDD-set of C_n. Hence if $n \neq 4$, but $n \equiv 0\ (mod\ 4)$, then C_n is an EDD-graph.

Case (ii): $n \equiv 1, 2, 3\ (mod\ 4)$. If $n = 4k+1$ or $4k+2$ or $4k+3$, then $\gamma_2^d(C_n) = k+1$. Hence in any disjunctive dominating set there exist at least one pair of vertices u, v such that $d(u, v) < 4$. It follows from lemma 2.1 that C_n is not an EDD- graph in these cases. □

Theorem 2.2. *For every positive integer n, P_n is an EDD-graph unless $n \equiv 0\ (mod\ 4)$.*

Proof. In [9] it is proved that, $\gamma_2^d(P_n) = \lceil \frac{n+1}{4} \rceil$ for all n . Hence an EDD-set in P_n, if it exists, has cardinality $\lceil \frac{n+1}{4} \rceil$. Let $\{1, 2, 3, \ldots, n\}$ be the vertices of P_n.

Case (i): $n \equiv 1, 2\ (mod\ 4)$. In this case $n = 4k+1$ or $4k+2$ and $D = \{1, 5, 9, \ldots, 4k+1\}$ is an EDD-set.

Case (ii): $n \equiv 3\ (mod\ 4)$. In this case $n = 4k+3$ and $D = \{2, 6, 10, \ldots, 4k+2\}$ is an EDD-set.
Cases (i) and (ii) show that P_n, $n \equiv 1, 2, 3\ (mod\ 4)$ is an EDD-graph.

Case (iii): $n \equiv 0\ (mod\ 4)$. Let $n = 4k$. We may note that any γ_2^d-set D of P_n has $k+1$ vertices and hence there must be at least one pair of vertices $u, v \in D$ with $d(u, v) < 4$. This is not possible for an EDD-set. Hence P_n for $n \equiv 0\ (mod\ 4)$ is not an EDD-graph. □

Theorem 2.3. *$G_{2,m} = P_2 \Box P_m$ is an EDD-graph if and only if $m \equiv 1\ (mod\ 3)$.*

Proof. W. Goddard et al. proves in [9] that $\gamma_2^d(G_{2,m}) = \lceil \frac{m+2}{3} \rceil$ and a γ_2^d-set of $G_{2,m}$ contains one vertex from every third column, taken from alternating rows, together with a vertex in the last column if no vertex is already taken from there. Let $V(P_2) = \{1, 2\}$ and $V(P_m) = \{1, 2, \ldots, m\}$. Then,
$V(G_{2,m}) = \{(1, 1), (1, 2), \ldots, (1, m), (2, 1), (2, 2), \ldots, (2, m)\}$

Case (i): $m \equiv 1 \ (mod\, 3)$. If $m = 3k+1$, then $D = \{(1, 1), (2, 4), (1, 7), (2, 10), \ldots, (1, 3k+1)\}$ or $\{(1, 1), (2, 4), (1, 7), (2, 10), \ldots, (2, 3k+1)\}$ is an EDD-set depending on k is even or odd. The case when $m = 10$ is illustrated in Figure 2.

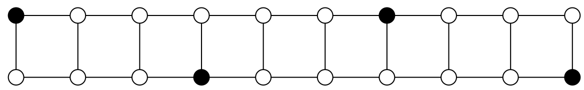

FIGURE 2. An EDD-set of $G_{2,10}$

Case (ii): $m \equiv 0, 2 \ (mod\ 3)$. If $m = 3k$ or $3k + 2$, then the construction of a γ_2^d-set of $G_{2,m}$ given in [9] shows that, any γ_2^d-set of $G_{2,m}$ contains two vertices within a distance less than 4 between them. Hence it follows from lemma 2.1 that these are not EDD-graphs. □

Theorem 2.4. *$G = P_3 \square P_3$ is an EDD-graph.*

Proof. Let $V(P_3) = \{1, 2, 3\}$. Then,

$$V(P_3 \square P_3) = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}.$$

$D = \{(1, 1), (3, 3)\}$ is an EDD-set of G. Hence $P_3 \square P_3$ is an EDD-graph. □

