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ON COMMUTING PAIR AND CENTRALISING PAIR OF AUTOMORPHISMS OF RINGS

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ABSTRACT. Let R be a ring and S and T be non trivial automorphims of R. If $[S(x), T(x)] = 0$ for all $x \in R$, then R is a commutative ring. If $[S(x), T(x)] \in Z$, the centre of R, then R is a commutative integral domain.

1. INTRODUCTION

Let R be an associative ring. An automorphism T of R is called a commuting automorphism,if $T(x)x = xT(x)$ for every $x \in R, In[3]$ Drivinskey showed that a semisimple artirian ring must be commutative if it posseses a non trivial commuting automorphism. Luh [4] extended this result by proving that a prime ring R possessing a non-trivial commuting automorphism T must be an integral domain. Mayne [5] generalised this result further by proving that a prime ring R possessing a non-triavial automorphism T.such that $T(x)x - xT(x)$ is in the centre of R,for every $x \in R$ must necessarily commutative.L.O.Chung and J.Luh [2] called an automorphism T of R, a semi commuting automorphism if $T(x) \cdot x = \pm xT(x)$ for each $x \in R$. They also proved that a prime ring R with

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characteristic $\neq 2, 3$ possessing a non-trivial semi-commuting automorphism is necessarily a commutative integral domain.

In this paper we generalise the result of J. Mayne [5] by defining commuting pair of automorphisms and centralising pair of automorphisms and more general results are obtained. Throughout this paper R denote an associative ring unless otherwise specifically stated. Z denote the centre of R.

2. Preliminary

Definition 2.1. *Let R be an associative ring and T be an automorphism of $R.T$ is called.*

- (i) *a commuting automorphism if $x\ T(x) = T(x) \times \forall x \in R$ i.e, $[xT(x)] = 0 \forall x \in R$;*
- (ii) *an anti commuting automorphism if $xT(x) = -T(x)x \forall x \in R$;*
- (iii) *a semi-commuting automorphism if either $xT(x) = T(x)x($ or $) \times T(x) = -T(x)x$ i.e, $\times T(x) = \pm T(x) \times \forall x \in R$;*
- (iv) *a strong commuting automorphism if $[x, T(y)] = [T(x), y] \forall x, y \in R$;*
- (v) *a strong semi commuting automorphism if $[x, T(y)] = \pm[T(x), y] \forall x, y \in R$;*
- (vi) *a centralising automorphism if $[x, T(x)] \in Z \forall x \in R$. i.e, $xT(x) - T(x)x \in Z, \forall x \in R$;*
- (vii) *an anti-centralising automorphism if $x\ T(x) + T(x)x \in Z, \forall x \in R$;*
- $(viii)$ *a semi-centralising automorphism if $xT(x) - T(x)x \in Z($ or $) \times T(x) + T(x)x \in Z, \forall x \in R$;*
- (ix) *a strong centralising automorphism if $[x, T(y)] - [T(x), y] \in Z, \forall x, y \in R$;*
- (x) *a strong semi-centralising automorphism if $[x, T(y)] \pm [T(x), y] \in Z, \forall x, y \in R$.*

Remark 2.1. *Let R be any ring. Then*

- (i) $[x, y + z] = [x, y] + [x, z], \forall x, y, z \in R$;
- (ii) $[x + y, z] = [x, z] + [y, z], \forall x, y, z \in R$;
- (iii) $[x, y] = -[y \times x], \forall x, y \in R$;
- (iv) $[xy, z] = x[y, z] + [x, z]y, \forall x, y, z \in R$; v $[x, y] = 0$ *if* $x = y$.

Definition 2.2. *Let R be an associative ring and T be an automorphism of R. Let*

$$R_- = \{x \in R/[x, T(y)] - [y, T(x)] \in Z, \forall x, y \in R\};$$
$$R_+ = \{x \in R/[x, T(y)] + [y, T(x)] \in Z, \forall x, y \in R\}.$$

3. Main Results

We generalize the above definitions and prove many interesting results.

