BIANCHI TYPE III NEW HOLOGRAPHIC DARK ENERGY MODEL WITH HYBRID EXPANSION LAW AND VARIABLE G AND Λ

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ABSTRACT. In this paper, we study a spatially homogeneous and anisotropic Bianchi Type III universe filled with cold dark matter and non-interacting holographic dark energy in the frame work of General Relativity. We consider both the parameters of the Einstein field equations, namely, the Newtonian gravitational constant G and the Einstein cosmological constant Λ to be time-varying quantities. To obtain exact solutions of Einstein's field equations, we assume the average scale factor a to be a combination of a power law and an exponential function, called hybrid expansion law, and the scalar of expansion to be proportional to the Eigen value of the shear tensor. Physical and geometrical properties of some parameters of cosmological importance including the jerk parameter of our model are explored. The properties are found to be consistent with cosmological scenarios of the present-day accelerated expanding universe.

1. INTRODUCTION

Various recent cosmological and astrophysical observations such as Supernovae Type Ia (SN Ia) [1-3], Cosmic Microwave Background (CMB) [4,5], Large

Scale Structure (LSS) [6-8], Baryon Acoustic Oscillation (BAO) [9] etc. reveal that the universe is dominated by some mysterious form of physical entity, dubbed dark energy, which is leading the present universe to pass through a phase of accelerated expansion. It is believed that there was also a cosmic acceleration, called inflation, which occurred at the very early times of the universe. Analysis of various cosmological observations such as Wilkinson Microwave Anisotropy Probe (WMAP) [4,10] provided strong support for the existence of an inflationary phase at an early epoch as well as the late time cosmic acceleration. The observational data indicate that dark energy comprises of more than sixty eight percent of the total energy of the universe and the equation of state (EoS) parameter ω_{HDE}, which is the ratio of the pressure of dark energy to its energy density, is very close to -1. This enforced the researchers to investigate the current phase of accelerated expansion of the universe and find out the true nature of dark energy which must have a high negative pressure. A number of dark energy candidates have been considered in the literature, ranging from the cosmological constant Λ, introduced by Einstein in his field equations, to a variety of dynamically evolving scalar fields such as quintessence [11], phantom [12], k-essence [13], tachyon [14], dilatonic ghost condensate [15] etc. and exotic fluid models like Chaplygin gas models [16].

Holographic dark energy is another important candidate of dark energy which emerges from a quantum gravitational principle called the Holographic Principle. The basic idea of Holographic Principle is that the number of degrees of freedom directly related to entropy of a physical system scales with the enclosing surface area of the system rather than with its volume. It was first put forwarded by G.'t Hooft [17] to explain the thermodynamics of black hole physics. Later Fischler and Susskind [18] extended this principle to the cosmological setting by stating that the gravitational entropy within a closed surface should not be always larger than the particle entropy that passes through the past light-cone of that surface. Subsequently, various modifications of this version were proposed in the literature. Granda and Oliveros [19] proposed a new holographic dark energy density of the form $\rho_{HDE} \approx \alpha H^2 + \beta \dot{H}$; where H is the Hubble parameter and α and β are constants which must satisfy the margins imposed by recent observational data. Granda and Oliveros [20] also established correspondence between quintessence, tachyon, k-essence and dilaton dark energy models with this holographic dark energy in flat Friedmann-Robertson-Walker

(FRW) universe. Many other authors have studied several aspects of holographic dark energy in different contexts (Chattopadhyay, Farajollahi et al., Karami and Fehri, Malekjani, Rao et al., Mete et al., Ghaffari, Saridakis, Srivastava et al., Dixit et al.etc.) [21].

