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ROUGH FUZZY GRAPH CELLULAR AUTOMATON

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ABSTRACT. Combining the concept of graph cellular automaton and rough set theory, we introduce a new class of automaton rough graph cellular automaton to broaden the scope of this two complex systems. The generations of rough graph cellular automaton also predicted using basic linear rules of cellular automaton. This concept of rough graph cellular automaton is extended to define rough fuzzy graph cellular automaton whose generations are also predicted. We covered the defined concept with some suitable examples. Overall our study revealed another core of cellular automaton in different manner.

1. INTRODUCTION

Over the past 10 years, the development of cellular automaton attracted the researchers. Rough set theory and graph theory is another two important complex systems and the attention of several researchers turned towards the two dynamical systems. The journey of cellular automaton started in the year 1950 by John Von Neumann. John Conway developed the concept 'Game of Life' [1] which influenced the research field of CA. Graph cellular automaton was started

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to develop in the year of 2001. David O'sullivan defined a model graph cellular automaton[2].In 2017, Krzysztof Malecki developed the concept of relation based neighbourhoods of cells by using graph cellular automaton[3].In 2014, Bartlomiej Placzek make the use of rule identification and neighbourhood selection to reduce a set of decision rules[4].The development of CA will increase rapidly in the upcoming years. V. Poupet defined the 3D cellular automaton with the help of 2D automaton [5]. The application of graph cellular automaton in generating languages is discussed by B. Praba and Saranya. R by using the time evolution function [6,7,8].Section 2 deals with some of the basic definitions which is required to study the rest of the sections, In Section 3 we defined the concept of Rough Graph Cellular Automaton with the aid of information system, Section 4 we defined Rough Fuzzy Graph Cellular Automaton followed by a conclusion.

2. Preliminaries

In this section we discuss the basic definitions which is required to study the rest of the sections.

Definition 2.1. *For any set $X \subseteq U$, the lower approximation is defined by $P_{-}(X) = \{x \in U | [X]_P \subseteq X\}$ and the upper approximation $P^{-}(X) = \{x \in U | [X]_P \cap X \neq \emptyset\}$. The rough set corresponding to X in the approximation space P, is defined by an ordered pair $RS(X) = (P_{-}(X), P^{-}(X))$* [7].

Definition 2.2. *I=(U,A,N) is an information system, where U is a nonempty finite set of objects and A is the nonempty finite set of fuzzy attributes. R is an indiscernibility equivalence relation on U and N is the finite set of decision variables $N = \{m_1, m_2, \ldots, m_r\}$ and $\forall m_i \in N$, $\mu_{m_i} : U \to [0,1]$ such that $\mu_{m_i}(x_i)$ also $\forall m_i \in N$, $\eta_{m_i} : U \to \mathcal{P}(U)$ is the neighbourhood function such that for any $x \in U, \eta_{m_i}(x)$ contains those elements of U, having almost the same opinion with respect to the decision m_i[7].$\eta_{m_i}(x) = \{y \in U | \ 0 \leq |\mu_{m_i}(x) - \mu_{m_i}(y)| \leq \delta_{m_i}\}$ $\forall m_i \in N, x \in U, \eta_{m_i}(x) \subseteq U$ and hence the corresponding Rough set is $RS(\eta_{m_i}(x)) = (\eta_{m_i}(x)_{-}, \eta_{m_i}(x)^{-})$ where, $\eta_{m_i}(x)_{-} = \{y \in U \mid [y]_p \subseteq \eta_{m_i}(x)\}$ and $\eta_{m_i}(x)^{-} = \{y \in U \mid [y]_p \cap \eta_{m_i}(x) \neq \phi\}$*

Definition 2.3. *Cellular automaton is a collection of cells which is in the form of grid. The colour of the grid indicated the life of each cell in the grid. Based on the neighbourhood cell a cell's life is assigned at any time.*

Rule 1, Rule 2, Rule 4, Rule 8 and Rule 16 are the fundamental basic rules. All the other rules can be obtained by using this 5 rules. Rule 32, Rule 64, Rule 128 and Rule 256 are also known as fundamental basic rules which was obtained with the transpose operation of the 1,2,4,8 and 16 respectively.

