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PRODUCTION PROBLEM WITH THE CONCEPTION OF TASK (JOB) BLOCK CRITERIA

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ABSTRACT. The following research study is an attempt at finding the solution regarding the ever present complication of scheduling of n tasks being prepared on the machines with a special focus on preparing time consisting of the shipping time as well as arbitrary lags. These lags include both or any of the start lag and stop lag. The main aim lies in finding an optimal order such that make span could be minimized. To support the conceptual viewpoint an illustrative example with numerical data entries has also been included.

1. INRODUCTION

Flow shop scheduling is an integral problems with every big or small organisation. No wonder it finds its applicability in industrial sector, the most. The essence of scheduling algorithms to reduce the total production time of tasks. Scheduling of operations is very difficult in itself. However without considering the important and practically fundamental are one of the widest known optimization techniques. The essence of scheduling algorithm is to reduce the total production time of tasks. Scheduling of operations is very difficult issues

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in itself. However without considering the important and practically fundamental aspects like shipping time, total elapsed time, arbitrary lags, job blocks etc. It finds its use in very limited situations. Thus by addition of such factors, the problem discussed here is much better adapted to real world scenarios. In general, an n job- m machine scheduling problem has [(n!).(m!)] possible outcome. Such a problem does not leave any space for a pen and paper solution. However by staying in the boundaries and limiting the number of machines to 'three' the study has been conducted. Hence for 3 - stage flow shop scheduling complication with considerable shipping times and arbitrary lags has been formulated and solved for the purpose of using it in the multiple organizations. The theory of job block is another important addition in this study. First of all in the field of scheduling introduced an algorithm by S.M. Johnson [1] taking a scheduling problem in this problem n tasks are prepared on two machines. Mitten [2]treated a problem with the concept of time lags. Maggu and Das [3] established equivalent job for job – blocks theorem for 2 stage problem. The conception of shipping (transportation) time is very crucial in flow shop scheduling problem when the machines are distantly placed. Singh. T.P [4] applied the conception of shipping time in scheduling. Gupta, D. and Singh, T.P. [5] worked on nx2 production problem in which processing time are correlated with their probabilities and set up time are examined. Singh, T.P. and Gupta, D.[6] classified scheduling problem in which n tasks are prepared on 3 machines. In it processing time of tasks and set up time of machines are correlated with probabilities including task (job) block.Gupta, D. and Kumar, R. [7] discribe the problem in which n tasks are processing on two machines and set up time are examined separately from preparing time. Gupta, D and Singla, P.[8] discussed the scheduling problem with arbitrary time lags. Here we boost the study of Gupta, D. and Singla, P. [8] by taking the concept of equivalent task for task (job) blocks.

Assumptions

1. Each machine is examined to be regularly available for the duty of tasks.
2. tasks are separate from each other.
3. At a time each machine can't be hold more than one task.

Notations

S: Sequence of tasks 1, 2, 3, ...,n.
M_j: Machine j, j= 1, 2, ...,m.
A_i: Processing time of i^{th} task on M_1.
B_i: Processing time of i^{th} task on M_2.
C_i: Processing time of i^{th} task on M_3.
$A\alpha_i$: Expected processing time of i^{th} task on M_1.
$B\alpha_i$: Expected processing time of i^{th} task on M_2.
$C\alpha_i$: Expected processing time of i^{th} task on M_3.
p_i: Probability correlated to the processing time A_i.
q_i: Probability correlated to the processing time B_i.
r_i: Probability correlated to the processing time C_i.
D_{i1}: Start lag of i^{th} task from $M_1 \to M_2$.
E_{i1}: Stop lag of i^{th} task from $M_1 \to M_2$.
D_{i2}: Start lag of i^{th} task from $M_2 \to M_3$.
E_{i2}: Stop lag of i^{th} task from $M_2 \to M_3$.
t_i: Shipping time of i^{th} task from $M_1 \to M_2$.
g_i: Shipping time of i^{th} task from $M_2 \to M_3$.
t_i': Effective Shipping time of i^{th} task from $M_1 \to M_2$.
g_i': Effective Shipping time of i^{th} task from $M_2 \to M_3$.