In the framework of General Relativity, the basis of all cosmological models is the Einstein field equations which contain the two constants G and Λ. The Newtonian gravitational constant G plays the role of coupling constant between the geometry of space and matter in Einstein's field equations while the Einstein cosmological constant Λ is now a well-established candidate for dark energy. The prevailing cosmological model with constant G and Λ also agrees with observational evidence such as the observed redshift and the current rate of accelerated expansion of the universe. However, a difficulty arises with the extremely small value of Λ leading to the fine-tuning problem. To overcome this difficulty attempts have been taken to construct cosmological models with time varying Λ. Recently, a time-varying cosmological constant term has become a focal point of interest as it could solve the cosmological constant problem in a natural way. In recent past, a number of authors have investigated cosmological scenarios with different decay laws for the time-dependent Λ [22]. A number of modifications of General Relativity are also available in the literature in which the gravitational constant term G varies with time so as to incorporate Mach's principle in General Relativity or to achieve potential unification between gravitation and elementary particle physics. In this line, Dirac [23] was the first to propose the idea of a time varying G. Since then various studies with variable G have been carried out by a number of authors. In an attempt to incorporate Mach's principle in General Relativity, Brans and Dicke [24] proposed an alternative approach leading to the Brans-Dicke theory of gravity in which the Newtonian gravitational constant G is replaced by a scalar field coupling to gravity through a new parameter representing a variable G. Justification of a time varying G is also established with the use of conformal invariance and the local transformations induced by it [25]. From the perspective of incorporating particle physics into Einstein's theory of gravity, Zeldovich [26] showed a simple way to interpret the cosmological constant term Λ in terms of quantum mechanics and the physics of the vacuum. But dimensional analysis [27] and cosmic constraints from action principle [28] require that either both G and Λ remain constant or both are variable. In an evolving universe, it is also very likely to look at these

parameters as functions of cosmic time. In this paper, we focus our attention to study the spatially homogeneous and anisotropic Bianchi Type III universe filled with non interacting new holographic dark energy and cold dark matter with variable gravitational and cosmological constant terms.

The paper is organized as follows: In Sect. 2, we derive the cosmic evolution equations from the Einstein field equations in the background of spatially homogeneous and anisotropic Bianchi Type III space-time. Cosmological solutions of the field equations are obtained in Sect. 3. For this purpose, we take the average scale factor a to be a combination of power law and exponential function, called hybrid expansion law, and the expansion scalar θ to be proportional to the Eigen value of the shear tensor σ_i^j. We also study the physical and geometrical properties of some cosmological parameters in the context of the solutions. Finally, we conclude the paper in Sect. 4 with a brief discussion.

2. METRIC AND FIELD EQUATIONS

Spatially homogeneous and anisotropic Bianchi Type III space-time is given by the metric

$$(2.1) \qquad ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2\alpha x} dy^2 + C^2 dz^2,$$

where A, B, C are functions of the cosmic time t and α is a positive constant.

The Einstein field equations for a universe filled with cold dark matter and non-interacting new holographic dark energy with time varying G and Λ are given by

$$(2.2) \qquad R_i^j - \frac{1}{2} R g_i^j = -8\pi G(t)(T_i^j + \bar{T}_i^j) + \Lambda(t) g_i^j,$$

where R_i^j is the Ricci tensor, R the Ricci scalar, and T_i^j and $\bar{T}_i^j$ are respectively the energy momentum tensors for cold dark matter and new holographic dark energy given by

$$(2.3) \qquad T_i^j = \rho_m u_i u^j$$

and

$$(2.4) \qquad \bar{T}_i^j = (\rho_{HDE} + p_{HDE}) u_i u^j + g_i^j p_{HDE},$$

ρ_m being the energy density of cold dark matter, and ρ_{HDE} and p_{HDE} are the energy density and pressure of the new holographic dark energy respectively.

Following Granda and Oliveros [24], we propose the new holographic dark energy density to be of the form

$$(2.5) \qquad \rho_{HDE} = k(\delta H^2 + \beta \dot{H}),$$

where k is a constant of proportionality, H, the Hubble parameter and δ and β are constants.

In comoving coordinates, for the metric 2.1 the equations 2.2 with 2.3 and 2.4 lead to the following system of field equations:

$$(2.6) \qquad \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\alpha^2}{A^2} = -8\pi G p_{HDE} + \Lambda,$$

$$(2.7) \qquad \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = -8\pi G p_{HDE} + \Lambda,$$

$$(2.8) \qquad \frac{\ddot{A}}{C} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = -8\pi G p_{HDE} + \Lambda,$$

$$(2.9) \qquad \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{C}\frac{\dot{C}}{C} - \frac{\alpha^2}{A^2} = 8\pi G(\rho_m + \rho_{HDE}) + \Lambda,$$

$$(2.10) \qquad \frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0,$$

where an over dot denotes differentiation with respect to t.