Theorem 2.5. *$G = P_4 \square P_3$ has no EDD-set.*

Proof. Consider the graph $G = P_4 \square P_3$ given in Figure 3. If possible let D be an EDD-set of G. It is clear that D must contain at least two vertices in G. Also from Lemma 2.1 it follows that there cannot be 3 vertices in D. Hence an EDD-set of G, if it exists, must be of order 2. If $(2, 2)$ or $(3, 2)$ is in D, Lemma 2.1 shows that D cannot contain a second vertex because all the other vertices of G are within a distance of 3 from these two vertices. Hence there are only two different possibilities for the set D. Without loss of generality we can assume the two different cases as $D = \{(1, 1), (3, 3)\}$ or $D = \{(1, 1), (4, 2)\}$.

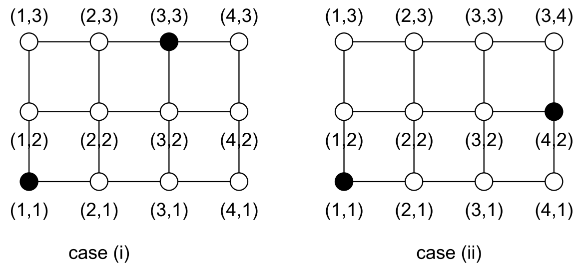

FIGURE 3. $P_4\Box P_3$

Case(i): $D = \{(1,1),(3,3)\}$. Consider the vertex $(4,1)$. It is at a distance 3 from both the vertices in D. So it is neither dominated nor disjunctively dominated by D. This is a contradiction to the choice of D.

Case(ii). $D = \{(1,1),(4,2)\}$. In this case the vertex $(2,3)$ is at a distance 3 from both the vertices in D. Hence it is neither dominated nor disjunctively dominated by D which is again a contradiction to the choice of D.

Thus in both cases we arrive at a contradiction to the assumption that D is an EDD-set of G. So we conclude that $G = P_4\Box P_3$ has no EDD-set. $\Box$

Theorem 2.6. *A two dimensional grid graph $G = P_n\Box P_m$ has no EDD-set if $n \geq 4$ and $m \geq 3$.*

Proof. Let $V(P_n) = \{u_1, u_2, \ldots, u_n\}$ and $V(P_m) = \{v_1, v_2, \ldots, v_m\}$. If possible let D be an EDD-set of G. It is clear that D must contain at least two vertices. Choose some vertex $(u_i, v_j) \in D$. Since G contains $P_4\Box P_3$ as a subgraph, with out loss of generality we can assume that there exist a path P_3 in G having vertices $(u_i, v_j), (u_{i+1}, v_j), (u_{i+2}, v_j)$. Relabel the vertices of G as $(u_i, v_j) = (0,0)$, $(u_{i\pm x}, v_{j\pm y}) = (\pm x, \pm y)$.

Vertex $(0,0) \in D$ dominates $(1,0), (-1,0), (0,1)$ and $(0,-1)$. Vertex $(2,0)$ is at a distance 2 from $(0,0) \in D$. For the disjunctive domination of this vertex, D must contain another vertex which is also at a distance 2 from $(2,0)$. Lemma 2.1 shows that vertices $(1,1)$ and $(1,-1)$ cannot be in D. Hence D must contain one vertex from the set $\{(4,0),(2,2),(2,-2),(3,1),(3,-1)\}$ whichever exists in G. Due to symmetry of G, we need to consider only one vertex from the vertices

$\{(2,-2),(2,2)\}$ and one vertex from the vertices $\{(3,1),(3,-1)\}$. Hence without loss of generality we can assume that there are only three possible cases: $(i)(4,0) \in D$, $(2,2) \in D$ or $(3,1) \in D$.

