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RANKING INTERVAL VALUED INTUITIONISTIC FUZZY SETS BY A NEW DISTANCE MEASURE

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ABSTRACT. Choosing the best alternative in decision-making problems is complex job. In this process ranking is one of the key components that have a vital role. In this paper a method is developed to rank Interval valued intuitionistic fuzzy sets (IVIFSs) by using a distance measure which takes membership, non-membership and hesitancy degree of IVIFSs into consideration. The competence of the proposed ranking is demonstrated through numerical examples along with counter-intuitive cases and also by comparing with the existing rankings.

1. INTRODUCTION

In 1975, Zadeh [8] introduced IVFSs, with the membership degree defined within a closed subinterval of [0, 1]. Due to uncertainty in membership of the elements later K.T. Atanassov [5] introduced Intuitionistic Fuzzy Sets (IFSs), with non-membership degree in the structure. Various researchers have applied the theory of IFSs in decision making through ranking [29–31]. Researchers

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[1, 4, 24] provided evidence of strong correlation between IFSs and IVFSs. Consequently K.T. Atanassov and Gargov [1] developed Interval-Valued Intuitionistic Fuzzy Set (IVIFS) theory which is a generalization of both IVFSs and IFSs. The structure of IVIFSs gives interval membership and non-membership rather than crisp numbers [2]. In literature [7], a considerable amount of study is reported on the relations and operations of IVIFSs.

The IVIFSs have the significant benefit of handling with incomplete and imperfect information. Thus, they were adequately used in different applications, particularly in decision-making through the ranking of IVIFSs [4, 15]. Many aggregation operators were introduced by various authors [11, 13, 15] to aggregate the interval-valued intuitionistic fuzzy information. Several accuracy functions [**?**, 20], distance measures [1, 12, 19], similarity measures [14, 16, 17] and entropy measures [19, 23] of IVIFSs are proposed by researchers, and employed them in decision making [6].

Distance measures and the similarity measures signify the extent of likeness between two sets, thus widely used for ranking of fuzzy sets. The objective of the paper is to propose a method to rank IVIFSs using Jaccard distance measure. The Jaccard distance is the complement of the Jaccard similarity co-efficient and measures dissimilarity of two sets. As any ranking method is said to be effective if it assigns a better rank to the set which is closer to ideal set, in the proposed method the IVIFSs are ranked based on the distance from the given set to the preferred ideal set (1,1,0,0).

The paper is arranged as follows. Basic definitions and operations on IVIFSs are stated within section 2. In section 3, the measure Jaccard distance on IVIFSs is introduced. In section 4, the ranking approach for IVIFSs based on this measure is discussed and is illustrated through numerical examples. A comparative study of ranking approaches is discussed in section 5. The conclusions are given in section 6.

2. Preliminaries

In this section, some definitions and operations on IVIFS are discussed.

Definition 2.1. *(Interval-valued Intuitionistic fuzzy Sets (IVIFSs) [1, 7]) Let* X *be a universe set and* $E = \{x_1, x_2, \ldots, x_n\}$ *be a subset of its elements, then an IVIFS* Ã *having the form:*

$$\tilde{\mathrm{A}} = \{< x_i, ([\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i)], [\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i)]) >, x_i \in E\},$$

$[\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i)] \subseteq S[0,1]$ *and* $[\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i)] \subseteq S[0,1]$*; where S[0,1] be closed subintervals of [0,1]. Satisfying* $0 \leq \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i) + \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i) \leq 1$ *and* $0 \leq \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i) + \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i) \leq 1$ *Note: If* $\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i) = \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i)$ *and* $\mu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i) = \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i)$ *then the IVIFS* $\tilde{\mathrm{A}}$ *reduces to IFS [1,5]*

Definition 2.2. *(Interval hesitancy degree [7]) For each* $x_i \in E$*, the interval hesitancy degree of any IVIFS* $\tilde{\mathrm{A}}$ *is defined as*

$$\pi_{\tilde{\mathrm{A}}}(x_i) = [1 - \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i) - \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}}(x_i), 1 - \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i) - \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}}(x_i)].$$