Vanishing divergence of the Einstein tensor yields,

$$(2.11) \qquad \begin{aligned} & 8\pi \dot{G}(\rho_m + \rho_{HDE}) + \dot{\Lambda} + 8\pi G[\dot{\rho}_m + \dot{\rho}_{HDE} \\ & + (\rho_m + \rho_{HDE} + p_{HDE})(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C})] = 0 \end{aligned}$$

Also, from continuity equation of cold dark matter and non-interacting holographic dark energy, we have

$$(2.12) \qquad \dot{\rho}_m + \rho_m(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) = 0,$$

$$(2.13) \qquad \dot{\rho}_{HDE} + (\rho_{HDE} + p_{HDE})(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) = 0.$$

Using 2.12 and 2.13 in 2.11, we obtain

$$(2.14) \qquad \dot{G}(t) = \frac{-\dot{\Lambda}(t)}{8\pi(\rho_m + \rho_{HDE})},$$

which shows that $G(t)$ increases or decreases according as $\Lambda(t)$ decreases or increases, and vice-versa.

For the metric 2.1, the average scale factor a, the spatial volume V and the mean Hubble parameter H are defined by

$$(2.15) \qquad\qquad a = (ABC)^{\frac{1}{3}}$$

$$(2.16) \qquad\qquad V = a^3 = ABC$$

$$(2.17) \qquad\qquad H = \frac{1}{3}(H_1 + H_2 + H_3),$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters along the three spatial directions x, y, z respectively.

The expansion scalar θ, the shear scalar σ, the mean anisotropy parameter A_m and the deceleration parameter q are given by

$$(2.18) \qquad\qquad \theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}$$

$$(2.19) \qquad\qquad \sigma^2 = \frac{1}{3}[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\dot{C}}{C}\frac{\dot{A}}{A}]$$

$$(2.20) \qquad\qquad A_m = \frac{1}{3}\sum_{i=1}^{3}(\frac{H_i - H}{H})^2$$

$$(2.21) \qquad\qquad q = \frac{d}{dt}(\frac{1}{H}) - 1.$$

3. Cosmological solutions of the field equations

On integration, the equation 2.10 yields

$$(3.1) \qquad\qquad A = mB$$

where m is a constant.

Now, in consideration of the number of independent equations involving the eight unknowns A, B, C, ρ_m, ρ_{HDE}, p_{HDE}, G and Λ, we need two more additional constraint equations to obtain an explicit solution of the system. So, we

first consider the hybrid expansion law put forwarded by Akarsu et al. [29] and take the average scale factor a as

$$(3.2) \qquad a(t) = a_0 \left(\frac{t}{t_0}\right)^\gamma e^{\xi\left(\frac{t}{t_0}-1\right)},$$

where γ and ξ are non-negative constants and a_0 and t_0 respectively denote the present value of the average scale factor and age of the universe. The relation 3.2 yields the exponential law cosmology for $\gamma=0$ and gives the power law cosmology for $\xi=0$. Further, this hybrid form of the average scale factor provided a simple transition of the universe from the early decelerating phase to the present accelerating phase.

Secondly, we consider the expansion scalar (θ) to be proportional to the Eigen value σ_2^2 of the shear tensor σ_i^j so that we can take

$$(3.3) \qquad B = l_1 (AC)^{m_1},$$

where $l_1 > 0$ and $m_1 > 0$ are constants. Then, from 2.15, 3.1, 3.2 and 3.3, we get the directional scale factors as

$$(3.4) \qquad A = m\left[l_1^{\frac{1}{m}} a_0^3 \left(\frac{t}{t_0}\right)^{3\gamma} e^{3\xi\left(\frac{t}{t_0}-1\right)}\right]^{\frac{m_1}{1+m_1}},$$

$$(3.5) \qquad B = \left[l_1^{\frac{1}{m}} a_0^3 \left(\frac{t}{t_0}\right)^{3\gamma} e^{3\xi\left(\frac{t}{t_0}-1\right)}\right]^{\frac{m_1}{1+m_1}},$$

$$(3.6) \qquad C = \frac{1}{ml_1^{\frac{2}{1+m_1}}}\left[a_0 \left(\frac{t}{t_0}\right)^\gamma e^{\xi\left(\frac{t}{t_0}-1\right)}\right]^{\left(3-\frac{6m_1}{1+m_1}\right)}.$$