Definition 3.1. *Let R be an associative ring and S and T be two non-trivial automorphisms of R. They are said to be*

- (i) *a commuting pair of automorphism if $S(x)T(x) = T(x)S(x) \forall x \in R$ i.e. $[S(x), T(x)] = 0 \forall x \in R$;*
- (ii) *an anti commuting pair of automorphism if $S(x)T(x) = -T(x)S(x) \forall x \in R$;*
- (iii) *a semi-commuting pair of automorphism if either $S(x)T(x) = T(x)S(x)$ (or) $S(x)T(x) = -T(x)S(x)$ i.e, $S(x)T(x) = \pm T(x)S(x) \forall x \in R$;*
- (iv) *a strong commuting pair of automorphism if $[S(x), T(y)] = [T(x), S(y)]$ $\forall x, y \in R$;*
- (v) *a strong semi commuting pair of automorphism if*

$$[S(x), T(y)] = \pm[T(x), S(y)], \quad \forall x, y \in R;$$

- (vi) *acentralising pair of automorphism if $[S(x), T(x)] \in Z \forall x \in R$ (or) $S(x)T(x) - T(x)S(x) \in z, \forall x \in R$;*
- (vii) *an anti-centralising pair of automorphism if $S(x)T(x) = -T(x)S(x), \forall x \in R$;*
- $(viii)$ *a semi-centralising pair of automorphism if $S(x)T(x) - T(x)S(x) \in z($ or $)S(x)T(x) + T(x)S(x) \in Z, \forall x \in R$;*
- (ix) *a strong centralizing pair of automorphism if $[S(x), T(y)] - [T(x), S(y)] \in Z$, $\forall x, y \in R$;*
- (x) *a strong semi-centralising pair of automorphism if*

$$[S(x), T(y)] \pm [T(x), S(y)] \in Z, \quad \forall x, y \in R.$$

Definition 3.2. *Let R be an associative ring and S and T be two automorphisms of R. Define*

$$\begin{aligned} R =& \{x \in R/[S(x), T(y)] - [T(x), S(y)] \in Z, \forall y \in R\}; \\ R_+ =& \{x \in R/[S(x), T(y)] + [T(x), S(y)] \in Z, \forall y \in R\}; \\ R_0 =& \{x \in R/[S(x), T(x)] = 0\}. \end{aligned}$$

Lemma 3.1. *Let R be an associative ring and S and T be a commuting pair of automorphisms of R. Then they are Strong commuting pair of automorphism of R.*

Proof. Let S and T be a commuting pair of automorphisms of R. Then

$$[S(x) \quad T(x)] = 0, \quad \forall x \in R. \tag{3.1}$$

So, $[S(x+y) \cdot T(x+y)] = 0 \forall x, y \in R$, i.e, $[S(x)+S(y), T(x)+T(y)] = 0 \forall x, y \in R$, i.e, $[S(x), T(x)] + [S(x), T(y)] + [S(y), T(x)] + [S(y), T(y)] = 0 \forall x, y \in R$.

Using equation (3.1) we get

$$[S(x), T(y)] + [S(y) - T(x)] = 0, \quad \forall x, y \in R,$$

i.e, $[S(x), T(y)] = -[S(y), T(x)] = [T(x), S(y)] \forall x, y \in R$ and

$$[S(x), T(y)] = [T(x), S(y)], \quad \forall x, y \in R.$$

This proves that S and T are strong commuting pair of automorphisms of R. □

Lemma 3.2. *Let R be an associative ring and S and T be a centralizing pair of automorphisms of R. Then they are Strong centralising pair of automorphisms of R.*

Proof. Let R be an associative ring andS and T be a centralisiing pair of automorphisms of R. Then

$$[S(x) - T(x)] \in Z, \quad \forall x \in R. \tag{3.2}$$

Then, $[S(x+y) - T(x+y)] \in Z \forall x, y \in R$ i.e, $[S(x), T(x)] + [S(x), T(y)] + [S(y), T(x)] + [S(y), T(y)] \in Z \forall x, y \in R$.

