ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **10** (2021), no.1, 1637–1654
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.1.47

WATSON CRICK FUZZY AUTOMATA WITH OUTPUT

N. Vijayaraghavan[1], N. Jansirani, and V.R. Dare

ABSTRACT. The concept of fuzziness is introduced in Watson Crick Mealy Automata and Watson Crick Moore Automata to give Watson Crick Fuzzy Mealy Automata and Watson Crick Fuzzy Moore Automata. The Fuzzy sets were introduced as an extension of classical sets. The uncertain sets or the Fuzzy sets have elements with certain degrees of membership. The notion of Fuzzy sets is well incorporated in Automata theory as Fuzzy Automata. The ideas of Watson Crick Finite Automata and Fuzzy Automata are combined and studied as Watson Crick Fuzzy Automata. Here in this research article, Watson Crick Fuzzy Automata with output are introduced and studied. The Watson Crick Fuzzy Mealy Automata and Watson Crick Fuzzy Moore Automata are theoretical output producing model with certain degrees of membership or weightage for DNA based Computation. Further, some of the topological and algebraic properties of Watson Crick Fuzzy Automata with Output are established.

1. INTRODUCTION

Rosenberg introduced and studied Watson Crick Finite Automaton which was designed to work on tapes that are double stranded sequences of symbols connected and related by a complementarity relation much like in a DNA molecule

[1]*corresponding author*

2020 *Mathematics Subject Classification.* O3D99, O3E72.
Key words and phrases. Fuzzy, Homomorphism, Output, Successor, Watson Crick.
Submitted: 09.02.2021; *Accepted:* 13.03.2021; *Published:* 24.03.2021.

[1, 3, 4, 6]. The Fuzzy Automaton was mathematically formulated with the introduction of Fuzzy set theoretical ideas in formal languages and Automata Theory [2, 5, 7]. Various modifications of Fuzzy Automata were done and their topological, algebraic and characteristic properties were also studied [9]. In order to study the imprecise languages and its usage in DNA Computing, the notion of fuzziness is introduced in Watson crick Finite Automata by Jansirani et.al to find Watson Crick Fuzzy Automata [10]. They also introduced the Equivalence Grammar part of Watson Crick Fuzzy Automata known as Watson Crick Fuzzy Regular Grammar. Since all modifications and developments on Watson Crick Fuzzy Automata are done only on Language recognition, this prompts us to introduce Watson Crick Fuzzy Automata with output by incorporating the Moore machine and Mealy machine ideas with Watson Crick Automata and study their characteristics. The corresponding Fuzzy languages of Watson Crick Fuzzy Automata with output are also studied.Here Watson Crick Fuzzy Automata with output is introduced for a double tape input. Hence on reading the double tape input, the Watson crick Fuzzy Automata with output not only moves to the next state but also produces an output with certain degree of membership.

The second section recalls the Preliminaries and Basic Definitions required. In the Third section, Watson Crick Fuzzy Automata with output are introduced and their Characterizations are studied. The Fourth section deals with the introduction of Homomorphism in Watson Crick Fuzzy Automata with output and its usage. The Final section gives the Conclusion and Future Work.

2. PRELIMINARIES

In this section, the Preliminaries and the Basic Definitions needed to introduce Watson Crick Fuzzy Automata with output are recalled.

A symmetric relation $\rho \subseteq \Sigma \times \Sigma$, called the Watson-Crick complementarity relation on Σ, inspired by the Watson-Crick complementarity of nucleotides in the double stranded DNA molecule. The symmetric relation ρ is injective if for any $a \in \Sigma$ there exists a unique complementary symbol $b \in \Sigma$ with $(a, b) \in \rho$.In accordance with the representation of DNA molecules, viewed as two strings written one over the other, it is written $\begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}$ instead of $\Sigma^* \times \Sigma^*$ and $\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$ instead of the element$(w_1, w_2) \in \Sigma^* \times \Sigma^*$. $\begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}_\rho$ is denoted as $\begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}_\rho$ = $\{ \begin{pmatrix} a \\ b \end{pmatrix} |(a, b) \in \rho \}$ and WK$\rho(\Sigma) = \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}_\rho^*$.The set WK$\rho(\Sigma)$ is called Watson-Crick domain associated to Σ and ρ.

A Watson Crick Finite Automaton is a 6-tuple $M = (Q, \Sigma, \rho, \delta, q_o, F)$ where, Q is a finite nonempty set of states, Σ is a finite set of Inputs called the alphabet,$\rho \subseteq \Sigma \times \Sigma$ is the complimentary relation, $\delta : Q \times \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix} \to Q(2^Q)$ is called the transition function such that $\delta \left(q, \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \right) \neq \phi$, only for finitely many triples $(q, w_1, w_2) \in Q \times \Sigma^* \times \Sigma^*, q_0$ is an element of Q called the initial state, $F \subseteq Q$ is the set of final states of M. **[8]** . The transition function can be replaced with rewriting rules, by using $s \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \to \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} s'$ instead of $s' \in \delta \left(s, \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \right)$.The transitions in a Watson-Crick finite automaton can be defined as follows. For $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \in \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}$ such that $\begin{pmatrix} x_1 y_1 w_1 \\ x_2 y_2 w_2 \end{pmatrix} \in WK_\rho(\Sigma)$ and $s, s' \in Q$, then $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} s \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} s' \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$ if anly only if $s' \in \delta \left(s, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right)$.
The language accepted by a Watson-Crick automaton is: $L(M) =$
$\{w_1 \in \Sigma^* | q_0 \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \Rightarrow * \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} s\}$, with $s \in F, w_2 \in \Sigma^*$, $\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \in WK_\rho(\Sigma)$.