Definition 2.3. *(Set operation on IVIFSs [11]) For any two IVIFSs* $\tilde{\mathrm{A}}_1$ *and* $\tilde{\mathrm{A}}_2$*,*

$\tilde{\mathrm{A}}_1 = \{< x_i, ([\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i)], [\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i)]) >, x_i \in E\}$,
$\tilde{\mathrm{A}}_2 = \{< x_i, ([\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i)], [\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i)]) >, x_i \in E\}$,

(*i*) $\tilde{\mathrm{A}}_1 \bigcap \tilde{\mathrm{A}}_2 = \{(x_i, [\min(\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), \min(\mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [\max(\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), \max(\nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))])\}$;
(*ii*) $\tilde{\mathrm{A}}_1 \bigcup \tilde{\mathrm{A}}_2 = \{(x_i, [\max(\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), \max(\mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [\min(\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), \min(\nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))])\}$;
(*iii*) $\tilde{\mathrm{A}}_1 + \tilde{\mathrm{A}}_2 = \{(x_i, [\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i) + \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)) - \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i) \cdot \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i) + \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i)) - \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i) \cdot \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i)], [\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i) \cdot \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i), (\nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i) \cdot \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i)])\}$;
(*iv*) $\tilde{\mathrm{A}}_1 \cdot \tilde{\mathrm{A}}_2 = \{(x_i, [\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i) \cdot \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i) \cdot \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i) + \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i) - \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i) \cdot \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), (\nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i) + \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i) - \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i) \cdot \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i)])\}$.

Definition 2.4. *(Distance measure [12]) Let* X *be universal set. For any three IVIFSs* $\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2, \tilde{\mathrm{A}}_3$ *defined on* X*, a map* $d : IVIFSs(X) \times IVIFSs(X) \rightarrow [0,1]$ *is measure on IVIFSs if it satisfies the conditions:*

(*i*) $d(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) \in [0,1]$;
(*ii*) $d(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) = 0 iff \tilde{\mathrm{A}}_1 = \tilde{\mathrm{A}}_2$;
(*iii*) $d(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_3) \leq d(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) + d(\tilde{\mathrm{A}}_2, \tilde{\mathrm{A}}_3)$.

Definition 2.5. *(Jaccard distance [9]) Jaccard distance is a measure of dissimilarity between two sets, given by* $d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) = 1 - S_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2)$*, where* $S_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) = \frac{|\tilde{\mathrm{A}}_1 \bigcap \tilde{\mathrm{A}}_2|}{|\tilde{\mathrm{A}}_1 \bigcup \tilde{\mathrm{A}}_2|}$ *is Jaccard similarity co-efficient.*

3. PROPOSED DISTANCE MEASURE FOR RANKING IVIFSS

Jaccard similarity measure gives a straightforward and innate measure of similarity between data sets and has been proved to be quite useful in decision support systems with various domains. In this section, Jaccard distance measure on IVIFSs is proposed. The measure is structured considering the interval hesitancy degree along with membership functions and non-membership functions which is given below.

Definition 3.1. ***Jaccard distance measure on IVIFSs*** *For any IVIFSs* $\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2$ *on E, the proposed Jaccard distance on* $\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2$ *is*

$$d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) = 1-$$
$$\Big\{ \mid [\min(\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), \min(\mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [\max(\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)),$$
$$\max(\nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [\min(\pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), \max(\pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))] \mid \Big\} \Big\backslash$$
$$\Big\{ \mid [\max(\mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), \max(\mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \mu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [\min(\nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)),$$
$$\min(\nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \nu^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))], [\min(\pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{L}}_{\tilde{\mathrm{A}}_2}(x_i)), \max(\pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_1}(x_i), \pi^{\mathrm{U}}_{\tilde{\mathrm{A}}_2}(x_i))] \mid \Big\}.$$

Here mod defines the Euclidean distance from the set to the origin.

Proposition 3.1. *The Jaccard distance* $d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2)$ *satisfies the distance measure axioms:*

(i) $0 \leq d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) \leq 1$;
(ii) $d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) = 0 iff \tilde{\mathrm{A}}_1 = \tilde{\mathrm{A}}_2$;
(iii) $d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) = d_J(\tilde{\mathrm{A}}_2, \tilde{\mathrm{A}}_1)$;
(iv) $d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) = 0, d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_3) = 0 then d_J(\tilde{\mathrm{A}}_2, \tilde{\mathrm{A}}_3) = 0$ *for all* $\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2, \tilde{\mathrm{A}}_3$ *IVIFSs on* X.

