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FUZZY RADIO RECIPROCAL LABELING NUMBER ON CERTAIN CYCLE FREE GRAPHS

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ABSTRACT. A new concept of fuzzy radio reciprocal labeling number are introduced. Fuzzy radio reciprocal labeling and fuzzy radio reciprocal labeling number for certain cycle free are newly defined. Every radio frequency and radio stations are fixed by the membership value [0,1]. Generalized formula has been determined for the Y-tree, star graph to fix membership value of the lines and points.

1. INTRODUCTION

Fix any structure of a fuzzy graph as a region like districts, state and etc. In the fuzzy reciprocal radio labeling, lines of the every pair of the points is the radio frequency as well as the points defined as radio stations.The membership value of the radio frequency and the radio stations are in certain sequence,If the radio frequency is very high were the difference of the radio stations is very small. Here the radio frequency is any two radio stations of $\mu-$distance [1-4].

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2. Preliminaries

Definition 2.1. *A fuzzy graph $G = (V, \sigma, \mu)$ of a nonempty set V together with a pair of function $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$, it satisfies the following condition $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ [2].*

Definition 2.2. *A graph $G = (V, \sigma, \mu)$ is said to be a fuzzy labeling graph, if $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ is bijective such that the membership value of lines and nodes are specific such that $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ and for all $u, v \in V$ [3].*

Definition 2.3. *If ρ is the path with vertices $x_1, x_2, \ldots, x_n$ in a fuzzy graph G,then the $\mu-$ length fuzzy graph $l(\rho)$ is defined for two points x, y in G for the $\mu-$ length of all paths joining x and y. The μ - distance $\delta(u, v)$ is the smallest $\mu-$length of any $u - v$ path and δ is metric [4,6].*

Definition 2.4. *A radio labeling of G is an assignment of positive integers to the nodes of G satisfying*

$$d(u, v) + |f(u) - f(v)| \leq 1 + diam(G) \tag{2.1}$$

for every two specific nodes u and v. The maximum integer in the range of the labeling is its span The radio number of G, $rn(G)$, is the minimum possible span of any radio labeling for G and the diameter of the graph G,$diam(G)$ is the distance between the points u and v [5].

Definition 2.5. *A Y-tree Y_{n+1} is a graph obtained from path P_n by appending a pendent line to the vertex adjacent to an end point of the path [8].*

Definition 2.6. *A star S_k is the complete bipartite graph $K_{1,n}$, a tree with one internal node and k leaves [8].*

3. Methodology

Definition 3.1. *A fuzzy graph $G = (u, v, \gamma, \tau)$ is a non-empty set V together with a pair of functions $\tau : V \to [0, 1]$ and $\gamma : E \to [0, 1]$, is said to be a fuzzy radio reciprocal labeling, then the following condition satisfy*

$$\delta(u, v) + \frac{1}{\mid \tau(u) - \tau(v) \mid} \geq diam(G), \tag{3.1}$$

where $\delta(u,v)$ is μ -distance of a fuzzy graph and $diam(G)$ is the maximum of μ-distance of every pair of the points. Fuzzy radio reciprocal labeling is denoted by FR_L^{-1}. The new concept is introduced that is Fuzzy radio reciprocal labeling number is the highest membership value of the fuzzy radio reciprocal labeling of fuzzy graph is denoted by $FR_L^{-1}(n)$ [7].

Definition 3.2. *A fuzzy star graph $k_{1,n} = (V, \sigma, \mu)$ of a nonempty set V together with a pair of function $\sigma : V \to [0,1]$ and $\mu : E \to [0,1]$ if $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$, denoted by $Fk_{1,n}$.*

Definition 3.3. *A fuzzy Ytree $G = (V, \sigma, \mu)$ of a nonempty set V together with a pair of function $\sigma : V \to [0,1]$ and $\mu : E \to [0,1]$ if $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ denoted by FY_n.*

4. RESULT

Lemma 4.1. *For any fuzzy radio reciprocal labeling graph, if the difference of point membership value is very small then sum of the membership value of lines is very high in the same pair of points.*

Lemma 4.2. *For any fuzzy radio reciprocal labeling graph, if the fuzzy radio reciprocal labeling number exceeds from their k - decimals then all the $m(t)$ and $m(s)$ are in k- decimals.*

Theorem 4.1. *Every star graph $K_{1,n}$ admits FR_L^{-1}.*

Proof. Let $K_{1,n}$ be the $n-$ dimensional star graph which has the set of points $\{s_1, s_2, \ldots, s_{n+1}\}$ the set of lines $\{t_1, t_2, \ldots, t_n\}$, there is only one way of path to every pair of the points $\{s_i, s_j\}$of $K_{1,n}$. so any path with pair of vertices are connected by lines and maximum possible lines between any pair of points is $1 \leq| T |\leq 2$, where $m(t)$ is defined

$$\gamma(t_i) = \frac{n+i-3}{10^k}. \tag{4.1}$$

Here k is by defined lemma 2.