A Fuzzy Finite Automaton is a 5-tuple $M = \{Q, \Sigma, \delta, q_0, F\}$ where Q is a finite nonempty set of states, Σ is a finite set of Inputs called the alphabet, $\delta : Q \times \Sigma \to f(Q)$ is called the fuzzy transition function, q_o is an element of Q, it is called the initial state, $F \in f(Q)$ is the fuzzy set of final states of Q. The fuzzy language L(M) recognized by M is a fuzzy subset of Σ^*, with the membership function defined by $L(M)s = \bigcup (\delta (q_0, s) \cap F)$.

A Watson Crick Fuzzy Automaton is a 6-tuple $M = \{Q, \Sigma, \rho, \delta, q_0, F\}$where, Q is a finite nonempty set of states, Σ is a finite set of Inputs called the alphabets, $\rho \subseteq \Sigma \times \Sigma$ is the complimentary relation, $\delta : Q \times \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix} \to f(Q)$ called the fuzzy transition function or $\delta : Q \times \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix} \times Q \to [0,1]$ is the fuzzy transition function, q_0 is an element of Q called the initial state, $F \in f(Q)$ is the fuzzy set of final states of Q. The Language L(M) accepted by a Watson Crick Fuzzy Automata M is a fuzzy subset of Σ^* defined by $L(M)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \bigcup \left(\delta \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, p \right) \cap F \right)$ for all $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}$ and $x_2 = \rho(x_1)$. Let $M = (Q, \Sigma, \rho, \delta, q_0, F)$ beWatson Crick Fuzzy Automata, then the extension of Watson Crick Fuzzy transition function from $Q \times \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix} \times Q$ to [0,1] is defined as **[10]**

$\delta^* \left(q, \begin{pmatrix} \lambda \\ \lambda \end{pmatrix}, p \right) = 1$ if q=p and $\delta^* \left(q, \begin{pmatrix} \lambda \\ \lambda \end{pmatrix}, p \right) = 0$ if $q \neq p$.

$$\delta^* \left(q, \begin{pmatrix} x_1 a_1 \\ x_2 a_2 \end{pmatrix}, p \right) = \bigcup_{r \in Q} \left(\delta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, r \right) \cap \delta^* \left(r, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, p \right) \right)$$

for all $p, q \in Q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}$

3. Watson Crick Fuzzy Automata with Output

In this section, the types of Watson Crick Fuzzy Automata with output are introduced and their equivalence is established. The Transition map and output map extensions of Watson Crick Fuzzy Automata with output are also done. The characteristic property known as the successor property of Watson Crick Fuzzy Automata with output and its usage is also dealt in detail. Finally the Watson Crick Fuzzy subsystem of Watson Crick Fuzzy Automaton with output is designed and its closure properties under Boolean operations are also studied. Here the theorems are proved only for Watson Crick Fuzzy Mealy Machines as it is equivalent to Watson Crick Fuzzy Moore Machine

Definition 3.1. *A Watson Crick fuzzy Mealy machine is a quintuple $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$, where Q is a finite non-empty set called the set of states, Σ is a finite non-empty set called the set of inputs, $\rho \subseteq \Sigma \times \Sigma$ is the complimentary relation, O is a finite non-empty set called the set of outputs,q_0 is an element of Q called the initial* *state, δ is called the transition function, which is a fuzzy subset of $Q \times \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix} \times Q, \theta$ is called the output function, which is a fuzzy subset of $Q \times \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix} \times 0$ with the following condition getting satisfied:*

$$(\forall q \in Q), \left(\forall \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}\right), (\exists p \in Q), \delta \left(q, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, p\right) > 0$$

$$\leftrightarrow (\exists b \in O), \theta \left(q, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, b\right) > 0.$$

Let $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$be a Watson Crick Fuzzy Mealy machine, then $\delta^* : Q \times \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix} \times Q \to [0, 1]$ is defined as

$$\forall p, q \in Q, \forall \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}, \forall \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}.$$

(i) $\delta^* \left(q, \begin{pmatrix} \lambda \\ \lambda \end{pmatrix}, p\right) = \begin{cases} 1 & p = q \\ 0 & p \neq q \end{cases}$

(ii) $\delta^* \left(q, \begin{pmatrix} x_1 a_1 \\ x_2 a_2 \end{pmatrix}, p\right) = \bigcup_{r \in Q} \delta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, r\right) \cap \delta \left(q, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, p\right)$

(iii) $\delta^* \left(q, \begin{pmatrix} x_1 y_1 \\ x_2 y_2 \end{pmatrix}, p\right) = \bigcup_{r \in Q} \delta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, r\right) \cap \delta \left(q, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, p\right)$

Let $M = (Q, \Sigma, \rho, O, \delta, q_o, \theta)$ be a Watson Watson Crick Fuzzy Mealy Machine, then $\theta^* : Q \times \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix} \times O^* \to [0,]$ is defined as $\forall q \in Q, \forall \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}$, $\forall \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}, \forall b \in O, \forall o \in O^*$.