Definition 2.4. *Graph Cellular Automaton* (GCA) *is defined as* $G = (V, I, \delta)$ *where* V *is the non empty set of vertices/cells in the grid.* I *is the input symbol and* δ *is the time evolution function defined by* $\delta : V \times I \to V$.

3. Rough Graph Cellular Automaton

In this section we extend the concept of GCA to define rough graph cellular automaton $(RGCA)$. Also we find the generation of RGCA using basic linear fundamental rules[6].

Definition 3.1. *Let* $I_0 = \left(G, \tilde{R}, \eta\right)$ *where* G *is a graph cellular automaton* (V, I, δ) *and* $\tilde{R}$ *is an equivalence relation defined on the vertices of* G*. For each* $x \in I$*,*$\eta_x : V(G) \to P(V(G))$*. This* η_x *acts as a neighbourhood function with respect to the input symbol* x*.*

Definition 3.2. *Consider a graph information system* I_0*. For* $q_0 \in V$*,* $x \in I$*,* $\eta_x(q_0) \subseteq V$*, its corresponding roughset with respect to* R *is defined by,* $RS(\eta_x(q_0)) = \left(\eta_x(q_0)_-, \eta_x(q_0)^-\right)$ *where* $\eta_x(q_0)_- = \left\{y \in V | \, [y]_p \subseteq \eta_x(q_0)\right\}$ *and* $\eta_x(q)^- = \left\{y \in V | \, [y]_p \cap \eta_x(q_0) \neq \emptyset\right\}$*. We call* $RS(\eta_x(q_0))$ *as the rough set induced by* I_0*. Note that a graph information system basically consists of a GCA, equipped with an equivalence relation on its set of vertices together with the neighbourhood function defined for each of the input symbol.Let* $\eta_- = \left\{\eta_x(q_0)_- | x \in I, q_0 \in V\right\}$ *and* $\eta^- = \left\{\eta_x(q_0)^- | x \in I, q_0 \in V\right\}$*. Using this* η_- *and* η^-*, the following definition can be obtained.*

Definition 3.3. *Let* I_0 *is a graph information system.Rough Graph Cellular Automaton* $(RGCA)$ *corresponding to* I_0 *is defined by an ordered pair* $G(I_0)$

$=\left(G\left(I_0\right)_{-}, G\left(I_0\right)^{-}\right)$ *where* $G\left(I_0\right)_{-}$ *is the lower approximation rough graph cellular automaton defined by* $G\left(I_0\right)_{-} = \{V, I, \delta_{-}\}$ *where* V *is the set of vertices in* G*,* I *is the non empty set of input symbol on* G *and* δ_{-} *is the transition function defined by* η_{-} *and* $G\left(I_0\right)^{-}$*is the upper approximation rough graph cellular automaton defined by* $G\left(I_0\right)^{-} = \{V, I, \delta^{-}\}$ *where* V *is the set of vertices in* G*,* I *is the non empty set of input symbol on* G *and* δ^{-} *is the transition function defined by* η^{-}*.This can be used to generate the lower approximation rough graph cellular automaton* $\left(G\left(I_0\right)_{-}\right)$ *and the upper approximation rough graph cellular automaton* $\left(G\left(I_0\right)^{-}\right)$*.*