The directional Hubble parameters H_1, H_2, H_3 and the mean Hubble parameter H are obtained as

$$(3.7) \qquad H_1 = \frac{\dot{A}}{A} = \frac{3m_1}{1+m_1}\left[\frac{\gamma}{t} + \frac{\xi}{t_0}\right]$$

$$(3.8) \qquad H_2 = \frac{\dot{B}}{B} = \frac{3m_1}{1+m_1}\left[\frac{\gamma}{t} + \frac{\xi}{t_0}\right]$$

$$(3.9) \qquad H_3 = \frac{\dot{C}}{C} = \frac{3(1-m_1)}{1+m_1}\left[\frac{\gamma}{t} + \frac{\xi}{t_0}\right]$$

$$(3.10) \qquad H = \frac{\gamma}{t} + \frac{\xi}{t_0}.$$

The expressions for the expansion scalar (θ), the shear scalar (σ), the mean anisotropy parameter (A_m) and deceleration parameter (q) are obtained as

$$(3.11) \qquad \theta = 3\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)$$

$$(3.12) \qquad \sigma^2 = \left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 \frac{(12m_1^2 - 12m_1 + 3)}{(1+m_1)^2}$$

$$(3.13) \qquad A_m = 2\left(\frac{2m_1 - 1}{1 + m_1}\right)^2$$

$$(3.14) \qquad q = -1 + \gamma^2 t_0^2 (\gamma t_0 + \xi t)^{-2}.$$

Now, from 2.5, we have

$$(3.15) \qquad \rho_{HDE} = k\left[\delta\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 - \frac{\beta\gamma}{t^2}\right].$$

Using 3.10 in 2.12, we obtain

$$(3.16) \qquad \rho_m = M t^{-3\xi} e^{\frac{-\xi t}{t_0}},$$

where M is a constant of integration. Using 3.10 and 3.15 in 2.13, we get

$$(3.17) \qquad p_{HDE} = \frac{2k\gamma\delta}{3t^2} - \frac{2k\beta\gamma}{3t^3\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)} - k\delta\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 + \frac{k\beta\gamma}{t^2}.$$

Subsequently, we obtain

$$(3.18) \qquad G(t) = \frac{\frac{18m_1 - 36m_1^2}{(1+m_1)^2}\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 + \frac{6m_1\gamma}{(1+m_1)t^2}}{8\pi\left[\frac{2k\gamma\delta}{3t^2} - \frac{2k\gamma\beta}{3t^3\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)} + M t^{-3\xi} e^{\frac{-\xi t}{t_0}}\right]}$$

$$(3.19)$$

$$\Lambda(t) = \frac{9(m_1^2 - m_1 + 1)}{(1+m_1)^2}\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 - \frac{3\gamma}{t^2(1+m_1)}$$

$$+ \frac{\left[\frac{18m_1 - 36m_1^2}{(1+m_1)^2}\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 + \frac{6m_1\gamma}{t^2(1+m_1)}\right]\left[\frac{2k\gamma\delta}{3t^2} - \frac{2k\beta\gamma}{3t^3\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)} - k\delta\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 + \frac{k\gamma\beta}{t^2}\right]}{\left[\frac{2k\gamma\delta}{3t^2} - \frac{2k\gamma\beta}{3t^3\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)} + M t^{-3\xi} e^{\frac{-\xi t}{t_0}}\right]}.$$

The EoS parameter ω_{HDE} of holographic dark energy and the total energy density parameter Ω are obtained as

$$(3.20) \qquad \omega_{HDE} = \frac{p_{HDE}}{\rho_{HDE}} = \frac{\frac{2k\gamma\delta}{3t^2} - \frac{2k\beta\gamma}{3t^3(\frac{\gamma}{t}+\frac{\xi}{t_0})} - k\delta(\frac{\gamma}{t}+\frac{\xi}{t_0})^2 + \frac{k\gamma\beta}{t^2}}{k[\delta(\frac{\gamma}{t}+\frac{\xi}{t_0})^2 - \frac{\beta\gamma}{t^2}]},$$