(i) $\theta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right) =$

$$\begin{cases} 1 & \text{if } \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \lambda \\ \lambda \end{pmatrix} \text{ and } o = \lambda \\ 0 & \text{if } \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \lambda \\ \lambda \end{pmatrix} \text{ and } o \neq \lambda \text{ or } \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \neq \begin{pmatrix} \lambda \\ \lambda \end{pmatrix} \text{ and } o = \lambda \end{cases}$$

(ii) $$\theta^* \left(q, \begin{pmatrix} x_1 a_1 \\ x_2 a_2 \end{pmatrix}, ob \right) = \bigcup_{r \in Q} \theta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o \right) \cap \delta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, r \right) \cap \theta \left(r, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, b \right)$$

$$= \theta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o \right) \cap \left(\bigcup_{r \in Q} \delta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, r \right) \cap \theta \left(r, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, b \right) \right)$$

Definition 3.2. *A Watson Crick Fuzzy Moore machine is a quintuple $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$, where Q is a finite non-empty set called the set of states, Σ is a finite non-empty set called the set of inputs,$\rho \subseteq \Sigma \times \Sigma$ is the complimentary relation, O is a finite non-empty set called the set of outputs,q_0 is an element of Q called the initial state, δ is called the transition function, which is a fuzzy subset of $Q \times \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix} \times Q$, θ is called the output function, which is a fuzzy subset of $Q \times O$.*

Theorem 3.1. *Watson Crick Fuzzy Mealy machine is equivalent to a Watson Crick Fuzzy Moore machine.*

Proof. Let $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$ be given Watson crick fuzzy Mealy machine. Construct a fuzzy Moore machines as $M = (Q \times O, \Sigma, \rho, O, \delta_1, q_0, \theta)$, where

$$\delta_1 \left((p, \alpha), \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, (q, \beta) \right) = \delta \left(p, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, q \right) \cap \theta \left(p, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \beta \right),$$

for all $(p, \alpha), (q, \beta) \in Q \times O$ and

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}; \quad \theta_1 ((p, \alpha), \beta) = \begin{cases} 1 & \text{if } \alpha = \beta \\ 0 & \text{otherwise} \end{cases}.$$

The extension of δ_1 is the fuzzy set $\delta_1^* : (Q \times O) \times \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix} \times (Q \times O) \to [0, 1]$ is defined for all $(p, \alpha), (q, \beta) \in Q \times O$, $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}$, $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}$ as $\delta_1^* ((p, \alpha), \lambda, (q, \beta)) = 0;$

$$\delta_1^*\left((p,\alpha),\begin{pmatrix}a_1\\a_2\end{pmatrix},(q,\beta)\right)=\theta\left(p,\begin{pmatrix}a_1\\a_2\end{pmatrix},\beta\right)\text{ and}$$

$$\delta_1^*\left((p,\alpha),\begin{pmatrix}x_1a_1\\x_2a_2\end{pmatrix},(q,\beta)\right)=\bigcup\{\delta_1\left((p,\alpha)\begin{pmatrix}a_1\\a_2\end{pmatrix},(r,\gamma)\right)\cap\delta\left(p,\begin{pmatrix}a_1\\a_2\end{pmatrix},r\right)\cap$$
$$\delta_1\left((r,\gamma),\begin{pmatrix}x_1\\x_2\end{pmatrix},(q,\beta)\,|\,(r,\gamma)\in Q\times O\right).$$

The extension of θ_1^* of θ_1 is the fuzzy set $\theta_1^*:(Q\times O)\times\begin{pmatrix}\Sigma^*\\\Sigma^*\end{pmatrix}\times O^*\to[0,1]$ is defined as length preserving map and it is given by:

$$\theta_1^*\left((p,\alpha),\begin{pmatrix}x_1\\x_2\end{pmatrix},\lambda\right)=\theta_1^*\left((p,\alpha),\begin{pmatrix}\lambda\\\lambda\end{pmatrix},b\right)=0;$$

$$\theta_1^*\left((p,\alpha),\begin{pmatrix}x_1y_1\\x_2y_2\end{pmatrix},\tau_1\right)=\bigcup\left\{\delta_1\left((p,\alpha),\begin{pmatrix}x_1\\x_2\end{pmatrix},((q,\tau_1))\right)\right.$$
$$\left.\cap\,\theta_1\left((q,\tau_1),\tau_1\right)|q\in Q\right\}=\theta\left(p,\begin{pmatrix}x_1\\x_2\end{pmatrix},\tau_1\right);$$

$$\theta_1^*\left((p,\alpha),\begin{pmatrix}x_1y_1\\x_2y_2\end{pmatrix},\tau_1\tau_2\right)=\bigcup\left\{\delta_1\left((p,\alpha),\begin{pmatrix}x_1\\x_2\end{pmatrix},((q,\tau_1))\right)\right.$$
$$\left.\cap\,\delta\left(\left(p,\begin{pmatrix}x_1\\x_2\end{pmatrix},q\right)\cap\theta_1^*\left((q,\tau_1),\begin{pmatrix}y_1\\y_2\end{pmatrix},\tau_2\right)|q\in Q\right)\right\}$$
$$=\bigcup\{\theta\left(p,\begin{pmatrix}x_1\\x_2\end{pmatrix},\tau_1\right)\cap\left[\delta\left(p,\begin{pmatrix}x_1\\x_2\end{pmatrix},q\right)\cap\theta\left(q,\begin{pmatrix}y_1\\y_2\end{pmatrix},\tau_2\right)\right]|q\in Q;$$

$$\theta_1^*\left((p,\alpha),\begin{pmatrix}x_1y_1z_1\\x_2y_2z_2\end{pmatrix},\tau_1\tau_2\tau_3\right)=\bigcup\{\delta_1\left((p,\alpha),\begin{pmatrix}x_1\\x_2\end{pmatrix},(q,\tau_1)\right)$$
$$\cap\,\delta\left[\delta\left(p,\begin{pmatrix}x_1\\x_2\end{pmatrix},\tau_1\right)\cap\theta_1^*\left(q,\begin{pmatrix}y_1z_1\\y_2z_2\end{pmatrix},\tau_2\tau_3\right),\tau_2\right)|q\in Q\};$$
$$=\bigcup\{\theta\left(p,\begin{pmatrix}x_1\\x_2\end{pmatrix},\tau_1\right)\cap\left[\delta\left(p,\begin{pmatrix}x_1\\x_2\end{pmatrix},q\right)\cap\theta\left(q,\begin{pmatrix}y_1\\y_2\end{pmatrix},\tau_2\right)\right]$$
$$\cap\left[\delta\left(q,\begin{pmatrix}y_1\\y_2\end{pmatrix},r\right)\cap\theta\left(r,\begin{pmatrix}z_1\\z_2\end{pmatrix},\tau_2\right)\right]|q,r\in Q\}.\qquad\square$$