The properties of distance measure are verified and the proofs are omitted.

Proposition 3.2. *The Jaccard distance between any two IVIFSs* $\tilde{\mathrm{A}}_1 and \tilde{\mathrm{A}}_2$ *is 0, if the Jaccard similarity of* $\tilde{\mathrm{A}}_1$ *and* $\tilde{\mathrm{A}}_2$ *is 1, i.e., if* $S_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) = 1$ *then* $d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2) = 0$.

Example 1. *Let us consider two IVIFSs* $\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2$ *as follows* $\tilde{\mathrm{A}}_1 = ([0.5, 0.6], [0.1, 0.3])$ *and* $\tilde{\mathrm{A}}_2 = ([0.35, 0.45], [0.2, 0.3])$. *Then the interval hesitancy degree of* $\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2$ *is given by* $\pi_{\tilde{\mathrm{A}}_1}(x_i) = [1-0.6-0.3, 1-0.5-0.1] = [0.1, 0.4]$ *and* $\pi_{\tilde{\mathrm{A}}_2}(x_i) = [1-0.45-$

$0.3, 1-0.35-0.2] = [0.25, 0.45]$. *The Jaccard distance of* $\tilde{A}_1, \tilde{A}_2$ *is*

$$d_J(\tilde{A}_1, \tilde{A}_2)$$
$$= 1 - \Big\{ \mid [\min(0.5, 0.35), \min(0.6, 0.45)], [\max(0.1, 0.2), \max(0.3, 0.3)],$$
$$[\min(0.1, 0.25), \max(0.4, 0.45)] \mid \Big\} \Big\backslash$$
$$\Big\{ \mid [\max(0.5, 0.35), \max(0.6, 0.45)], [\min(0.1, 0.2), \min(0.3, 0.3)],$$
$$[\min(0.1, 0.25), \max(0.4, 0.45)] \mid \Big\}$$
$$= 1 - \frac{\mid [0.35, 0.45], [0.2, 0.3], [0.1, 0.45] \mid}{\mid [0.5, 0.6], [0.1, 0.3], [0.1, 0.45] \mid}$$
$$= 1 - \frac{\sqrt{0.4168}}{\sqrt{0.5306}} = 1 - \frac{0.645}{0.728} = 1 - 0.886 = 0.113.$$

Therefore, $d_J(\tilde{A}_1, \tilde{A}_2) = 0.113$.

Definition 3.2. *(Ranking of IVIFSs) For any two IVIFSs* $\tilde{A}_1, \tilde{A}_2$ *the ranking is given as follows:*

(i) *If* $d_J(\tilde{A}_1, \tilde{I}) < d_J(\tilde{A}_2, \tilde{I})$ *then* $\tilde{A}_1 > \tilde{A}_2$.
(ii) *If* $d_J(\tilde{A}_1, \tilde{I}) > d_J(\tilde{A}_2, \tilde{I})$ *then* $\tilde{A}_1 < \tilde{A}_2$.
(iii) *If* $d_J(\tilde{A}_1, \tilde{I}) = d_J(\tilde{A}_2, \tilde{I})$ *then* $\tilde{A}_1 = \tilde{A}_2$.

Here $\tilde{I} = ([1,1],[0,0])$ *is the Ideal IVIFN and* $d_J(\tilde{A}_1, \tilde{I})$ *is given by*

