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A CONCEPT OF AN OPTIMAL SOLUTION OF THE TRANSPORTATION PROBLEM USING THE WEIGHTED ARITHMETIC MEAN

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ABSTRACT. The transportation problem is a special category of the linear programming problem and has many applications in the optimization theory to achieve the optimal cost. The present paper explores a new method to solve the transportation problem for optimality. We utilized the concept of the Weighted Arithmetic Mean in the transportation problem. The objective of this paper is to find the initial basic feasible solution as well as the optimal solution (or close to the optimal solution) of the transportation problem. The method, Weighted Arithmetic Mean is based on weights which are assigned to the cells in the transportation cost matrix beginning with minimum weights to the maximum cost in the transportation matrix subsequently and finding a solution to the transportation problem. Compared to the MODI method, it provides a new method and also a new way to find the optimality.

1. INTRODUCTION

Transportation problems are widely studied in engineering science and Operations Research. It is one of the fundamental problems of network flow problem,

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which is usually used to minimize the transportation cost of transporting a single product from a given number of sources (e.g. factories) to a certain number of destinations (e.g.warehouses) while satisfying the supply limit and demand requirement. In logistics and supply-chain management, transportation models play an important role in cost reduction and service improvement.The transportation problem is a part of linear programming in the real world, also it is an interesting feature studied in operations research which is intended to transport different quantities of the same processed products from several sources to separate destinations where the transportation cost is minimized.

As such, there are ways to get optimal solutions for reducing the cost. The Column Minima Method(CMM), Row Minima Method (RMM), North-West Corner Rule (NWCM), Least Cost Method (LCM), and Vogel's Approximation Method (VAM), etc. are looking for initial basic feasible solutions and required to go for the next step. Then by constantly improving the initial basic feasible solution, the MODI method, and the Stepping Stone (SS) method to achieve the optimal solution.

The transportation problem falls into two groups, viz a balanced transportation problem, and an unbalanced transportation problem. When the number of resources is equal to the total number of requests, we are talking about a balanced transportation problem. Otherwise, it is a case of an unbalanced transportation problem.

The transportation problem was first introduced in 1941 by FL Hitchcock [6]. In the area of the industry,it plays an important role to minimize transportation costs. Furthermore, TC Koopmans [7] contributed to the optimum transportation system in 1947. In the direction of formulating and solving linear programming work, TC Koopmans and G B Dantzig [4] have further improved it in 1951.

In 1954, the Stepping Stone approach was developed in Charnes and Copper [3] Abdul Quddoos et.al [1] and Sudhakar et.al [12] have identified two different approaches for seeking an optimum solution in 2012. Researchers have found linear programming problems with fuzzy numbers in the past few years and have used the simplex method to find the optimal solution. Recently, in 2017, Reena Patel et. al [10] described the method of optimality with less computation for transportation problems.

We developed a statistical method in this paper to find the optimal solution, called the Weighted Arithmetic Mean. Using this method, we have established an initial basic feasible solution and optimal solution or close to the optimal solution to the transportation problem. The optimality is tested by the Weighted Arithmetic Mean with the aid of the principle of stepping stone method, using the sum of the weights assigned in the transportation cost matrix and finding the net difference in the weights of empty cells, to check whether the solution is optimal or not. The new algorithm discussed here gives the idea of the optimality compared to the MODI method.We have also provided numerical examples that illustrate the new algorithm.

2. WEIGHTED ARITHMETIC MEAN (WAM)

The Weighted Arithmetic Mean is derived from the formula

$$\overline{x}_w = \frac{w_1x_{1+}w_2x_2 + w_3x_3 + \ldots + w_nx_n}{w_1 + w_{2+}w_3 + \ldots + w_n} = \frac{\sum_{i=1}^{n} w_ix_i}{\sum_{i=1}^{n} w_i}.$$

Here, w_i is the non-negative weight for non-negative data x_i, i=1,2,...,n. The Weighted Arithmetic Mean is an average computed by giving different weights to the set of observations. If the weights are equal, then the Weighted Arithmetic Mean is nothing but arithmetic mean. The Weighted Arithmetic Mean plays a significant role in the systems of data analysis, weighted differential, and integral calculus. We applied this arithmetic mean to the transportation problem in this paper.

3. TRANSPORTATION PROBLEM

3.1. **Mathematical Formulation of the Transportation Problem.** Let us consider the standard transportation problem with m sources S_i (with supplies a_i, i= 1, 2, 3,...,m) and n destinations D_j (with demands b_j,j= 1,2,3,...,n). Here,

c_{ij} = Transportation cost for transporting the load from the source S_i to destination D_j

x_{ij}= The number of load units moving from S_i to D_j.

Mathematically the problem can be stated as minimize $\mathbb{Z}=\sum_{i=1}^{i=m}\sum_{j=1}^{j=n} c_{ij}\, x_{ij}$

Subject to constrain:

$\sum_{j=1}^{n} x_{ij} = a_i$ for i= 1, 2, 3..., m (Supply constraints)

$\sum_{i=1}^{m} x_{\text{ij}} = \; b_i$ for j = 1,2,3,. . . , n (Demand constrains)
$x_{\text{ij}} > 0$ for all i & j

Transportation Problem

Source	**Destination**				**Supply**
	D_1	D_2	$\dots$	D_n	
S_1	(X_{11}) C_{11}	(X_{12}) C_{12}	$\dots$	(X_{1n}) C_{1n}	a_1
S_2	(X_{21}) C_{21}	(X_{22}) C_{22}	$\dots$	(X_{2n}) C_{2n}	a_2
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
S_m	(X_{m1}) C_{m1}	(X_{m2}) C_{m2}	$\dots$	(X_{mn}) C_{mn}	a_m
Demand	b_1	b_2	$\dots$	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

A transportation problem is called balanced if the total supplies of all sources equals to the total demands at all destinations, i.e, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Otherwise, it is called the unbalanced transportation problem.