Thus in general this gives rise to a Watson Crick Fuzzy Moore machine. Hence from a Watson Crick Fuzzy Mealy machine, A Watson crick fuzzy Moore machine can be obtained. The following corollary gives the converse part. From that we can establish the equivalence between Watson Crick Fuzzy Moore machine and Watson Crick Fuzzy Mealy machine.

Corollary 3.1. *Watson Crick Fuzzy Moore machine is equivalent to a Watson Crick Fuzzy Mealy machine.*

Proof. let $M = (Q, \Sigma, \rho, O, \delta, q_o, \theta)$ be given Watson Crick Fuzzy Moore machine. Construct a Watson Crick fuzzy Mealy machine M_1 as: $M_1 = (Q, \Sigma, \rho, O, \delta_1, q_o, \theta_1)$, where θ_1 is defined by

$$\theta_1 \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o \right) = \delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o \right), \forall p \in Q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix} \text{ and } o \in O^*.$$

Then, M_1 is equivalent to M. □

Definition 3.3. *Let $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$ be a Watson Crick Fuzzy Mealy machine. Let $p, q \in Q$, then q is called the immediate successor of p, if $\exists \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}, b \in O$ such that $\delta \left(p, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, q \right) \cap \theta \left(p, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, b \right) > 0$ and q is called the successor of p if $\exists \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}$ and $o \in O^*$ such that $\delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, q \right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o \right) > 0$*

Let $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$ be a Watson Crick Fuzzy Mealy machine and let $q \in Q$,the set of all successors of q is denoted as S(q). Let $A \subseteq Q$, then the set S(A), which represents set of all successors of A, is defined as $S(A) = (\bigcup (S(q)/q \in A))$. Let $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$ be a Watson Crick Fuzzy Mealy machine let $p, q \in Q$. Then M is called strongly connected, if$p \in S(q)$.

Theorem 3.2. *Let $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$ be a Watson Crick Fuzzy Mealy machine.Define a relation $\sim$ on Q as $p \sim q$ if and only if q is a successor of p. then the relation $\sim$ is not an equivalence relation.*

Proof. Let $p, q, r \in Q$. Consider $p \sim q \leftrightarrow q$ called the successor of

$$p \leftrightarrow \delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, q \right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o \right) > 0.$$

Consider $q \sim r \leftrightarrow r$ is called the successor of

$$p \leftrightarrow \delta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, r\right) \cap \theta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right) > 0.$$

To prove that $p \sim r \leftrightarrow r$ called the successor of $p \leftrightarrow \delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, r\right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right) > 0$,

$$\delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, q\right) \cap \delta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, r\right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right) \cap \theta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right)$$
$$= \delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, r\right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right) \cap \theta^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right) \cap \theta^* \left(q, \begin{pmatrix} \lambda \\ \lambda \end{pmatrix}, \lambda\right)$$
$$= \delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, r\right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right) > 0.$$

Hence it is transitive.

Clearly $p \sim p$, hence it is reflexive.

Suppose if $p \sim q$, then q cannot be a successor of p, hence it is not symmetric. Hence it is established that the successor property satisfies the reflexive and transitive axioms but not symmetric, therefore the relation defined is not an equivalence relation. □

Theorem 3.3. *Let $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$ be a Watson Crick Fuzzy Mealy machine.Let $A, B \subseteq Q$*

(1) if $A \subseteq B$ then $S(A) \subseteq S(B)$;

(2) $A \subseteq S(A)$;

(3) $S(S(A)) = A$;

(4) $S(A \cup B) = S(A) \cup S(B)$;

(5) $S(A \cap B) \subset S(A) \cup S(B)$.

Proof.

(1) Given $A \subseteq B$, to prove $S(A) \subseteq S(B)$. Consider $q \in S(A)$, then $q \in S(p)$ for some $p \in A$. Since $A \subseteq B, p \in B$, therefore $q \in S(p)$ for some $p \in B$. Hence $q \in S(B)$, this implies that $S(A) \subseteq S(B)$.

(2) From the definition 3.3, it is obvious that $A \subseteq S(A)$.

(3) By (2), we have $A \subseteq S(A)$, then $S(A) \subseteq S\left(S(A)\right)$,which proves the first part. To prove the second part $S\left(S\left(A\right)\right) \subseteq S\left(A\right)$. Consider $q \in S\left(S\left(A\right)\right)$. Then it is obvious that $q \in S(r)$, for some $r \in S(A)$. Hence $r \in S(t)$, for some $t \in A$. Now, q is a successor of r and r is successor of t, hence by Theorem 3.2, q is a successor of t. Thus $q \in S(t) \subseteq S(A)$. Therefore $q \in S(A)$. Hence,$S\left(S\left(A\right)\right) \subseteq S\left(A\right)$.