Example 1. *Consider the following graph cellular automaton,*

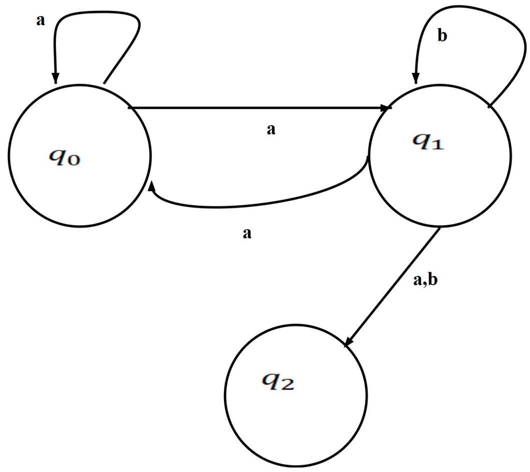

FIGURE 1. GCA $F\left(G_0\right)$ with input symbols $I = \{a, b\}$

In the graph cellular automaton, $V = \{q_0, q_1, q_2\}$*,*$I = \{a, b\}$*. The corresponding adjacency matrix is* $A\left(G\right) = \begin{bmatrix} a & a & 0 \\ a & b & a \\ 0 & 0 & a,b \end{bmatrix}$*;* $I_0 = \left(G, \tilde{R}, \eta\right)$ *is a graph information system with input symbol where the equivalence classes induced by the neighbourhood function are* $X_1 = \{q_0, q_1\}$ *and* $X_2 = \{q_2\}$*. To calculate the neighbourhood function for* η_a *and* η_b*. Let us adopt the following procedure. For* $a \in I$*, let* $\mu_a\left(q_0\right) =$ *num. of incoming edges with input symbol* $a to q_0$*+num. of outgoing edges with input symbol* $a to q_0$*,*$\mu_a\left(q_0\right) = 1$*, similarly we calculate the value which is given in the following table.*

TABLE 1. Table description

V	μ_a	μ_b	η_a	η_b
q_0	1	0	q_0, q_1, q_2	q_0
q_1	1	1	q_0, q_1, q_2	q_1, q_2
q_2	2	1	q_0, q_1, q_2	q_1, q_2

Now we define the function η_a by $\eta_a : V(G) \to P(V(G))$ defined by $\eta_a(q_0) = \{q \in V | \mu_a(q_0) - \mu_a(q) \leq \delta\}$ where δ is a threshold. On taking $\delta_a = 1$ and $\delta_b = 0.5$, η_a and $\eta_b$is given in table 1.$(table2)$. Therefore $\eta_- = \{X_1 \cup X_2, U, U\}$ and $\eta^- = \{X_1 \cup X_2,)U, U\}$.

Hence $RS(\eta_a(q_0)) = \{X_1 \cup X_2, X_1 \cup X_2\}$, $RS(\eta_a(q_1)) = \{U, U\}$ and so on. Then the lower approximation rough graph cellular automaton and the upper approximation rough graph cellular automaton are shown in fig 2. and fig. 3 respectively.

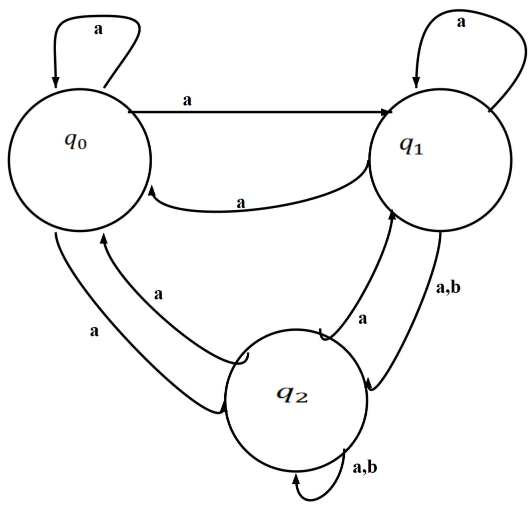

FIGURE 2. Lower Approximation Rough Graph Cellular Automaton $(G(I_0)_-)$

Hence $A(G(I_0)_-) = \begin{bmatrix} a & a & a \\ a & a & a,b \\ a & a & a,b \end{bmatrix}$ *and* $A(G(I_0)^-) = \begin{bmatrix} a,b & a,b & a \\ a,b & a,b & a,b \\ a,b & a,b & a,b \end{bmatrix}$.