Every path is connected by lines and the maximum possible points of the path is $2 \leq| T |\leq 3$, where $m(s)$ is defined

$$\tau(s_i) = \frac{n+i+(|T|-3)}{10^k}. \tag{4.2}$$

Here k is by defined lemma 2. Fix the membership of lines and points in certain order by equations (4.1), (4.2) which satisfies fuzzy labeling as well as fuzzy graph, i.e.,

$$\gamma(s_i, s_j) < \tau(s_i) \wedge \tau(s_j), \tag{4.3}$$

$$\gamma(s_i, s_j) \leq \tau(s_i) \wedge \tau(s_j), \tag{4.4}$$

where $i = 1, 2, 3, \ldots, n$ and $j = 1, 2, 3, \ldots, n$z.

To prove: Adding the reciprocal difference of any two pair of points and μ-distance of the same points need to be greater then or equal to the diameter of $k_{1,n}$.

Fix the membership to any vertex which is adjacent to the maximum number of adjacent points of $k_{1,n}$ as follows, if the total number of points is odd, then mid value of the point is fixed, if the total number of points is even then any one set of mid value of the point is fixed.

The μ - distance of the graph always differs by allocating lines by using equation (4.1) which minimize the value of $diam(k_{1,n})$ as well as the value of the $|\tau(s_i) - \tau(s_j)|$ differs by using equation (4.2) which not decides the value of $diam(k_{1,n})$.

In the case of one or two lines in the path then $m(t)$ or the sum of $m(t)$ is greater than $|\tau(s_i) - \tau(s_j)|$. Moreover $10 < \mu - distance < 10^k$, where k is defined by lemma 2. The adding reciprocal of $|\tau(s_i) - \tau(s_j)|$ and μ-distance of any two pair of (s_i, s_j) is always grater then or equal to diameter of $k_{1,n}$. □

Algorithm 4.1. *Fuzzy radio reciprocal labeling on star graph $K_{1,n}$*

Step 1: *Fix the $m(t)$ to the lines in certain sequence of $(t_1, t_2, t_3, \ldots, t_n)$, $t_i = \frac{n+i-3}{10^k}$, $n \geq 3$, k is fixed by number of decimals.*

Step 2: *Fix the m(s) to the points in certain sequence of $(s_1, s_2, s_3, \ldots, s_{n+1})$, $s_i = \frac{n+(|T|-3)+i}{10^k}$, $n \geq 3$, k is fixed by number of decimals.*

Step 3: *Find $\mu-$distance for every pair of points $\delta(v_i, v_j)$ by step 1.*

Step 4: *Find $diam(G)$, by using step 3.*

Step 5: *Adding $\delta(v_i, v_j)$ and reciprocal of $|\tau(a_i) - \tau(a_j)|$, if $\delta(v_i, v_j) + \frac{1}{|\tau(a_i)-\tau(a_j)|} \geq diam(K_{1,n})$ the go to step 6. if not go to step 1.*

Step 6: *The highest membership value is $FR_L^{-1}(n)$.*

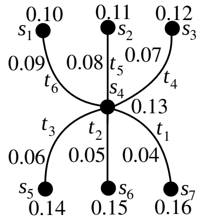

FIGURE 1.

$$\delta(s_i, s_j) + \frac{1}{|\tau(s_i) - \tau(s_j)|} \geq diam(k_{1,n})$$
$$\delta(s_1, s_7) + \frac{1}{|\tau(s_1) - \tau(s_7)|} \geq diam(k_{1,6})$$
$$31.11 + \frac{1}{0.06} \geq 45$$
$$47.78 \geq 45$$

Theorem 4.2. *Any $Y-$tree satisfies fuzzy radio reciprocal labeling.*

Proof. Let Y_{n+1} be the $n-$ dimensional tree which has the set of points $\{s_1, s_2, \ldots, s_{n+1}\}$ and the set of lines $\{t_1, t_2, \ldots, t_n\}$,then there is only one path to every pair of points $\{s_i, s_j\}$ of Y_{n+1}. Any pair of every path are connected by lines and the maximum possible lines between any pair of points is $1 \leq |T| \leq t$. where t is the maximum number of line in the path of any two points.