(4) Since Q is non empty, Let$A, B \subseteq Q$, then either A or B is non empty, which implies $A \cup B$ is nonempty. To prove $S\left(A \cup B\right) = S\left(A\right) \cup S\left(B\right)$, consider $q \in S\left(A \cup B\right)$, then $q \in S(r)$, for some $r \in A \cup B$. Then $q \in S(r)$, for some $r \in A \cup B$, $r \in A \cup B$ implies $r \in A$ or $r \in B$, thus $q \in S(r)$, for some $r \in A$ or $r \in B$ hence $q \in S(A) \cup S(B)$. Therefore, $S(A \cup B) \subseteq S(A) \cup S(B) \ldots (i)$. Retracing the above steps, we get $S(A) \cup S(B) \subseteq S(A \cup B)$. Therefore $S(A \cup B) = S(A) \cup S(B)$.

(5) Proof is similar to (4), □

Definition 3.4. *Let $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$ be a Watson Crick Fuzzy Mealy Machine. Let ω be a fuzzy subset of Q. We say that ω is called a Watson Crick Fuzzy subsystem of M, if*

$$\omega\left(q\right) \geq \omega\left(p\right) \cap \delta\left(p, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, q\right) \cap \theta\left(p, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, b\right),$$

for all $p, q \in Q$, $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}, b \in O$.

Theorem 3.4. *Let $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$ be a Watson Crick Fuzzy Mealy machine. Then ω is a Watson Crick Fuzzy subsystem of M, if, and only if,*

$$\omega\left(q\right) \geq \omega\left(p\right) \cap \delta^*\left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, q\right) \cap \theta^*\left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right),$$

for all $p, q \in Q$, $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}$, $o \in O^*$.

Proof. Suppose ω is a Watson Crick Fuzzy subsystem of M. Let $p, q \in Q$, $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}$ and $o \in O^*$.

To prove this theorem, Let us use the concept of mathematical induction on $\begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = |o| = n$

Basis: If n = 0, then $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \lambda \\ \lambda \end{pmatrix}$ and $o = \lambda$. Now, if q=p, then

$$\omega(p) \cap \delta^* \left(q, \begin{pmatrix} \lambda \\ \lambda \end{pmatrix}, q \right) \cap \theta^* \left(q, \begin{pmatrix} \lambda \\ \lambda \end{pmatrix}, \lambda \right) = \omega(q).$$

Now, if q $\neq$ p, then

$$\omega(p) \cap \delta^* \left(p, \begin{pmatrix} \lambda \\ \lambda \end{pmatrix}, q \right) \cap \theta^* \left(p, \begin{pmatrix} \lambda \\ \lambda \end{pmatrix}, \lambda \right) = 0 \leq \omega(q).$$

Hence the theorem is true for the basis.

Induction Hypothesis: Let us Assume that the theorem holds true for all $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}$ and $0 \in O^*$ such that $\begin{vmatrix} u_1 \\ u_2 \end{vmatrix} = |v| = n-1, n > 1.$

Induction: Let $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} u_1 a_1 \\ u_2 a_2 \end{pmatrix}$ and o=bv, where $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}, b \in O$ and $\begin{vmatrix} u_1 \\ u_2 \\ - \end{vmatrix} = |v| = n-1$. Then

$$\begin{aligned}
&\omega(p) \cap \delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, q \right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o \right) \\
&\quad = \omega(p) \cap \delta^* \left(p, \begin{pmatrix} u_1 a_1 \\ u_2 a_2 \end{pmatrix}, q \right) \cap \theta^* \left(p, \begin{pmatrix} u_1 a_1 \\ u_2 a_2 \end{pmatrix}, bv \right) \\
&\quad = \omega(p) \cap \left\{ \bigcup_{r \in Q} \delta^* \left(p, \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, r \right) \cap \delta \left(r, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, q \right) \right\} \\
&\qquad \cap \left\{ \theta^* \left(p, \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, r \right) \cap \left(\bigcup_{r \in Q} \delta^* \left(p, \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, r \right) \cap \theta \left(r, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, b \right) \right) \right\} \\
&\quad = \omega(p) \cap \left\{ \bigcup_{r \in Q} \delta^* \left(p, \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, r \right) \cap \delta \left(r, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, q \right) \right. \\
&\qquad \left. \cap\, \theta^* \left(p, \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, v \right) \cap \delta^* \left(p, \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, r \right) \cap \theta \left(r, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, b \right) \right\}
\end{aligned}$$

$$= \left\{ \bigcup_{r\in Q} \omega\left(p\right) \cap \delta^* \left(p, \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, r\right) \cap \delta\left(r, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, q\right) \cap \theta^* \left(p, \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, v\right) \right.$$
$$\left. \cap\, \theta^* \left(r, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, b\right) \right\}$$
$$\leq \bigcup_{r\in Q} \omega\left(r\right) \cap \delta^* \left(p, \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, r\right) \cap \theta^* \left(p, \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, v\right) \leq w\left(q\right).$$

Therefore $\omega\left(q\right) \geq \omega\left(p\right) \cap \delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, q\right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right)$.

Retracing the above steps will establish the converse part. □

Theorem 3.5.

a) Every constant Watson Crick Fuzzy set ω on Q defines a Watson Crick Fuzzy subsystem of a Watson Crick Fuzzy Mealy machine $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$.

b) $M = (Q, \Sigma, \rho, O, \delta, q_0, \theta)$be a Watson Crick Fuzzy Mealy machine . Let ω_1 and ω_2 be a Watson Crick fuzzy subsystems of M, Then (1) $\omega_1 \cap \omega_2$ Watson Crick Fuzzy subsystem of M and (2) $\omega_1 \cup \omega_2$ is Watson Crick Fuzzy subsystem of M.

Proof.