Using equation (3.2) we get

$$[S(x), T(y)] + [S(y)] = [T(x)] \in Z, \quad \forall x, y \in R,$$

i.e, $[S(x), T(y)] = -[S(y), T(x)] = [T(x), S(y)] \forall x, y \in R$, and

$$[S(x), T(y)] = [T(x), S(y)], \quad \forall x, y \in R.$$

This proves that S and T are strong centralising pair of automorphisms of R. □

Lemma 3.3. *Let R be an associative ring and S and T be a strong semi centralizing pair of automorphisms of R. If a, be R Then $a+b \in R$ and $a-b \in R, \forall x, y \in R$.*

Proof. Let $a, b \in R$. Then

$$[S(a), T(y)] - [T(a), S(y)] \in Z, \quad \forall y \in R, \tag{3.3}$$

$$[S(b), T(y)] - [T(b), S(y)] \in Z, \quad \forall y \in R. \tag{3.4}$$

Expressions (3.3) and (3.4) give

$$[S(a)+S(b), T(y)] - [T(a)+T(b), S(y)] \in Z, \quad \forall y \in R,$$

i.e,

$$[S(a+b), T(y)] - [T(a+b), S(y)] \in Z, \forall y \in R.$$

This implies $a+b \in R_-$. Similarly (3.3) and (3.4) give $a-b \in R_-$. □

Lemma 3.4. *Let R be any ring and S and T be a strong semi centralizing pair of automorphisms of R. If a, $b \in R_+$ Then $a+b \in R_+$ and $a-b \in R_+$*

Proof. Similar to the proof of lemma 3.3. □

Lemma 3.5. *Let R be a 2-torision free ring. Let S and T be a strong commuting pair of automorphisms of R. Then the v are commuting pair of automorphisms of R.*

Proof. Let S and T be a strong commuting pair of automorphisms of R. Then

$$[S(x), T(y)] = [T(x), S(y)], \quad \forall x, y \in R, \tag{3.5}$$

i.e, $[S(x), T(x)] = [T(x), S(x)] = -[S(x), T(x)], \quad \forall x \in R$, i.e. S and T are commuting pair of automorphisms of R. □

Lemma 3.6. *Let R be a 2-torision free ring. Let S and T be a strong centralising pair of automorphisms of R, then they are centralising pair of automorphisms of R.*

Proof. Let S and T be a strong centralising pair of automorphisms of R. Then

$$[S(x), T(y)] - [T(x), S(y)] \in Z, \quad \forall x, y \in R, \tag{3.6}$$

$$\Rightarrow \quad [S(x),T(x)] - [T(x),S(x)] \in Z, \quad \forall x \in Z$$
$$\Rightarrow \quad [S(x),T(x)] + [S(x),T(x)] \in Z, \quad \forall x \in Z$$
$$\Rightarrow \quad 2[S(x),T(x)] \in Z, \quad \forall x \in Z$$
$$\Rightarrow \quad [S(x),T(x)] \in Z, \quad \forall x \in Z.$$

Finally, S and T are centralising automorphisms of R. □

Theorem 3.1. *Let R be a Prime ring and S and T be two non-trivial automorphisms of R, such that $S \neq T$. If S and T are commuting pair of automorphisms of R, then R is a commutative integral domain.*

Proof. Let S and T be two non-trivial commuting pair of automorphisms of R, such that $S \neq T$. Then

$$[S(x),T(x)] = 0, \quad \forall x \in R. \tag{3.7}$$

Replace x by $x+y$ in (3.7) we get,

$$[S(x+y),T(x+y)] = 0, \quad \forall x,y \in R,$$

i.e,

$$[S(x),T(x)] + [S(x),T(y)] + [S(y),T(x)] + [S(y),T(y)] = 0, \quad \forall x,y \in R.$$

Using (3.7) we get,

$$[S(x),T(y)] + [S(y)T(x)] = 0, \quad \forall x,y \in R,$$
$$[S(x),T(y)] = -[S(y),T(x)], \quad \forall x,y \in R,$$
$$[S(x),T(y)] = -[T(x),S(y)], \quad \forall x,y \in R. \tag{3.8}$$

Replace y by xy in (3.8) we get,

$$[S(x),T(xy)] = [T(x),S(xy)], \quad \forall x,y \in R;$$
$$[S(x),T(x)T(y)] = [T(x),S(x)S(y)], \quad \forall x,y \in R;$$
$$T(x)[S(x),T(y)] + [S(x),T(rx)]T(y) = S(x)[T(x),S(y)];$$
$$+ [T(x),S(x)]S(y), \quad \forall x,y \in R.$$