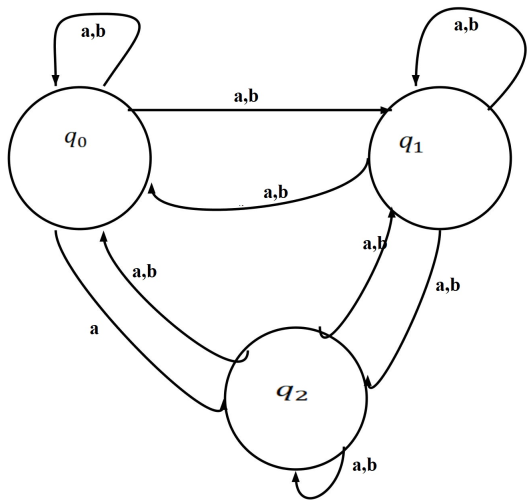

FIGURE 3. Upper Approximation Rough Graph Cellular Automaton $\left(G\left(I_0\right)^{-}\right)$

4. GENERATIONS ROUGH GRAPH CELLULAR AUTOMATON

In this section we predict the generation of rough graph cellular automaton using basic linear rules.

Definition 4.1. *K^{th} generation rough graph cellular automaton which is defined by $G_k\left(I_0\right)=\left(G_{k-1}\left(I_0\right)_{-}, G_{k-1}\left(I_0\right)^{-}, R\right)$ where, K^{th} generation lower approximation rough graph cellular automaton is $G_k\left(I_0\right)_{-}=\left(G_{k-1}\left(I_0\right)_{-}, R\right)$ defined by $A\left(G_k\left(I_0\right)_{-}\right)=R \circ A\left(G_{k-1}\left(I_0\right)_{-}\right)$ where $A\left(G_{k-1}\left(I_0\right)_{-}\right)$ is the $(k-1)^{th}$ generation of lower approximation rough graph cellular automaton and $A\left(G_k\left(I_0\right)_{-}\right)$ is the k^{th} generation lower approximation rough graph cellular automaton, R is the rule matrix. Similarly to find the k^{th} generation upper approximation automaton is defined by $G_k\left(I_0\right)^{-}=\left(G_{k-1}\left(I_0\right)^{-}, R\right)$ and it is calculate by the adjacency matrix rule. The RGCA is said to be definable at time t if $G_t\left(I_0\right)_{-}=G_t\left(I_0\right)^{-}$.*

Example 2. *For the above lower approximation rough graph cellular automaton $\left(G\left(I_0\right)_{-}\right)(figure.2)$, by using rule 4 of cellular automaton the next generation of figure 2 is obtained using the following way.*

$A\left(G_1\left(I_0\right)_-\right) = M_4 \circ A\left(G_0\left(I_0\right)_-\right).$

$$\begin{bmatrix} 0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{bmatrix} \begin{bmatrix} a\\ a\\ a\\ a\\ a\\ a,b\\ a\\ a\\ a,b \end{bmatrix} = \begin{bmatrix} a\\ a,b\\ 0\\ a\\ a\\ 0\\ 0\\ 0\\ 0 \end{bmatrix}.$$

Hence $A\left(G_1\left(I_0\right)_-\right) = \begin{bmatrix} a & a,b & 0\\ a & a & 0\\ 0 & 0 & 0 \end{bmatrix}.$

Similarly for upper approximation rough graph cellular automaton,

$$A\left(G_1\left(I_0\right)^-\right) = \begin{bmatrix} a,b & a,b & 0\\ a,b & a,b & 0\\ 0 & 0 & 0 \end{bmatrix},$$

and so on. By repeatedly applying rule 4 the generations of RGCA can be obtained and it can be noted that $A\left(G_4\left(I_0\right)_-\right) = A\left(G_4\left(I_0\right)^-\right)$*. Hence G is definable at time* $t = 4$.