$$d_J(\tilde{A}_1, \tilde{I}) = 1-$$
$$\Big\{ \mid [\min(\mu^L_{\tilde{A}_1}(x_i), \mu^L_{\tilde{I}}(x_i)), \min(\mu^U_{\tilde{A}_1}(x_i), \mu^U_{\tilde{I}}(x_i))], [\max(\nu^L_{\tilde{A}_1}(x_i), \nu^L_{\tilde{I}}(x_i)),$$
$$\max(\nu^U_{\tilde{A}_1}(x_i), \nu^U_{\tilde{I}}(x_i))], [\min(\pi^L_{\tilde{A}_1}(x_i), \pi^L_{\tilde{I}}(x_i)), \max(\pi^U_{\tilde{A}_1}(x_i), \pi^U_{\tilde{I}}(x_i))] \mid \Big\} \Big\backslash$$
$$\Big\{ \mid [\max(\mu^L_{\tilde{A}_1}(x_i), \mu^L_{\tilde{I}}(x_i)), \max(\mu^U_{\tilde{A}_1}(x_i), \mu^U_{\tilde{I}}(x_i))], [\min(\nu^L_{\tilde{A}_1}(x_i), \nu^L_{\tilde{I}}(x_i)),$$
$$\min(\nu^U_{\tilde{A}_1}(x_i), \nu^U_{\tilde{I}}(x_i))], [\min(\pi^L_{\tilde{A}_1}(x_i), \pi^L_{\tilde{I}}(x_i)), \max(\pi^U_{\tilde{A}_1}(x_i), \pi^U_{\tilde{I}}(x_i))] \mid \Big\}.$$

Therefore, the set with the lowest distance (close to 0) from $\tilde{I}$ *is the best alternative for the ideal solution.*

Example 2. *Let* $\tilde{P} = ([1,1],[0,0])$ *and* $\tilde{N} = ([[0,0],[1,1])$ *and* $\tilde{I} = ([1,1],[0,0])$ *is the Ideal IVIFN. By definition (2.2)* $\pi_{\tilde{P}}(x) = [0,0]$ *and* $\pi_{\tilde{N}}(x) = [0,0]$ *and* $\pi_{\tilde{I}}(x) = [0,0]$. *and by Definition(3.2)* $d_J(\tilde{P}, \tilde{I}) = 0$ *and* $d_J(\tilde{N}, \tilde{I}) = 1$. *Therefore,* $d_J(\tilde{P}, \tilde{I}) < d_J(\tilde{N}, \tilde{I})$. *Thus,* $\tilde{P} > \tilde{N}$.

This shows that the distance between two entirely similar sets is lowest, i.e., 0 and the distance between two entirely dissimilar sets is highest, i.e., 1 which is a trivial property that is followed by the distance measures and has been proved here.

Example 3. *Consider two IVIFSs* $\tilde{\mathrm{A}}_1, \tilde{\mathrm{A}}_2$ *in X as follows* $\tilde{\mathrm{A}}_1 =< x, ([0.5, 0.6], [0.1, 0.3]) >$ *and* $\tilde{\mathrm{A}}_2 =< x, ([0.6, 0.7], [0.05, 0.15]) >$*. Then* $\pi_{\tilde{\mathrm{A}}_1}(x) = [0.1, 0.4]$ *and* $\pi_{\tilde{\mathrm{A}}_2}(x) = [0.15, 0.35]$ *and* $\pi_{\tilde{\mathrm{I}}}(x) = [0, 0]$*. Also,* $d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{I}}) = 0.29$ *and* $d_J(\tilde{\mathrm{A}}_2, \tilde{\mathrm{I}}) = 0.21$*. Therefore,* $d_J(\tilde{\mathrm{A}}_1, \tilde{\mathrm{I}}) > d_J(\tilde{\mathrm{A}}_2, \tilde{\mathrm{I}})$*. Thus,* $\tilde{\mathrm{A}}_2 > \tilde{\mathrm{A}}_1$*.*

4. Comparative study

In this section, the comparative study is done with the maximum available methods. The proposed method is compared with 16 existing methods of ranking attained on various concepts such as; score functions, accuracy functions and distance measures. These methods are applied on 12 different IVIFSs covering all possibility of occurrence and the comparative study is given in Table 1.