Problem formulation

In this problem n - tasks are processed on three machines with processing time correlated with their respective probabilities, shipping time and arbitrary time lags of tasks given below in table 1.

Obtained an optimal algorithm of tasks to minimize the make span by taking (k,m) as a task (job) block.

ALGORITHM

Step1: Describe expected processing time $A\alpha_i$, $B\alpha_i$ and $C\alpha_i$ on machine M_1, M_2 and M_3 respectively as below:

1. $A\alpha_i = A_i \times p_i$
2. $B\alpha_i = B_i \times q_i$
3. $C\alpha_i = C_i \times r_i$

TABLE 1.

Tasks	Machine M_1		Shipping Time	Machine M_2		Shipping Time	Machine M_3		*Start* lag		*Stop* lag	
I	A_i	p_i	t_i	B_i	q_i	g_i	C_i	r_i	D_{i1}	D_{i2}	E_{i1}	E_{i2}
1.	A_1	p_1	t_1	B_1	q_1	g_1	C_1	r_1	D_{11}	D_{12}	E_{11}	E_{12}
2.	A_2	p_2	t_2	B_2	q_2	g_2	C_2	r_2	D_{21}	D_{22}	E_{21}	E_{22}
3.	A_3	p_3	t_3	B_3	q_3	g_3	C_3	r_3	D_{31}	D_{32}	E_{31}	E_{32}
....	....	....	....	....	....	....	....	....	....	....	....	....
n.	A_n	p_n	t_n	B_n	q_n	g_n	C_n	r_n	D_{n1}	D_{n2}	E_{n1}	E_{n2}

Step 2: Describe effective shipping time t_i' and g_i' of i^{th} task from machine$M_1 \rightarrow M_2$. and machine $M_2 \rightarrow M_3$. respectively as follows:

1. $t_i' = max(D_{i1} - A\alpha_i, E_{i1} - B\alpha_i, t_i)$
2. $g_i' = max(D_{i2} - B\alpha_i, E_{i2} - C\alpha_i, g_i)$

Step 3: Calculate processing time by producing two factitious machines G and H with their processing time G_i and H_i respectively as follows:
$G_i = |A\alpha_i + B\alpha_i + t_i' + g_i'|$ and $H_i = |B\alpha_i + C\alpha_i + t_i' + g_i'|$
if, either min$(A\alpha_i + t_i') \geq max(B\alpha_i + t_i')$ or min$(C\alpha_i + g_i') \geq max(B\alpha_i + g_i')$. Or both satisfied the conditions.
Step 4: Describe expected processing time for the equivalent task (job) block β=(k,m) on factitious machine G and H as follow.

G_β = G_k + G_m – min (H_k, G_m) and H_β = H_k + H_m – min (H_k, G_m).

Step 5: Use Johnson's (1954) rule to attained an optimal algorithm of tasks for the new described problem in step 4.
Step 6: Framing the in - out table for the sequence attained in step 5.

2. NUMERIC EXAMPLE

Examine the 5 - tasks and three machines problem with processing time correlated with their respective probabilities, shipping time and arbitrary lags of tasks given below in table 2.

Obtained an optimal algorithm of tasks to minimize the make span by taking (2,4) as a task (job) block.

TABLE 2.

Tasks	Machine M_1		Shipping Time	Machine M_2		Shipping Time	Machine M_3		*Start* lag		*Stop* lag	
I	A_i	p_i	t_i	B_i	q_i	g_i	C_i	r_i	D_{i1}	D_{i2}	E_{i1}	E_{i2}
1.	45	.2	4	20	.3	3	20	.2	10	12	12	10
2.	20	.4	3	50	.1	2	10	.3	9	8	10	12
3.	60	.1	6	30	.1	2	70	.1	12	6	5	6
4.	20	.2	8	10	.2	3	20	.2	5	5	4	7
5.	70	.1	5	20	.3	4	15	.2	8	7	7	8

TABLE 3.