Case(i): $(4,0) \in D$. Since G contains $P_4 \Box P_3$ as subgraph, vertex $(2,1)$ or $(2,-1)$ will be in G. Both these vertices are at a distance three from $(0,0)$ and $(4,0)$. Due to symmetry of G we can assume, with out loss of generality, that $(2,1) \in G$. For the domination or disjunctive domination of this vertex, D must contain one vertex from its closed neighborhood

$$N[(2,1)] = \{(2,1),(1,1),(3,1),(2,0),(2,2)\}$$

or two vertices from its second neighborhood

$$N_2((2,1)) = \{(1,0),(3,0),(2,-1),(0,1),(4,1),(1,2),(3,2),(2,3)\}.$$

If D contains a vertex from $N[(2,1)]$, then $f_D(2,0)$ will be greater than one, which contradicts the definition of D. For disjunctive domination of $(2,1)$, if two vertices from its second neighborhood are chosen in D, these vertices together with the already chosen vertices $(0,0)$ and $(4,0)$ in D do not satisfy Lemma 2.1. Thus $(2,1)$ is neither dominated nor disjunctively dominated by D, which contradicts the definition of D.

Case(ii): $(2,2) \in D$. Since G contains $P_4 \Box P_3$ as subgraph, vertex $(3,0)$ or $(-1,2)$ will be in G. Due to symmetry of G we can assume that $(3,0) \in G$. For the domination of this vertex, D must contain one vertex from its closed neighbor set $N[(3,0)] = \{(3,0),(2,0),(4,0),(3,1),(3,-1)\}$. But any of these vertices in D makes $f_D(2,0) > 1$. Now for the disjunctive domination of $(3,0)$, there must be two vertices in D from its second neighborhood,

$$N_2((3,0)) = \{(4,-1),(4,1),(3,-2),(3,2),(2,-1),(2,1),(1,0),(5,0)\}.$$

But Lemma 2.1 shows that the only possible case is $(5,0),(3,-2) \in D$. Suppose these two vertices are in D. Then $(3,0)$ is disjunctively dominated by D. Now consider the vertex $(3,1) \in G$. This vertex is at a distance 2 from $(2,2) \in D$ and at a distance 3 from other vertices chosen in D. For the disjunctive domination of this vertex, D must contain another vertex from its second neighborhood. But all the vertices in its second neighborhood are at a distance less than 4 from the already chosen vertices, which is a contradiction to the choice of D.

Case(iii): $(3,1) \in D$. Vertex $(1,2)$ or $(2,-1)$ will be in G. We can assume, without loss of generality, that $(1,2) \in G$. For the domination of this vertex, D must contain one vertex from its closed neighbor set $N[(1,2)] = \{(1,2),(0,2),(2,2),(1,3),(1,1)\}$. But any of these vertices in D makes $f_D(1,1) > 1$. Now for the disjunctive domination of $(1,2)$, there must be two vertices in D from its second neighborhood,

$$N_2((1,2)) = \{(-1,2),(0,1),(0,3),(1,0),(1,4),(2,1),(2,3),(3,2)\}.$$

All these vertices except $(1,4)$ are at a distance less than 4 from the already chosen vertices in D. Hence disjunctive domination of $(1,2)$ is also not possible, contradicting our hypothesis on D.

From the above cases we can conclude that an EDD-set cannot exist in a grid graph which has an induced subgraph isomorphic to $P_4 \square P_3$. □

Theorem 2.7. *$G = P_n \square P_m$ is an EDD-graph, if, and only if,*

(i) *n= 2, m= 3k+1;*
(ii) *n= m= 3.*

Proof. It follows from Theorems 2.3, 2.4, 2.5 and 2.6. □

3. Nearly efficient disjunctive dominating sets

From the above theorem we can see that an infinite grid graph has no EDD-set, but it has a disjunctive dominating set with the following property.