5. Rough Fuzzy Graph Cellular Automaton $(RFGCA)$

In this section we consider a fuzzy graph $F = \left(V, I, \tilde{\delta}\right)$ where V is the non empty set of vertices, I is the set of input symbols and $\tilde{\delta}$ is the fuzzy transition function defined by $\tilde{\delta} : V \times V \times I \to [0,1]$ which represents the grade of membership of the edges (ie). $\tilde{\delta}\left(q_0, q_1, a\right)$ represents the grade of membership of the edge from q_0 to q_1. Let F be a fuzzy graph. Then we define a fuzzy set μ on the set of verties V as follows $\mu_x : V \to [0,1]$ defined by a fuzzy set $\mu_x\left(V\right) = max_{V_i}\left(\tilde{\delta}\left(V, V_i, x\right)\right)$. Note that a grade of membership of vertex V.

Definition 5.1. *Consider a fuzzy graph* F*, then the fuzzy graph information system is defined by* $\tilde{I}_0 = \left(F, \tilde{R}, \tilde{\eta}\right)$ *where* F *is a graph defined by* $\left(V, I, \tilde{\delta}\right)$*,* $\tilde{R}$ *is an equivalence relation defined on the vertices of F. For each* $x \in I$*,*$\eta_x : V\left(F\right) \to$

$\mathcal{P}(V(F))$. *This* η_x *acts as a neighbourhood function with respect to the input symbol* x *and the neighbourhood function is defined by*

$$\tilde{\eta}_x(V) = V_i \in V|\mu_x(V_i - \mu_x(V_j) \leq \delta_x)$$

where the fuzzy vertex membership value μ *is a mapping* $\mu : V \to [0,1]$ *and it is defined by* $\mu(V_i) = max_{V_j}(\mu(V_i, V_j))$. *Fuzzy graph information system induced by an input symbol with fuzzy vertex membership value is known as fuzzy graph cellular automaton* $\tilde{F}\left(\tilde{I}_0\right)$. *The corresponding fuzzy adjacency matrix for* $\tilde{F}\left(\tilde{I}_0\right)$ *fuzzy graph cellular automaton is denoted by* $\tilde{A}\left(\tilde{F}\left(\tilde{I}_0\right)\right)$.

Let $\tilde{I}_0$ *is a fuzzy graph information system and* $\tilde{F}$ *ia a fuzzy graph cellular automaton, then for* $q_0 \in V, x \in I$*,*$\eta_x(q_0) \subseteq V$*, its corresponding rough set with respect to R is defined by* $RS(\eta_x(q_0)) = \left(\eta_x(q_0)_-, \eta_x(q_0)^-\right)$*, where* $\eta_x(q_0)_- = \{y \in V|\, [y]_R \subseteq \eta_x(q_0)\}$ *and* $\eta_x(q_0)^- = \{y \in V|\, [y]_R \cap \eta_x(q_0) \neq \phi\}$. *We call* $RS(\eta_x(q_0))$ *as the rough set induced by* I_0. *Let* $\eta_- = \left\{\left(\eta_x(q_0)_-\right)|x \in I, q_0 \in V\right\}$ *and* $\eta^- = \left\{\left(\eta_x(q_0)^-\right)|x \in I, q_0 \in V\right\}$. *Using this* η_- *and* η^-*, the following definition can be obtained.*

Definition 5.2. *Let* $\tilde{I}_0 = \left(F, \tilde{R}, \tilde{\eta}\right)$ *is a fuzzy graph information system and* $\tilde{F}$ *is a fuzzy graph cellular automaton, then the Rough Fuzzy Graph Cellular Automaton is defined by and ordered pair* $\tilde{F}\left(\tilde{I}_0\right) = \left(\tilde{F}\left(\tilde{I}_0\right)_-, \tilde{F}\left(\tilde{I}_0\right)^-\right)$ *where* $\tilde{F}\left(\tilde{I}_0\right)_-$ *is a lower approximation rough fuzzy graph cellular automaton and* $\tilde{F}\left(\tilde{I}_0\right)^-$ *is an upper approximation rough fuzzy graph cellular automaton.* $\tilde{A}\left(\tilde{F}\left(\tilde{I}_0\right)_-\right)$ *is the fuzzy adjacency matrix of the lower approximation rough fuzzy graph cellular automaton and* $\tilde{A}\left(\tilde{F}\left(\tilde{I}_0\right)^-\right)$ *is the adjacency matrix of the upper approximation rough fuzzy graph cellular automaton.*