$$(3.21) \qquad \Omega = \Omega_m + \Omega_{HDE} = \frac{\rho_m}{3H^2} + \frac{\rho_{HDE}}{3H^2} = \frac{Mt^{-3\xi}e^{\frac{-\xi t}{t_0}} + k[\delta(\frac{\gamma}{t}+\frac{\xi}{t_0})^2 - \frac{\beta\gamma}{t^2}]}{3(\frac{\gamma}{t}+\frac{\xi}{t_0})^2}.$$
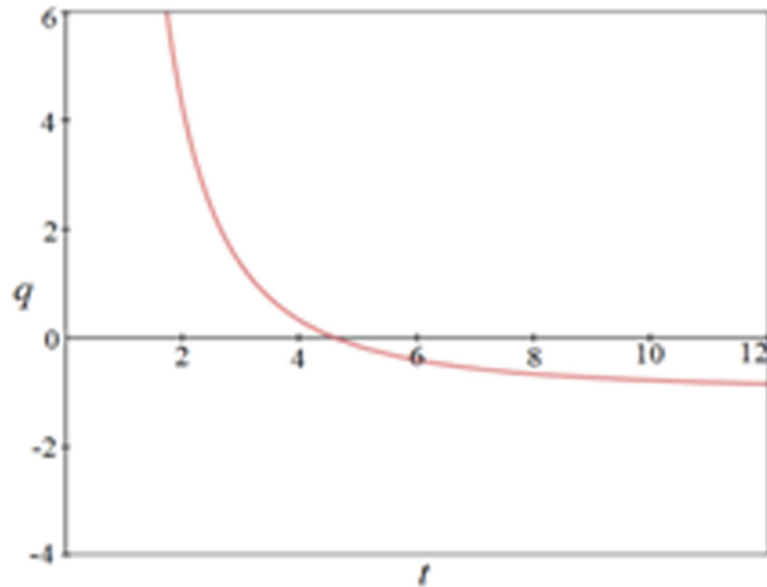

FIGURE 1. The variation of the deceleration parameter q vs.Cosmic time t with $\xi=1$, $\gamma=0.5$, $to=13.8$

Fig.3 shows that the deceleration parameter q is a decreasing function of the cosmic time t. Its value is positive at early times of the evolution of the universe and decreases rapidly approaching -1 asymptotically at late times. The negative values of the deceleration parameter show acceleration in the expansion of the universe. Thus, our model describes a transition from deceleration to acceleration in an evolving universe which is in agreement with the present observational data. Fig.2 shows that both ρ_{HDE} and ρ_m are decreasing functions of cosmic time t. At late times, ρ_m tends to zero and ρ_{HDE} is near to zero.
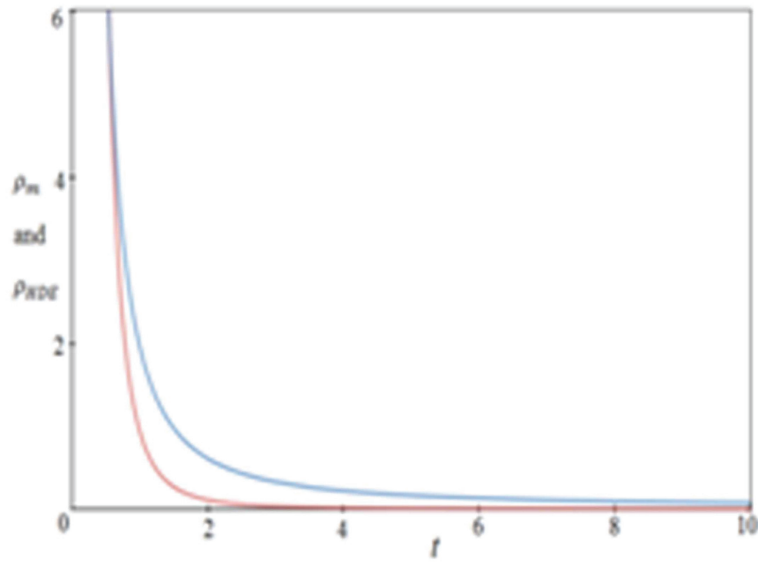
FIGURE 2. The variation of holographic dark energy (HDE) density ρ_{HDE} and cold dark matter energy density ρ_m vs. cosmic time t with $\gamma = 0.5$, $\delta=2$, $\beta=0.99$, $\xi=1$, $k=3$, $t_0=13.8$, $M=6$. The blue line represents the HDE density and red line represents the cold dark matter density
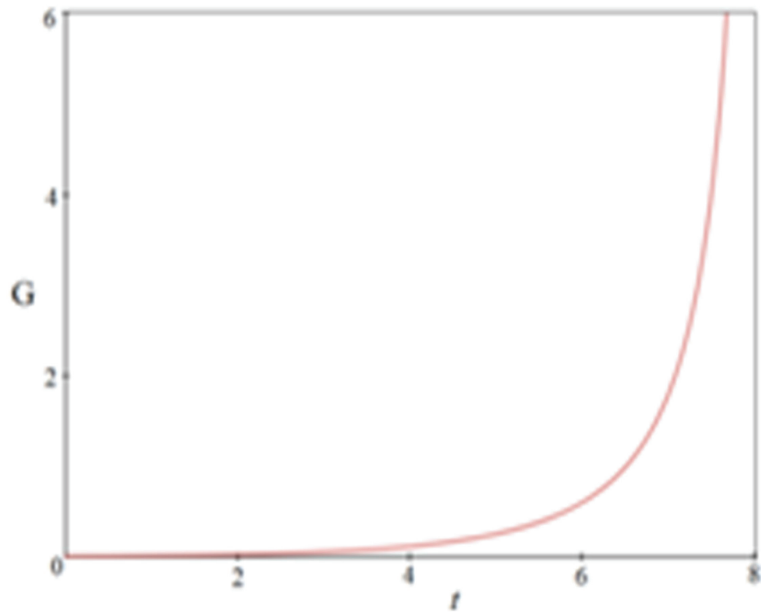