$Ex1 : \tilde{A} = ([0.5, 0.6], [0.1, 0.3])$ and $\tilde{B} = ([0.6, 0.7], [0.05, 0.15])$
$Ex2 : \tilde{A} = ([0.35, 0.45], [0.2, 0.3])$ and $\tilde{B} = ([0.35, 0.45], [0.15, 0.35])$
$Ex3 : \tilde{A} = ([0.4, 0.5], [0.15, 0.3])$ and $\tilde{B} = ([0.45, 0.45], [0.2, 0.25])$
$Ex4 : \tilde{A} = ([0.3, 0.5], [0.1, 0.3])$ and $\tilde{B} = ([0.3, 0.5], [0.15, 0.25])$
$Ex5 : \tilde{A} = ([0.2, 0.8], [0.1, 0.2]))$ and $\tilde{B} = ([0.3, 0.7], [0, 0.3])$
$Ex6 : \tilde{A} = ([0.2, 0.2], [0.3, 0.4])$ and $\tilde{B} = ([0.2, 0.2], [0.35, 0.35])$
$Ex7 : \tilde{A} = ([0.1, 0.1], [0.1, 0.1])$ and $\tilde{B} = ([0.05, 0.15], [0.05, 0.15])$
$Ex8 : \tilde{A} = ([0.2, 0.3], [0.2, 0.3])$ and $\tilde{B} = ([0.2, 0.3], [0.3, 0.4])$
$Ex9 : \tilde{A} = ([0.18, 0.29], [0.25, 0.46])$ and $\tilde{B} = ([0.19, 0.33], [0.32, 0.37])$
$Ex10 : \tilde{A} = ([0.14, 0.24], [0.15, 0.46])$ and $\tilde{B} = ([0.1, 0.4], [0.2, 0.3])$
$Ex11 : \tilde{A} = ([0.35, 0.45], [0.2, 0.3])$ and $\tilde{B} = ([0.3, 0.5], [0.15, 0.35])$
$Ex12 : \tilde{A} = ([0.2, 0.4], [0, 0])$ and $\tilde{B} = ([0.375, 0.4], [0.3, 0.4])$

Table 1: Table description

S.No	Methods	Ex1	Ex2	Ex3	Ex4	Ex5	Ex6	Ex7	Ex8	Ex9	Ex10	Ex11	Ex12
1	Xu[11]	$\tilde{A} < \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} > \tilde{B}$
2	Ye[20]	$\tilde{A} < \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} < \tilde{B}$
3	Wang [24]	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} = \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B}$

4	Lee [28]	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$
5	Nayagam [21]	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$
6	Chen[25]	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$
7	Bai[22]	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$
8	Sivaraman [3]	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$
9	Sahin[18]	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$
10	Chen[27]	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$
11	Joshi[26]	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$
12	$XuED^*$[10]	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}=\tilde{B}$
13	$XuHD^*$[10]	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$
14	$XuHH^*$[10]	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}<\tilde{B}$
15	Zhang[19]	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$
16	Liu[12]	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}=\tilde{B}$
12	Proposed	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$

*ED- Euclidean distance; *HD- Hamming distance; *HH- Hausdorff Hamming distance

Through this study it is observed that the proposed ranking is strictly ordering the IVIFSs in all the tested cases. Analyzing separately based on the concepts the rankings are defined, the following results are observed.

The comparative analysis with existing rakings defined by various distance measures [10, 12, 19] illustrates that the proposed ranking is more effective to Xu HD [10], Xu HH [10], Liu [12] and Zhang dist [19] in several cases and is almost coinciding with Xu ED [10]. While in the case of Ex. 12, it is observed that the proposed method is giving better result than Xu ED [10] as the proposed distance measure considers interval hesitancy degree.

The analysis with other stated methods [3, 11, 18, 20–22, 24–28] show that in many tested cases these methods are unable to order the IVIFSs. In contrast, the proposed distance measure is effectively ordering in those cases. Moreover, the proposed distance measure gives the amount of dissimilarity between the sets as it is defined based on similarity measure, where as the other distances measures only the distance between the sets.

5. CONCLUSION

In this paper, IVIFSs are ranked using a new distance measure-Jaccard. The key advantage of the proposed method is measuring the amount of dissimilarity from the preferred ideal solution and taking interval hesitancy degree into consideration while ranking. Hence it reduces the information loss while ranking. The supremacy of the proposed ranking approach is demonstrated by tanking various possible cases on IVIFSs. It is observed the proposed method helps in strictly ranking the IVIFSs. Hence, the proposed procedure effectively solves the decision making problems therefore can be widely applied.

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