Example 3. *Consider the following FGCA* $(fig.5)$ *Where* $V = \{q_1, q_2 q_3\}$*,*$I = \{x, y\}$ *and* $\tilde{\delta}$ *is defined by* $\tilde{\delta}(q_1, x) = (q_1, q_2)/0.5$ *;*$\delta(q_3, x) = (q_3, q_1)/1 + (q_3, q_2)/0.3 + (q_3, q_3)/0.3$*;* $\delta(q_2, y) = (q_2, q_2)/0.2 + (q_2, q_3)/0.8$*;*$\delta(q_3, q_2)/0.5 + (q_3, q_3)/0.5$.
$\tilde{A}\left(\left(\tilde{F}\left(\tilde{I}_0\right)_x\right)\right) = \begin{bmatrix} 1 & 1 & 0.7 \\ 0 & 0 & 0.3 \\ 0 & 0 & 0 \end{bmatrix}$*;* $\tilde{A}\left(\left(\tilde{F}\left(\tilde{I}_0\right)_y\right)\right) = \begin{bmatrix} 0 & 0.5 & 0.2 \\ 0 & 0 & 0.2 \\ 0 & 0 & 0 \end{bmatrix}$*;* $\tilde{I}_0$ *is an fuzzy graph information system. The fuzzy vertex membership value is* $\mu_x(q_0) = \max\{1, 1, 0.7\} = 1$. *Similarly for the remaining* μ *value is given in table 3.*

TABLE 2. table description

V	μ_x	μ_y	η_a	η_b
q_1	1	0.5	q_1	q_1, q_2
q_2	0.3	0.2	q_2, q_3	q_1, q_2
q_3	0	0	q_2, q_3	q_2, q_3

Consider the equivalence classes are $X_1 = \{q_1, q_2\}$ *and* $X_2 = \{q_3\}$ *and* $RS(\eta_a(q_1)) = \{\emptyset, X_1\}$, $RS(\eta_a(q_2)) = \{X_2, U\}$*,and so on. Then the lower approximation Rough Fuzzy Graph Cellular Automaton and the upper approximation RFGCA are shown in fig4. and fig.5 respectively.*

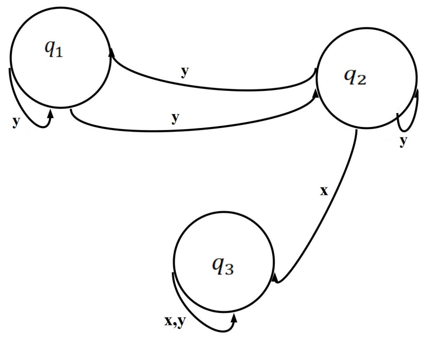

FIGURE 4. Lower Approximation Rough Graph Cellular Automaton $\tilde{F}\left(\tilde{I}_0\right)_-$

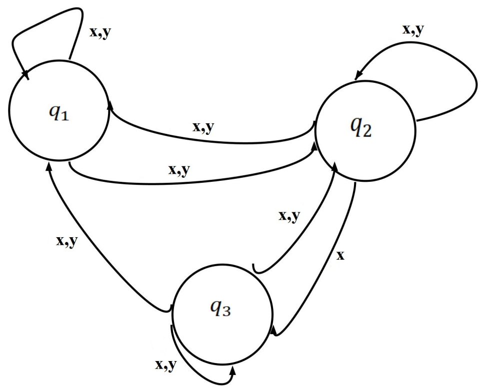

FIGURE 5. Upper Approximation Rough Graph Cellular Automaton $\tilde{F}\left(\tilde{I}_0\right)_-$

6. Generations of Rough Fuzzy Graph Cellular Automaton

In this section the generation of Rough Fuzzy Graph Cellular Automaton is predicted. As discussed above the k^{th} generation rough fuzzy graph cellular automaton is predicted using the basic linear rules of cellular automaton.