Theorem 3.1. *An infinite grid graph G has a disjunctive dominating set D such that for each vertex $u \in V$ in G, $1 \le f_D(u) < 2$.*

Proof. Let $\mathbb{Z}$ denote the additive group of integers, $\mathbb{Z}^2 = \mathbb{Z} \times \mathbb{Z}$ the product of $\mathbb{Z}$ with itself and $\mathbb{Z}_8$ the group of integers modulo 8. Let

$$f : \mathbb{Z}^2 \to \mathbb{Z}_8$$

be the homomorphism given by

$$f(x,y) = x + 3y \; for \; (x,y) \in \mathbb{Z}^2.$$

Let $e_1 = (1, 0)$, $e_2 = (0, 1)$. Then $f(e_1) = 1$ and $f(e_2) = 3$. For all $u = (x, y) \in \mathbb{Z}^2$,

$$f(u \pm e_1) = f(u) \pm 1 = f(u) + 1 \text{ or } f(u) + 7 in \mathbb{Z}_8,$$
$$f(u \pm e_2) = f(u) \pm 3 = f(u) + 3 \text{ or } f(u) + 5,$$
$$f(u \pm 2e_1) = f(u) \pm 2 = f(u) + 2 \text{ or } f(u) + 6,$$
$$f(u \pm 2e_2) = f(u) \pm 6 = f(u) + 6 \text{ or } f(u) + 2,$$
$$f(u + e_1 + e_2) = f(u) + 4,$$
$$f(u + e_1 - e_2) = f(u) - 2 = f(u) + 6,$$
$$f(u + e_2 - e_1) = f(u) + 2,$$
$$f(u - e_1 - e_2) = f(u) - 4 = f(u) + 4.$$

The unit ball $B(u)$ about a vertex $u \in \mathbb{Z}^2$ is defined as the set

$$B(u) = \{v : d(u, v) \leq 1\}.$$

The ball $B^2(u)$ about $u \in \mathbb{Z}^2$ and of radius 2 is defined as the set

$$B^2(u) = \{v : d(v, u) \leq 2\}.$$

So

$$B(u) = \{u, u \pm e_1, u \pm e_2\}$$

and

$$B^2(u) = \{u, u \pm e_1, u \pm e_2, u \pm 2e_1, u \pm 2e_2, u + e_1 + e_2, u + e_1 - e_2, u + e_2 - e_1, u - e_1 - e_2\}.$$

Hence

$$f(B(u)) = \{f(u), f(u) + 1, f(u) + 3, f(u) + 5, f(u) + 7\}$$

and

$$f(B^2(u)) = \{f(u), f(u)+1, f(u)+2, f(u)+3, f(u)+4, f(u)+5, f(u)+6, f(u)+7\}.$$

Thus

$$f(B^2(u)) = f(u) + \mathbb{Z}_8 = \mathbb{Z}_8.$$

So f restricted to $B(u)$ is a bijection to a subset of $\mathbb{Z}_8$ and its restriction to $B^2(u)$ is an onto map from $B^2(u)$ to $\mathbb{Z}_8$.

Consider the subset $D = f^{-1}(0)$ of V. It is easy to see that,

$$B(u) \cap B^2(v) = \phi \; if \; u \neq v \; and \; u, v \in f^{-1}(0).$$

We can show that D is a disjunctive dominating set of G such that each vertex in V is either dominated by exactly one vertex in D or disjunctively dominated by 2 or 3 vertices in D so that $f_D(u) = 1 \; or \; \frac{3}{2}$ for all $u \in V$.

Let $u = (x, y)$ be any element of V. Following are the different possibilities for u.

(i) If $f(u) = 0$, then $u \in D$ and $f_D(u) = 1$.

(ii) If $f(u) = 1$, then $f(u - e_1) = f(x - 1, y) = 0$ and so $u - e_1 \in D$. Also $u - e_1$ is at a distance one from u. Hence u is dominated by $u - e_1 \in D$ and $f_D(u) = 1$.