FIGURE 3. The variation of gravitational constant term G vs. cosmic time t with $m_1=0.1$, $\gamma = 0.5$, $\delta=.5$, $\beta=0.99$, $\xi=1$, $k=3$, $t_0=13.8$, $M=6$.
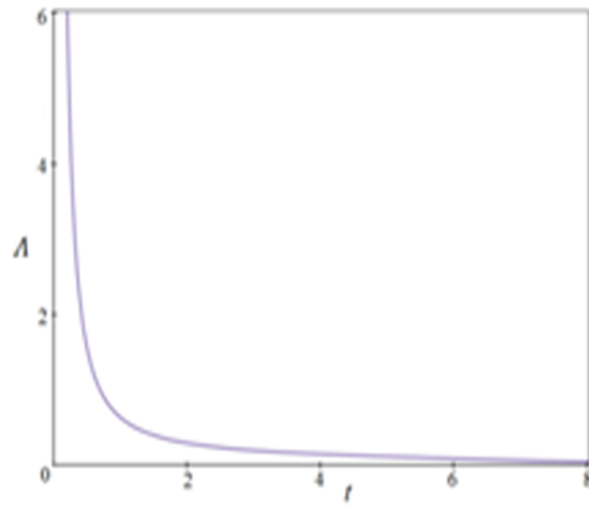
FIGURE 4. The variation of gravitational constant term Λ vs. cosmic time t with $m_1 = 0.1$, $\gamma = 0.5$, $\delta = .5$, $\beta = 0.99$, $\xi = 1$, $k = 3$, $t_0 = 13.8$, $M = 6$.

Fig.3 shows that G is an increasing function of cosmic time t while Fig.4 shows that Λ is a decreasing function of t. Thus, Fig.3 and Fig.4 justify the inference of the equation 2.14.
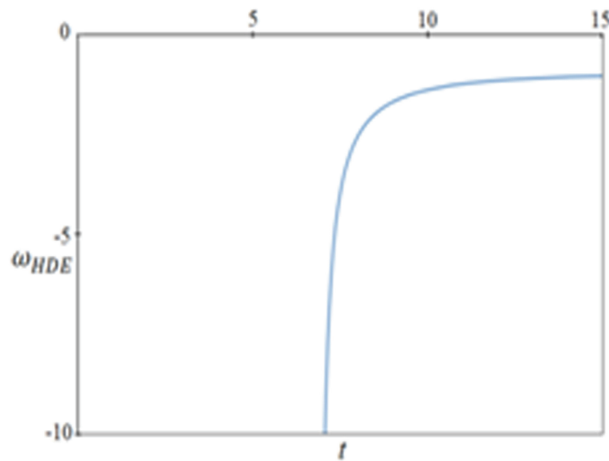