Definition 6.1. *K^{th} generation lower approximation rough fuzzy graph cellular automaton is defined by $\tilde{F}_k\left(\tilde{I}_0\right) = \left(\left(\tilde{F}_k\left(\tilde{I}_0\right)_-, \tilde{F}_k\left(\tilde{I}_0\right)^-\right), R\right)$ where k^{th} generation lower aproximation rough fuzzy graph cellular automaton is defined by $\tilde{F}_k\left(\tilde{I}_0\right)_- = \left(\tilde{F}_{k-1}\left(\tilde{I}_0\right)_-, R\right)$ it is calculated by*

$$\tilde{A}\left(\tilde{F}_k\left(\tilde{I}_0\right)_-\right) = R \circ \tilde{A}\left(\tilde{F}_{k-1}\left(\tilde{I}_0\right)_-\right),$$

where $\tilde{A}\left(\tilde{F}_{k-1}\left(\tilde{I}_0\right)_-\right)$ is the $(k-1)^{th}$ generation lower approximation rough graph cellular automaton and $\tilde{A}\left(\tilde{F}_k\left(\tilde{I}_0\right)_-\right)$ is the k^{th}generation lower approximation rough fuzzy graph cellular automaton, R is the rule matrix. Similarly, the k^{th} generation upper approximation rough fuzzy graph cellular automaton is defined by $\tilde{F}_k\left(\tilde{I}_0\right)^- = \left(\tilde{F}_{k-1}\left(\tilde{I}_0\right)^-, R\right)$ and it calculated using the adjacency matrix.

Example 4. *For the lower approximation rough fuzzy graph cellular automaton $(figure.6)$ the adjacency matrix is $\tilde{A}\left(\tilde{F}_0\left(\tilde{I}_0\right)_-\right) = \begin{bmatrix} b & b & 0 \\ b & b & a \\ 0 & 0 & a,b \end{bmatrix}$;*

$$\tilde{A}\left(\tilde{F}_1\left(\tilde{I}_0\right)_-\right) = M_4 \circ \tilde{A}\left(\tilde{F}_0\left(\tilde{I}_0\right)_-\right).$$

Hence $\tilde{A}\left(\tilde{F}_1\left(\tilde{I}_0\right)_-\right) = \begin{bmatrix} b & 0 & 0 \\ b & a & 0 \\ 0 & a,b & 0 \end{bmatrix}$. Similarly for upper approximation rough graph cellular automaton,

$$\tilde{A}\left(\tilde{F}_0\left(\tilde{I}_0\right)^-\right) = \begin{bmatrix} a,b & a,b & 0 \\ a,b & a,b & a \\ a,b & a,b & a,b \end{bmatrix};$$

$$\tilde{A}\left(\tilde{F}_1\left(\tilde{I}_0\right)^-\right) = \begin{bmatrix} b & 0 & 0 \\ b & a & 0 \\ 0 & a,b & 0 \end{bmatrix}.$$

7. Conclusion

We defined rough graph cellular automaton and the generations of rough graph cellular automaton is predicted.The rough fuzzy graph cellular automaton enables us to compare two machines. In order to demonstrate the usefulness of fuzzy graph cellular automaton and to strengthen the theoretical approach of FGCA, the first study of this kind was performed in the area of rough set theory and also the fuzzy vertex membership value specified in the information table reduce the level of uncertainity. Nonetheless, the progressive development of cellular automaton, the extended version of this defined concept with real world simulation will take place in our next implementation.

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