(a) Suppose ω is a constant Watson Crick Fuzzy set on Q. Then for any $p, q \in Q$ we have $\omega(p) = \omega(q)$. Therefore for any $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in \begin{pmatrix} \Sigma \\ \Sigma \end{pmatrix}, b \in O$, we have clearly,

$$\omega\left(q\right) = \omega\left(p\right) \geq \omega\left(p\right) \cap \delta\left(q, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, p\right) \cap \theta\left(q, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, b\right).$$

Therefore ω is Watson Crick Fuzzy subsystem of a Watson Crick Fuzzy Mealy Machine M.

(b) Since ω_1 and ω_2 are Watson Crick Fuzzy subsystems of M, for $p, q \in Q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \Sigma^* \\ \Sigma^* \end{pmatrix}$ and $o \in O^*$, we have $\omega_1\left(q\right) \geq \omega_1\left(p\right) \cap \delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, q\right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right)$ and $\omega_2\left(q\right) \geq \omega_2\left(p\right) \cap \delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, q\right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right)$. We consider $\left(\omega_1 \cap \omega_2\right)\left(q\right) = \omega_1\left(q\right) \cap \omega_2\left(q\right) \geq \left(\omega_1\left(p\right) \cap \omega_2\left(p\right)\right) \cap \delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, q\right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o\right)$, therefore $\omega_1 \cap \omega_2$ is Watson Crick Fuzzy subsystem of M. Also

$$\omega_1 \cup \omega_2 (q) = \omega_1 (q) \cup \omega_2 (q) \geq (\omega_1 (p) \cup \omega_2 (p)) \cap \delta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, q \right) \cap \theta^* \left(p, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o \right).$$

This implies that $\omega_1 \cup \omega_2$ is also Watson Crick Fuzzy subsystem of M. □

4. Homomorphism on Watson Crick Fuzzy Automata with Output

In this section, the algebraic concept of homomorphism is introduced on Watson Crick Fuzzy Automata with Output. The applications of the homomorphism in Successor property, topological property and Watson Crick Fuzzy subsystem of Watson Crick Fuzzy Automata with output are studied. The Kernel of the homomorphism is also defined with kernel being an equivalence relation is also established. Here also the theorems are proved only for Watson Crick Fuzzy Mealy Machine as it is equivalent to Watson Crick Fuzzy Moore Machine.

Definition 4.1. *Let $M_1 = (Q_1, \Sigma_1, \rho, O_1, \delta_1, q_o, \theta_1)$ and $M_2 = (Q_2, \Sigma_2, \rho, O_2, \delta_2, q_o, \theta_2)$ be Watson Crick Fuzzy Mealy Machines .A triplet (f,g,h) of mappings, $f : Q_1 \to Q_2, g : \begin{pmatrix} \Sigma_1 \\ \Sigma_1 \end{pmatrix} \to \begin{pmatrix} \Sigma_2 \\ \Sigma_2 \end{pmatrix}$ and $h : O_1 \to O_2$, is called a Watson Crick Fuzzy Mealy machine homomorphism from M_1 to M_2,denoted by*

(i) $\delta_1 \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, p \right) \leq \delta_2 \left(f(q), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right), f(p) \right)$

(ii) $\theta_1^* \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o \right) \leq \theta_2^* \left(f(q), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right), h(o) \right) \forall q, p \in Q_1$,

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \sum_1^* \\ \sum_1^* \end{pmatrix}$ *and $o \in O_1^*$.*

Theorem 4.1. *Let $M_1 = (Q_1, \Sigma_1, \rho, O_1, \delta_1, q_o, \theta_1)$ and $M_2 = (Q_2, \Sigma_2, \rho, O_2, \delta_2, q_o, \theta_2)$ be Watson Crick Fuzzy Mealy Machines. Let $(f, g, h) : M_1 \to M_2$ be a Watson Crick Fuzzy Mealy machine homomorphism from M_1 to M_2 if p is a successor of q in M_1,then f(p) is a successor of f(q) in M_2.*

Proof. $(f, g, h) : M_1 \to M_2$ a Watson Crick Fuzzy Mealy machine homomorphism from M_1 to M_2 . Therefore by definition, we have

(i) $\delta_1 \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, p \right) \leq \delta_2 \left(f(q), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right), f(p) \right)$

(ii) $\theta_1^*\left(q,\begin{pmatrix}x_1\\x_2\end{pmatrix},o\right)\leq\theta_2^*\left(f\left(q\right),g\left(\left(\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)\right),h\left(o\right)\right)\forall q,p\in Q_1$, $\begin{pmatrix}x_1\\x_2\end{pmatrix}\in\begin{pmatrix}\Sigma_1^*\\\Sigma_1^*\end{pmatrix}$ and $o\in O_1^*$.

Also it is given that p is a successor of q in M_1, then $\exists\begin{pmatrix}x_1\\x_2\end{pmatrix}\in\begin{pmatrix}\Sigma_1^*\\\Sigma_1^*\end{pmatrix}$ and $o\in O_1^*$ such that $\delta_1^*\left(q,\begin{pmatrix}x_1\\x_2\end{pmatrix},p\right)\cap\theta_1^*\left(q,\begin{pmatrix}x_1\\x_2\end{pmatrix},o\right)>0$.