Tasks	Machine M_1	Shipping Time	Machine M_2	Shipping Time	Machine M_3	*Start* lag		*Stop* lag	
I	$A\alpha_i$	t_i	$B\alpha_i$	g_i	$C\alpha_i$	D_{i1}	D_{i2}	E_{i1}	E_{i2}
1.	9	4	6	3	4	10	12	12	10
2.	8	3	5	2	3	9	8	10	12
3.	6	6	3	2	7	12	6	5	6
4.	4	8	2	3	4	5	5	4	7
5.	7	5	6	4	3	8	7	7	8

Solution:

Step1: Describe expected processing time $A\alpha_i$, $B\alpha_i$ and $C\alpha_i$ on machine M_1, M_2 and M_3 respectively as below in table 3.

Step 2: Define effective shipping time t'_i and g'_i of i^{th} task from machine $M_1 \to M_2$ and machine $M_2 \to M_3$ respectively in table 4:

1. $t'_i = max(D_{i1} - A\alpha_i, E_{i1} - B\alpha_i, t_i)$
2. $g'_i = max(D_{i2} - B\alpha_i, E_{i2} - C\alpha_i, g_i)$.

Step 3: Now we go through the conditions which we discussed in the algorithm, and here min$(A\alpha_i + t'_i) \geq max(B\alpha_i + t'_i)$ condition is fulfilled. Soin table 5 we discover two factious machines G and H with their processing time G_i and H_i as follows:

$G_i = |A\alpha_i + B\alpha_i + t'_i + g'_i|$ and $H_i = |B\alpha_i + C\alpha_i + t'_i + g'_i|$.

Step 4: Adopting task block criteria β over job (2,4) as follows in table 6:

G_β = 27 + 17 – min (22, 17) = 27 and H_β = 22 + 17 – min (22,17) = 22.

TABLE 4.

Tasks	Machine M_1	Effective Shipping Time	Machine M_2	Effective Shipping Time	Machine M_3
I	$A\alpha_i$	t'_i	$B\alpha_i$	g'_i	$C\alpha_i$
1.	9	6	6	6	4
2.	8	5	5	9	3
3.	6	6	3	3	7
4.	4	8	2	3	4
5.	7	5	6	5	3

TABLE 5.

Tasks	Factious Machine G	Factious Machine H
I	G_i	H_i
1.	27	22
2.	27	22
3.	18	19
4.	17	17
5.	23	19

TABLE 6.

Tasks	Factious Machine G	Factious Machine H
I	G_i	H_i
1.	27	22
β.	27	22
3.	18	19
5.	23	19

Step5: Now by adopting Johnson's technique, optimal sequence is S = 3, β, 1, 5. or 3, 1, β, 5, i.e. 3, 2, 4, 1, 5. or 3, 1, 2, 4, 5.
Step 6: In table 7 and 8 we framing the in - out table for the sequence attained in step 5.

Or

Minimum total production time for the given complication is 53 units.

TABLE 7.

Tasks	Machine M_1	Effective Shipping Time	Machine M_2	Effective Shipping Time	Machine M_3
I	In-Out	t'_i	In-Out	g'_i	In-Out
3.	0 - 6	6	12 - 15	3	18 - 25
2.	6 - 14	5	19 - 24	9	33 - 36
4.	14 - 18	8	26 - 28	3	36 - 40
1.	18 - 27	6	33 - 39	6	45 - 49
5.	27 - 34	5	39 - 45	5	50 - 53

TABLE 8.

Tasks	Machine M_1	Effective Shipping Time	Machine M_2	Effective Shipping Time	Machine M_3
I	In-Out	t'_i	In-Out	g'_i	In-Out
3.	0 - 6	6	12 - 15	3	18 - 25
1.	6 - 15	6	21 - 27	9	33 - 36
2.	15 - 23	5	28 - 33	9	42 - 45
4.	23 - 27	8	35 - 37	3	45 - 48
5.	27 - 34	5	39 - 45	5	50 - 53

Remark 2.1. *This study deal with the production scheduling problem with the leading intention to reduce the total elapsed time of tasks. The work can be boosted by taking varied parameters like setup times, break-down interval, mean weightage time etc.*

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