Using (3.7) we get,

$$T(x)[S(x),T(y)] = S(x)[T(x),S(y)], \quad \forall x,y \in R.$$

Using (3.8) we get

$$T(x)[S(x),T(y)] = S(x)[S(x),T(y)], \quad \forall x,y \in R,$$

i.e,

$$(S(x)-T(x))[S(x),T(y)] = 0, \quad \forall x,y \in R. \tag{3.9}$$

Since T is an automorphism we have

$$(S(x)-T(x))[S(x),z] = 0, \quad \forall x,z \in R. \tag{3.10}$$

Now,

$$y[S(x),z] = [S(x),yz] - [S(x),y]z, \quad \forall x,y,z \in R,$$

i.e.,

$$\begin{aligned}(S(x)-T(x))y[S(x),z] =&(S(x)-T(x))[S(x),yz] \\ &- (S(x)-T(x))[S(x),y]z = 0.\end{aligned}$$

This is true for all $y \in R$. Hence

$$(S(x)-T(x))R[S(x),z] = 0, \quad \forall x,z \in R. \tag{3.11}$$

Since $S \neq T$ there must be atleast one $x_0 \in R$ such that $S(x_0) \neq T(x_0)$. Since R is Prime $[S(x_0), z] = 0 \forall z \in R$ i.e. $S(x_0) \in z$. Suppose $S(y) \notin Z$ for some $y \in R$ Thus $S(x_0) + S(y) \notin Z$. Using (3.11) we get

$$[S(y)-T(y)]R[S(y),z] = 0, \quad \forall z \in R$$

Since

$$S(y) \notin Z, [\ S(y),z] \neq 0. \tag{3.12}$$

Since R is Prime. $S(y)-T(y) = 0$ i.e, $S(y) = T(y)$. Similarly,

$$S(x_0+y) = T(x_0+y) \tag{3.13}$$

(3.12) and (3.13) gives $S(x_0) = T(x_0)$ So $S(y) \in Z$ is contradictives. Since S is an automorphism of $Rx \in Z \forall x \in R$ and R is commutative. □

Remark 3.1. *Taking S as identity automorphism of R, we get the Theorem of J. Luh [4].*

Theorem 3.2. *Let R be a 2 torision free prime ring and S and T are non-trivial automorphisms of R. If S and T are strong commuting automorphisms of R then R is commutative.*

Proof. Let S and T be strong commuting pair of automorphismsof R. By lemma 3.5, They are commuting pair of automorphisms of R.

R is commutative follows from theorem 3.1. □

Lemma 3.7. *Let R be a prime ring and $x, y \in R$ such that $0 \neq x \in Z$ If $xy = 0$, then $y = 0$.*

Proof. Let $z \in R$ be any element then $zxy = 0 \Rightarrow xzy = 0$ (since $x \in Z$) $\Rightarrow xRy = 0$. Since R is prime $x = 0$ or $y = 0$. Now As $x \neq 0$, we get $y = 0$. □

Lemma 3.8. *Let b and ab be in the centre of a prime ring R. If b is not zero,then a is in the centre of R.*

Proof. Let $x \in R$ be any element. Now

$$\begin{aligned}(ax - xa)b =& axb - xab\\ =& abx - abx \ (\because b \in Z \text{ and } ab \in Z)\\ =& 0\end{aligned}$$

By lemma 3.7, we get $ax - xa = 0$ i.e, $ax = xa \forall x \in R$, i.e, $a \in Z$. □

Theorem 3.3. *Let R be a prime ring with non-trivial centralizing pair of automorphisms S and T such that $S \neq T$. Then R is commutative integral domains.*

Proof. Let S and T be non trivial centralizing automorphisms of R such that $S \neq T$. Now,

(3.14) $$[S(x), T(x)] \in z, \quad \forall x \in R.$$

We will first prove that S and T are commuting pair of automorphisms of R. Suppose there exists $x_0 \in R$ such that

(3.15) $$[S(x_0), T(x_0)] \neq 0.$$

Replacing x by $x_0 + y$ in (3.14) we get, $[S(x_0 + y), T(x_0 + y)] \in Z, \forall y \in R$,

$$[S(x_0), T(x_0)] + [S(x_0), T(y)] + [S(y), T(x_0)] + [S(y), T(y)] \in Z, \quad \forall y \in R.$$