To prove that f(p) is a Successor of f(q) in M_2, consider

$$\delta_1^*\left(q,\begin{pmatrix}x_1\\x_2\end{pmatrix},p\right)\cap\theta_1^*\left(q,\begin{pmatrix}x_1\\x_2\end{pmatrix},o\right)>0\geq\delta_2^*\left(f\left(q\right),g\left(\left(\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)\right),f\left(p\right)\right)$$
$$\cap\,\theta_2^*\left(f\left(q\right),g\left(\left(\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)\right),h\left(o\right)\right)\text{ (by definition).}$$

Hence f(p) is a successor of f(q) □

Theorem 4.2. *Let* $M_1=(Q_1,\Sigma_1,\rho,O_1,\delta_1,q_o,\theta_1)$ *and* $M_2=(Q_2,\Sigma_2,\rho,O_2,\delta_2,q_o,\theta_2)$ *be Watson Crick Fuzzy Mealy Machines. Let* $(f,g,h):M_1\to M_2$ *be a Watson Crick Fuzzy Mealy machine homomorphism from* M_1 *satisfying the conditions*

(i) $\delta_2\left(f\left(q\right),g\left(\left(\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)\right),f\left(p\right)\right)=\delta_1\left(q,\begin{pmatrix}x_1\\x_2\end{pmatrix},p\right)$

(ii) $\theta_2^*\left(f\left(q\right),g\left(\left(\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)\right),h\left(o\right)\right)=\theta_1^*\left(q,\begin{pmatrix}x_1\\x_2\end{pmatrix},o\right),\forall p,q\in Q_1,\begin{pmatrix}x_1\\x_2\end{pmatrix}\in\begin{pmatrix}\Sigma_1^*\\\Sigma_1^*\end{pmatrix}$ *and* $o\in O_1^*$, *then* $S\left(f\left(q\right)\right)=f\left(S\left(q\right)\right),\forall q\in Q_1$

Proof.

$$f\left(p\right)\in f\left(S\left(q\right)\right)\Leftrightarrow p\in S\left(q\right)$$
$$\Leftrightarrow\delta_1^*\left(q,\begin{pmatrix}x_1\\x_2\end{pmatrix},p\right)\cap\theta_1^*\left(q,\begin{pmatrix}x_1\\x_2\end{pmatrix},\right)>0$$
$$\Leftrightarrow\delta_1^*\left(q,\begin{pmatrix}x_1\\x_2\end{pmatrix},p\right)>0\text{ and }\theta_1^*\left(q,\begin{pmatrix}x_1\\x_2\end{pmatrix},o\right)>0$$
$$\Leftrightarrow\delta_2^*\left(f\left(q\right),g\left(\left(\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)\right),f\left(p\right)\right)>0\text{ and }\theta_2^*\left(f\left(q\right),g\left(\left(\begin{pmatrix}x_1\\x_2\end{pmatrix}\right)\right),h\left(o\right)\right)>0$$

$\Leftrightarrow \delta_2^* \left(f(q), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right), f(p) \right) \cap \theta_2^* \left(f(q), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right), h(o) \right) > 0$

$\Leftrightarrow f(p) \in f(S(q))$.

Hence $S(f(q)) = f(S(q))$. ☐

Theorem 4.3. *Let $M_1 = (Q_1, \Sigma_1, \rho, O_1, \delta_1, q_o, \theta_1)$ and $M_2 = (Q_2, \Sigma_2, \rho, O_2, \delta_2, q_o, \theta_2)$ be Watson Crick Fuzzy Mealy Machines. Let $(f, g, h) : M_1 \to M_2$ be a Watson Crick Fuzzy Mealy machine homomorphism from M_1 to M_2.If M_1 is strongly connected, then M_2 is also strongly connected.*

Proof. Let $r, s \in Q_2$. Then there exists $p, q \in Q_1$ such that f(p)= r and f(q)=s. Since M_1 is given to be strongly connected, we have $p \in S(q)$. Then $f(p) \in f(S(q))$. By the previous Theorem, we have $f(p) \in S(f(q))$, which implies that $r \in S(s)$. Therefore M_2 also is strongly connected. ☐

Theorem 4.4. *Let $M_1 = (Q_1, \Sigma_1, \rho, O_1, \delta_1, q_o, \theta_1)$ and $M_2 = (Q_2, \Sigma_2, \rho, O_2, \delta_2, q_o, \theta_2)$ be Watson Crick Fuzzy Mealy Machines. Let $(f, g, h) : M_1 \to M_2$ be a Watson Crick Fuzzy Mealy machine homomorphism from M_1 to M_2 satisfying the conditions*

(i) $\delta_2 \left(f(q), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right), f(p) \right) = \delta_1 \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, p \right)$

(ii) $\theta_2^* \left(f(q), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right), h(o) \right) = \theta_1 * \left(q, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o \right), \forall p, q \in Q_1, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \Sigma_1^* \\ \Sigma_1^* \end{pmatrix}$ *and $o \in O_1^*$, then then if ω is a Watson Crick Fuzzy subsystem of M_1,then $f(\omega)$ is a Watson Crick Fuzzy subsystem of M_2.*

Proof. Let $q_1, q_2 \in Q_2$ and $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \begin{pmatrix} \Sigma_2^* \\ \Sigma_2^* \end{pmatrix}$ and $o_2 \in O_2^*$. Since f is given to be onto, then there exist $p_1, p_2 \in Q_1$, such that $f(p_1) = q_1$ and $f(p_2) = q_2$ .Also it is given that g and h are onto, then there exists $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \begin{pmatrix} \Sigma_1^* \\ \Sigma_1^* \end{pmatrix}$ and $o \in O_1^*$, such that $g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ and $h(o_1) = o_2$.