(iii) If $f(u) = 2$, then $f(u - 2e_1) = f(x - 2, y) = 0$, $f(u + 2e_2) = f(x, y + 2) = 0 \; in \; \mathbb{Z}_8$ and $f(u+e_1-e_2) = f(x+1, y-1) = 0$. Hence $u-2e_1, u+2e_2, u+e_1-e_2 \in D$. These vertices are at a distance 2 from u. So u is disjunctively dominated by these three vertices in D and so $f_D(u) = \frac{3}{2} < 2$.

(iv) If $f(u) = 3$, then $f(u - e_2) = f(x, y - 1) = 0$ and so $u - e_2 \in D$. Then u is dominated by $u - e_2 \in D$ and $f_D(u) = 1$.

(v) If $f(u) = 4$, then $f(u + e_1 + e_2) = f(x + 1, y + 1) = 0$ in $\mathbb{Z}_8$ and $f(u - e_1 - e_2) = f(x - 1, y - 1) = 0$ So $u + e_1 + e_2, u - e_1 - e_2 \in D$. Thus u is at a distance of 2 from two different vertices in D or it is disjunctively dominated by two vertices in D and so $f_D(u) = 1$.

(vi) If $f(u) = 5$, then $f(u + e_2) = f(x, y + 1) = 0$ in $\mathbb{Z}_8$ and so $u + e_2 \in D$. Then u is dominated by $u + e_2 \in D$ and $f_D(u) = 1$.

(vii) If $f(u) = 6$, then $f(u + 2e_1) = f(x + 2, y) = 0$ in $\mathbb{Z}_8$ $f(u - 2e_2) = f(x, y - 2) = 0$ and $f(u - e_1 + e_2) = f(x - 1, y + 1) = 0$ in $\mathbb{Z}_8$. Hence $u + 2e_1, u - 2e_2, u - e_1 + e_2 \in D$. Thus u is at a distance of 2 from three different vertices in D. Hence it is disjunctively dominated by three vertices in D and so $f_D(u) = \frac{3}{2}$.

(viii) If $f(u) = 7$, then $f(u + e_1) = f(x + 1, y) = 0$ in $\mathbb{Z}_8$ and so $u + e_1 \in D$. Then u is dominated by $u + e_1 \in D$ and $f_D(u) = 1$.

Thus in all the cases $u \in V$ is either dominated exactly once or disjunctively dominated by 2 or 3 vertices in D and so $1 \leq f_D(u) \leq \frac{3}{2} < 2$ for all $u \in V$. □

The above theorem motivated us to define a **nearly efficient disjunctive dominating set** in a graph.

Definition 3.1. *Let $G = (V, E)$. A subset D of V for which $B(u) \cap B^2(v) = \phi$ for every $u, v \in D$ and $1 \leq f_D(u) < 2$ for every $u \in V$ is called a* ***nearly efficient disjunctive dominating set*** *or* ***NEDD****-set. A graph having an NEDD-set is called an* ***NEDD-graph.***

Example 2. *NEDD-sets of $P_5 \Box P_3$, $P_5 \Box P_5$ and $P_5 \Box P_2 \Box P_2$ are shown in Figure 4.*

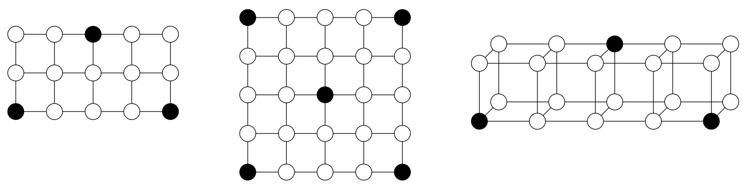

FIGURE 4. NEDD-graphs

Even though an infinite two dimensional grid graph has an NEDD-set, it can be observed that all finite two dimensional grid graphs are not NEDD-graphs.

4. CONCLUSION

In this paper we studied the existence of EDD-sets and NEDD-sets in some graphs, especially in two dimensional grid graphs. Existence of EDD-sets and NEDD-sets in three dimensional grid graphs are interesting topics for further study. Study of existence of these sets in other graphs also have further scope for research.

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