Define, $m(t)$ is

$$\gamma(t_i) = \frac{n+i-2}{10^k}, \tag{4.5}$$

where k is by defined lemma 2.

Every path is connected by edges and maximum possible vertices of the path is $2 \leq |A| \leq t+1$.

Define, $m(s)$ is

$$\tau(s_i) = \frac{n+i+(|T|-2)}{10^k}, \tag{4.6}$$

where k is by defined lemma 2. Fix the membership of lines and points in certain order by equation (4.5) and (4.6),it satisfies fuzzy labeling by as well as fuzzy graph that is

$$\gamma(s_i, s_j) < min\{\tau(a_i), \tau(a_j)\} \tag{4.7}$$

$$\gamma(s_i, s_j) \leq min\{\tau(a_i), \tau(a_j), \} \tag{4.8}$$

where $i = 1, 2, 3, \ldots, n$ and $j = 1, 2, 3, \ldots, n$.

To prove: $\delta(s_i, s_j) + \frac{1}{|\tau(s_i) - \tau(s_j)|} \geq diam(Y_{n+1})$. Fix the membership function for both lines and points by equation (4.5) and (4.6). The sequence of the points is $\{s_1, s_2, \ldots, s_{n+1}\}$, where there is only one pendent line from the set $\{t_1, t_2, \ldots, t_n\}$ and which is the highest value of of $m(t)$ of Y_{n+1} then continuously fix $m(s)$ to the every point of Y - tree, sequence of second and third points connected by only pendent lines,next to an end points of the path in Y - tree.

case i): if there is only one lines in the path then $m(t)$ is also greater than $|\tau(s_i) - \tau(s_j)|$.

case ii): if the length of the path is an increases then $|\tau(s_i) - \tau(s_j)|$ is also increase (i,e) μ-distance is greater then $|\tau(s_i) - \tau(s_j)|$.

For μ-distance of $k_{1,n}$ is lies between $10 < \mu - distance < 10^k$. where k is defined by lemma 2.Adding reciprocal of $|\tau(s_i) - \tau(s_j)|$ and μ-distance of any two pair of (s_i, s_j) is always greater than or equal to diameter of Y_{n+1}. Hence $\delta(s_i, s_j) + \frac{1}{|\tau(s_i) - \tau(s_j)|} \geq diam(Y_{n+1})$. □

Algorithm 4.2. *Fuzzy radio reciprocal labeling number on Y-tree (Y_{n+1}).*

Step 1: *Fix $m(t)$, to the lines in certain sequence of $(t_1, t_2, t_3, \ldots, t_n)$ by $t_i = \frac{n+i-2}{10^k}$, $n \geq 3$, k is fixed by number of decimals.*

Step 2: *Fix $m(s)$, to the points in certain sequence of $(s_1, s_2, s_3, \ldots, s_{n+1})$ by $s_i = \frac{n+(|T|-2)+i}{10^k}$, $n \geq 3$, k is fixed by number of decimals.*

Step 3: *Find $\mu-$ distance for every pair of points $\delta(v_i, v_j)$.*

Step 4: *Find $diam(Y_{n+1})$, by using step 3*

Step 5: *Adding $\delta(v_i, v_j)$ and reciprocal of $|\tau(a_i) - \tau(a_j)|$ is greater than or equal to $diam(Y_{n+1})$,then go to step 6. if not then go to step 1.*

Step 6: *The highest membership value is $FR_L^{-1}(n)$.*

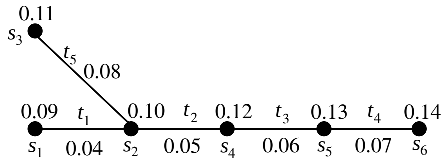

FIGURE 2.

For example, let us take any pair points is (s_1, s_4), $\delta(s_1, s_4) = 45$, $diam(Y_n) = 75.96$ and $|\tau(s_1) - \tau(s_4)| = 0.03$.

$$\begin{aligned}
\delta(s_i, s_j) + \frac{1}{|\tau(s_i) - \tau(s_j)|} &\geq diam(Y_{n+1}) \\
\delta(s_1, s_4) + \frac{1}{|\tau(s_1) - \tau(s_4)|} &\geq diam(Y_5) \\
45 + \frac{1}{0.03} &\geq 75.96 \\
78.33 &\geq 75.96
\end{aligned}$$

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