Using (3.14) we get

(3.16) $$[S(x_0), T(y)] + [S(y), T(x_0)] \in Z, \quad \forall y \in R.$$

So,

(3.17) $$[S(x_0), [S(x_0) T(y)] + [S(y), T(x_0)]] = 0, \quad \forall y \in R.$$

Replace y by x_0^2 in (iv) we get,

$$\text{i.e } \left[S\left(x_0\right), \left[S\left(x_0\right), T\left(x_0^2\right)\right] + \left[S\left(x_0^2\right), T\left(x_0\right)\right]\right] = 0$$

$$\text{i.e. } \left[S\left(x_0\right), S\left(x_0\right), T\left(x_0\right) T\left(x_0\right)\right] + S\left(x_0\right)\left[S\left(x_0\right) S\left(x_0\right), T\left(x_0\right)\right] = 0$$

$$\left[S\left(x_0\right), T\left(x_0\right)\left[S\left(x_0\right), T\left(x_0\right)\right] + \left[S\left(x_0\right), T\left(x_0\right)\right], T\left(x_0\right)\right]$$

$$+ \left[S\left(x_0\right), S\left(x_0\right)\left[S\left(x_0\right), T\left(x_0\right)\right] + \left[S\left(x_0\right), T\left(x_0\right)\right], S\left(x_0\right)\right] = 0.$$

Using (3.14) we get,

$$\left[S\left(x_0\right), 2\, T\left(x_0\right)\left[S\left(x_0\right), T\left(x_0\right)\right] + \left[S\left(x_0\right), 2\, S\left(x_0\right)\left[S\left(x_0\right), T\left(x_0\right)\right]\right]\right] = 0$$

$$2\, T\left(x_0\right)\left[S\left(x_0\right)\left[S\left(x_0\right), T\left(x_0\right)\right] + 2\left[\, S\left(x_0\right), T\left(x_0\right)\right]\left[S\left(x_0\right), T\left(x_0\right)\right]\right]$$

$$+2\, S\left(x_0\right)\left[S\left(x_0\right)\left[S\left(x_0\right), T\left(x_0\right)\right] + 2\left[\, S\left(x_0\right), S\left(x_0\right)\right]\left[S\left(x_0\right), T\left(x_0\right)\right]\right] = 0.$$

Using (3.14) we get, $0 + 2\left[\, S\left(x_0\right), T\left(x_0\right)\right]^2 + 0 + 0 = 0$ i.e, $2\left[\, S\left(x_0\right), T\left(x_0\right)\right]^2 = 0$.
If Char $R \neq 2$ then $\left[S\left(x_0\right), T\left(x_0\right)\right]^2 = 0$. Using lemma 3.8, $\left[S\left(x_0\right) T\left(x_0\right)\right] = 0$.
Contradicting (3.15), this contradiction proves that $[S(x), T(x)] = 0 \forall x \in R$ if Char $R \neq 2$. Assume Char $R = 2$, then $x = -x \forall x \in R$. Now,

$$[[S(x)S(y)]T(x)] + \left[S\left(x^2\right), T(y)\right]$$
$$= [S(x)S(y) - S(y)S(x), T(x)] + S(x)[S(x), T(y)] + [S(x), T(y)]S(x)$$
$$= [S(x)S(y), T(x)] - [S(y)S(x), T(x)] + S(x)[S(x), T(y)] + [S(x), T(y)]S(x)$$
$$= S(x)[S(y), T(x)] + [S(x)T(x)]S(y) - S(y)[S(x), T(x)] - [S(y), T(x)]S(x)$$
$$+ S(x)[S(x), T(y)] + [S(x), T(y)]S(x)$$

Using the fact $[S(x), T(x)] \in Z \forall x$ and $x = x \forall x \in R$. We get

$$S(x)[S(y), T(x)] + [S(y), T(x)]S(x) + S(x)[S(x), T(y)] + [S(x), T(y)]S(x)$$
$$= S(x)\{[S(y), T(x)] + [S(x), T(y)]\} + \{[S(y), T(x)] + [S(x), T(y)]\}S(x)$$
$$= 2\, S(x)\{[S(y), T(x)] + [S(x), T(y)]\}$$
$$= 0 \quad (\text{since char } R = 2).$$

Thus

(3.18) $$[[S(x), S(y)], T(x)] + \left[S\left(x^2\right), T(y)\right] = 0, \quad \forall x, y \in R.$$

Put $z = T(x)$,

(3.19) $$[S(x)_2\, S(y), z] + \left[S\left(x^2\right), T(y)\right] = 0, \quad \forall x, y \in R, z = T(x).$$

Put $x = y$ in (3.19) we get $[S(x^2), T(x)] = 0, \forall x \in R$, i.e.