Let us suppose that there exists a $p_3 \in Q_1$, such that $f(p_3) = q_1$. Then,

$$\delta_1^* \left(p_1, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, p_2 \right) = \delta_2^* \left(f(p_1), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right), f(p_2) \right)$$

$$= \delta_2^* \left(f\left(p_3\right), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right), f\left(p_2\right) \right) = \delta_1^* \left(p_3, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, p_2 \right)$$

$$\theta_1^* \left(p_1, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o_1 \right) = \theta_2^* \left(f\left(p_1\right), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right), h\left(o_1\right) \right)$$

$$= \theta_2^* \left(f\left(p_3\right), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right), h\left(o_1\right) \right) = \theta_1^* \left(p_3, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o_1 \right).$$

Now, let us prove that $f\left(\omega\right)$ is a Watson Crick Fuzzy subsystem of M_2. Consider

$$f(\omega)(q_1) \cap \delta_2^* \left(q_1, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, q_2 \right) \cap \theta_2^* \left(p_2, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, o_2 \right)$$

$$= \bigcup \left\{ \omega\left(p_3\right) | f\left(p_3\right) = q_1 \right\} \cap \delta_2^* \left(q_1, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, q_2 \right) \cap \theta_2^* \left(p_2, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, o_2 \right)$$

$$= \bigcup \left\{ \omega\left(p_3\right) \cap \delta_2^* \left(q_1, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, q_2 \right) \cap \theta_2^* \left(p_2, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, o_2 \right) | f\left(p_3\right) = q_1 \right\}$$

$$= \bigcup \left\{ \omega\left(p_3\right) \cap \delta_2^* \left(f\left(p_1\right), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right), f\left(p_2\right) \right) \right.$$
$$\left. \cap\, \theta_2^* \left(f\left(p_1\right), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right), h\left(o_1\right) \right) | f\left(p_3\right) = q_1 \right\}$$

$$= \bigcup \left\{ \omega\left(p_3\right) \cap \delta_2^* \left(f\left(p_3\right), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right), f\left(p_2\right) \right) \right.$$
$$\left. \cap\, \theta_2^* \left(f\left(p_3\right), g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right), h\left(o_1\right) \right) | f\left(p_3\right) = q_1 \right\}$$

$$= \bigcup \left\{ \omega\left(p_3\right) \cap \delta_1^* \left(p_3, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, p_2 \right) \cap \theta_1^* \left(p_3, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, o_1 \right) | f\left(p_3\right) = q_1 \right\}$$

$$\leq \bigcup \left\{ \omega(p_3) | f(p_3) = q_1 \right\}.$$

Since ω is a Watson Crick Fuzzy subsystem of $M_1 \leq \bigcup \left\{ f(\omega)(q_2) | f(p_3) = q_1 \right\} = f(\omega)(q_2)$. □

Definition 4.2. *Let $\pi : M_1 \to M_2$ be a Watson Crick Fuzzy Mealy machine homomorphism from M_1 to M_2. The kernel of π denoted by $ker(\pi)$ is defined as* $ker(\pi) = \left\{ (p, q) / \pi(p) = \pi(q) \right\}$.

Theorem 4.5. *Let* $M_1 = (Q_1, \Sigma_1, \rho, O_1, \delta_1, q_o, \theta_1)$ *and* $M_2 = (Q_2, \Sigma_2, \rho, O_2, \delta_2, q_o, \theta_2)$ *be Watson Crick Fuzzy Mealy Machines. Let* $\pi : M_1 \to M_2$ *be a Watson Crick Fuzzy Mealy machine homomorphism from* M_1 *to* M_2*. then The kernel of* π *denoted by* $ker(\pi)$ *is an equivalence relation.*

Proof. Suppose π is a Watson Crick Fuzzy Mealy machine homomorphism, then clearly by the definition of $ker(\pi)$, if $p, q \in Q_1$ and $(p, q) \in ker(\pi)$, then $\pi(p) = \pi(q)$ Clearly π is relexive. By the definition if if $p, q \in Q_1$ and $(p, q) \in ker(\pi)$, then $\pi(p) = \pi(q)$ That is $\pi(q) = \pi(p)$,which implies it is symmetric. To prove the transitivity if $p, q, r \in Q_1$ and $(p, q) \,\&\, (q, r) \in ker(\pi)$, then we have $\pi(p) = \pi(q)$ and $\pi(q) = \pi(r)$, hence $\pi(p) = \pi(q) = \pi(r)$. Therefore $\pi(p) = \pi(r)$. This implies that $(p, r) \in ker(\pi)$. Thus $ker(\pi)$ is an equivalence relation on Q. □

5. Conclusion and Future Scope

The Mealy machine and Moore machine ideas were introduced in Watson Crick Fuzzy Automata to get Watson Crick Fuzzy Automata with output. Their word processing takes less time when compared to ordinary Mealy and Moore machines. The equivalence of Watson Crick Fuzzy Moore Automata and Watson Crick Fuzzy mealy Automata is established. The characterizations of Watson Crick Fuzzy Automata with output are well studied along with its properties. Some Algebraic aspects of Watson Crick Fuzzy Automata with output are also discussed. The Future work aims to introduce Watson Crick Rough Automata with output and to study its characterizations.

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1.DEPARTMENT OF MATHEMATICS,
KCG COLLEGE OF TECHNOLOGY,CHENNAI.
2.DEPARTMENT OF MATHEMATICS,
QUEEN MARY'S COLLEGE-UNIVERSITY OF MADRAS, CHENNAI,
INDIA.
Email address: nvrnsb@gmail.com

DEPARTMENT OF MATHEMATICS
QUEEN MARY'S COLLEGE-UNIVERSITY OF MADRAS, CHENNAI,
INDIA.
Email address: njansirani@gmail.com

DEPARTMENT OF MATHEMATICS,
MADRAS CHRISTIAN COLLEGE-UNIVERSITY OF MADRAS, CHENNAI.
INDIA.
Email address: rajkumardare@yahoo.com