FIGURE 5. The variation of the EoS parameter ω_{HDE} vs. Cosmic time t with $\gamma = 0.5$, $\delta = .5$, $\beta = 0.99$, $\xi = 1$, $k = 3$, $t_0 = 13.8$, $M = 6$.
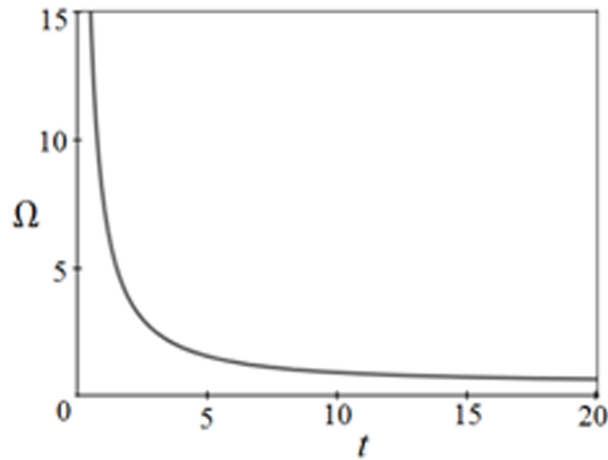
FIGURE 6. The variation of the total energy density parameter Ω vs. cosmic time t with $\gamma = 0.5$, $\delta = .5$, $\beta = 0.99$, $\xi = 1$, $k = 3$, $t_0 = 13.8$, $M = 6$.

From Fig. 5, it is clear that the EoS parameter ω_{HDE} evolves from phantom phase, increases rapidly and approaches -1 asymptotically at late times. Thus, at late times, our model behaves like ΛCDM model. Fig. 6 shows that the total energy density decreases rapidly in the early era of evolution of the universe and approaches 1 at late times. So, our model comes close to a flat universe at late times.

COSMIC JERK PARAMETER:

The cosmic jerk parameter is a dimensionless quantity defined by

$$(3.22) \qquad j(t) = \frac{1}{H^3} \frac{\dddot{A}}{A}.$$

For our model, cosmic jerk parameter is obtained as

$$(3.23) \qquad j(t) = 1 - \frac{\xi}{t^2(\frac{\gamma}{t} + \frac{\xi}{t_0})^2} - 2\frac{\gamma}{t^2(\frac{\gamma}{t} + \frac{\xi}{t_0})^2} - \frac{\gamma}{t^2(\frac{\gamma}{t} + \frac{\xi}{t_0})^3}.$$

From equation 3.23, it is clear that the cosmic jerk parameter tends to 1 at late times showing thereby that our model corresponds to ΛCDM model.

4. CONCLUSION

In this paper, we study a spatially homogeneous and anisotropic Bianchi Type III universe filled with cold dark matter and non-interacting new holographic dark energy with density $\rho_{HDE} = k(\delta H^2 + \beta \dot{H})$, where k is a constant of proportionality and δ and β are constants. Exact solutions of the Einstein field equations with time varying G and Λ are obtained by choosing a hybrid expansion law and by assuming the scalar of expansion to be proportional to the Eigen value of the shear tensor. We investigate physical and geometrical properties of some cosmological parameters of the model obtained and find that:

- At $t = 0$, the expansion scalar (θ), shear scalar (σ) diverge and scale factor (a), spatial volume (V) vanish. Hence, our model exhibits point type singularity.
- The universe evolves with a zero volume at $t = 0$ and $\frac{dH}{dt}$ tends to 0 at t tends to ∞
- The anisotropy parameter $A_m \neq 0$ except at $m_1 = \frac{1}{2}$. Hence our model will retain its anisotropic character throughout the evolution of the universe except for $m_1 = \frac{1}{2}$. The universe is isotropic only when $m_1 = \frac{1}{2}$.
- Figure 1 shows that the deceleration parameter q is positive at early stages of the evolution of the universe and decreases rapidly approaching -1 asymptotically at late times. Thus the model exhibits de-Sitter like expansion at late times and transition from decelerating expansion phase to the present accelerating phase. This is in agreement with the present observational data.
- Figure 2 exhibits that both new holographic dark energy density and cold dark matter energy density decrease as the universe evolves. At late times, the cold dark matter energy density tends to zero and holographic dark energy density is near to zero.
- From Figure 3, it is clear that the Newtonian gravitational term G increases rapidly as the time evolves and Figure 4 indicates that the cosmological constant term Λ decreases from a very large value to a small value as t increases. Therefore, the model starts with big bang at $t = 0$ and goes on expanding until it comes to halt at t tends to ∞.
- Figure 5 shows that the EoS parameter evolves from phantom phase, increases rapidly till the present epoch and approaches -1 asymptotically

at late times. So, our model represents a ΛCDM model for present as well as in the future time.

- The cosmic jerk parameter also tends to 1 at late times showing thereby that our model corresponds to ΛCDM model.

- Figure 6 shows that the value of the total energy density in our anisotropic background decreases rapidly and approaches 1 at late times. So, our model comes close to a flat universe at late times.

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