(3.20) $$\left[S\left(x^2\right), z\right] = 0.$$

Put $y = xS^{-1}zS$ in (3.19). Then $S(y) = S\left(xS^{-1}(z\ S)\right) = S(x)z$. So (3.19) becomes

$$[[S(x), S(x)z], z] + \left[S\left(x^2\right), T\left(xS^{-1}(zS)\right] = 0$$
$$[S(x)[S(x), z] + [S(x), S(x)]z, z] + \left[S\left(x^2\right), T(x)T\left(S^{-1}(z)\right] = 0$$
$$[S(x)[S(x), z], z] + \left[S\left(x^2\right), z^w\right] = 0,$$

where $w = T\left(s^{-1}(z)\right)$

$$S(x)[[S(x), z], z] + [S(x), z][S(x), z] + z\left[S\left(x^2\right)w\right] + \left[S\left(\left(x^2\right), z\right]w = 0\ ,$$
$$[S(x), z] = [S(x), T(x)] \in Z,$$

$\forall x \in R$. Using (3.20) we get $[S(x), z]^2 + z\left[S\left(x^2\right), w\right] = 0$, i.e.,

(3.21) $$[S(x), z]^2 = -z\left[S\left(x^2\right), w\right] = z\left[S\left(x^2\right), w\right], \quad \forall x \in R.$$

Put $y = xs^{-1}(z)x$ in (3.19). Then $S(y) = S\left(xs^{-1}(z)x\right) = S(x)zS(x)$. So, (3.19) becomes

$$[S(x), S(x)zS(x), z] + \left[S\left(x^2\right), T\left(xs^{-1}(z)x\right)\right] = 0$$
$$\left[S\left(x^2\right)zS(x) - S(x)zS\left(x^2\right), z\right] + \left[S\left(x^2\right), T(x)T\left(S^{-1}(zS)\right)T(x)\right] = 0$$
$$\left[S\left(x^2\right)zS(x), z\right] - \left[S(x)zS\left(x^2\right), z\right] + \left[S\left(x^2\right), zwz\right] = 0$$
$$S\left(x^2\right)[z\ S(x), z] + \left[S\left(x^2\right), z\right]zS(x) - S(x)z\left[S\left(x^2\right), z\right]$$
$$-[S(x)z, z]S\left(x^2\right) + \left[S\left(x^2\right), zwz\right] = 0.$$

Using (3.20) we get

$$S\left(x^2\right)[z\ S(x), z][S(x)z, z]S\left(x^2\right) + \left[S\left(x^2\right), zwz\right] = 0$$
$$S\left(x^2\right)\{z[\}S(x), z] + [z, z]S(x)\} - \{S(x), [z, z] + [S(x), z]z\}S(x)$$
$$+\left[S\left(x^2\right), zwz\right] = 0$$
$$\text{i.e. } S\left(x^2\right)z[\ S(x), z] - [S(x), z]zS\left(x^2\right) + \left[S\left(x^2\right), zwz\right] = 0.$$

Since,

$$[S(x), z] = [S(x), T(x) \in Z\forall x$$
$$[S(x), z]\left[S\left(x^2\right)z - zS\left(x^2\right)\right] + \left[S\left(x^2\right), zwz\right] = 0$$
$$\text{i.e. } [S(x), z]\left[S\left(x^2\right), z\right] + \left[S\left(x^2\right), zwz\right] = 0.$$

Using (3.20) we get $[S\ (x^2), zwz] = 0$ i.e,

$$z\left[S\left(x^2\right), wz\right] + \left[S\left(x^2\right), z\right] wz = 0.$$

Using (3.20) we get $z\ [S\ (x^2), wz] = 0$ i.e.

$$z\left\{w\left[S\left(x^2\right), z\right] + \left[S\left(x^2\right), w\right] z\right\} = 0.$$

Using (3.20) we get $z\ [S\ (x^2), w]\ z = 0$. Using (3.21) we get $[S(x), z]^2 \cdot z = 0$. Since $z = T(x) \neq 0$, we get $[S(x), z]^2 = 0 \forall x$, i.e., $[S(x), z] = 0 \forall x$, i.e., $[S(x), T(x)] = 0 \forall x \in R$ i.e, S and T are commuting pair of automorphim. Hence by Theorem 3.1, R is commutative. □

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