3.2. **WAM in Transportation Problem.** Now suppose w_{ij} be the assigned weights to the costs c_{ij} for each i= 1, 2, 3,. . . ,m and j= 1,2,3,. . . ,n. We assigned the weights to the costs c_{ij} from the highest to lowest transportation cost for each row and each column. That is, we assigned the weight 1 to the highest (maximum) cost, subsequent weights to the subsequent costs, and n to the lowest cost for each m row. Similarly, we assigned the weight 1 to the highest (maximum) cost, subsequent weights to the subsequent costs, and m to the lowest cost for each n column. We note that weights can be possibly assigned *chronologically or randomly* in case of tied costs.

We denote the weights assigned to the cost c_{ij} by w_{ij}^{r} considering the cell in a row and w_{ij}^{c} considering it in a column. We consider the sum of the weights for each cell in the transportation matrix for the optimality test later.

We define the sum of weights $S_{ij}=w_{ij}^{r}+w_{ij}^{c}$ for each cell in the transportation cost matrix. We define the net difference in the weights $d_{w_{ij}}$ for each unoccupied cell using the sum of weights S_{ij}.

4. ALGORITHM

This approach provides the procedure for the Weighted Arithmetic Mean to find the initial basic feasible solution.

Step 1: Determine whether the transportation problem is balanced or not. If it is balanced, then proceed further step.

Step 2: Using WAM in the transportation Matrix, compute the weighted arithmetic mean for each row and each column.

Step 3: Select the highest value from step 2 and allot the minimum of supply and demand to the lowest cost value of the corresponding row or column.

Step 4: Repeat steps 2 and 3 until the demands are met and all the supplies are exhausted.

Step 5: Total cost is calculated as the sum of the product of the cost and its allocated values of Supply or demand.

The basic feasible solution may take care of all sources and destinations but it need not give the least transportation cost. There can be several basic feasible solutions but the one that minimizes the total transportation cost is called the optimal solution. We can find the initial basic feasible solution by any one of the methods discussed earlier including WAM. This solution needs to be tested for optimality which will lead to the best solution and if not, it will lead us to a better solution. We provided a new optimality test using the weights in the Weighted Arithmetic Mean method.

THE OPTIMALITY TEST:

To verify the optimality,we should follow the following procedure:

Step: 1

1. Start with an initial basic feasible solution by the Weighted Arithmetic Mean containing $m + n - 1$ allocation in independent positions.
2. If the initial basic feasible solution is found by any method other than WAM, then we must assign the weights to this transportation cost matrix as discussed earlier in this paper.

Step: 2

1. Trace a closed path (or loop) starting from an unoccupied cell through at least three occupied cells, and then back to the selected unoccupied cell.
2. Assign plus (+) and minus (-) signs alternatively for each corner cells having the sum of weights of the closed path just being traced, starting with a plus (+) sign for the unoccupied cell.
3. Compute the net difference in the weights $(d_{w_{\text{ij}}})$ for each unoccupied cell by adding the sum of weights (S_{ij}) from the cells having plus sign and subtracting the sum of weights (S_{ij}) from the cells having minus sign along with the closed path traced in step-2.
4. Repeat this process for all other unoccupied cells in the matrix.

Step: 3

1. The optimal solution is achieved when the net difference in the weights $d_{w_{\text{ij}}} \leq 0$ for the unoccupied cells.
2. Otherwise, select the unoccupied cell with the highest positive net difference in the weights $(d_{w_{\text{ij}}} > 0)$.

Step: 4 Repeat step-2 and step-3 until the optimal solution is obtained.

Step: 5 the total transportation cost is calculated as the sum of the product of value and corresponding allotted cost of Supply or demand, i.e., Total cost=$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{\text{ij}}X_{\text{ij}}$.

This optimality test provides the optimal solution or close to the optimal solution by using the sum of the weights we assigned for each cell. We utilized the net difference in the weights $d_{w_{\text{ij}}}$ in this optimality test. The net difference in the weights $(d_{w_{\text{ij}}})$ is nothing but the resultant weight for each unoccupied cell which never is negative or zero. If the resultant weight for any unoccupied cells remains positive, it will contribute to the optimal cost of the transportation matrix.

5. Numerical examples (New Method)

1. Consider the following cost minimizing transportation problem with four sources and five destinations.

TABLE 1. Result Table

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	4	7	3	8	2	4
S_2	1	4	7	3	8	7
S_3	7	2	5	6	6	9
S_4	1	5	9	8	7	2
Demand	8	3	7	2	2	Total= 22

After Applying the Weighted Arithmetic Mean method for initial basic feasible Solution with weights, the independent allocations of the problem are obtained as follows:

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	4 3 2	7 2 1	3 [4] 4 4	8 1 1	2 5 4	4
S_2	1 [5] 5 3	4 3 3	7 2 2	3 [2] 4 4	8 1 1	7
S_3	7 [1] 1 1	2 [3] 5 4	5 [3] 4 3	6 2 3	6 [2] 3 3	9
S_4	1 [2] 5 4	5 4 2	9 1 1	8 2 2	7 3 2	2
Demand	8	3	7	2	2	

The transportation cost using WAM is obtained as follows:

Total Cost = (3*4) + (1*5) + (3*2) + (7*1) + (2*3) + (5*3)
+ (6*2) + (1*2) = 65

By the optimality test since $d_{w_{ij}} \leq 0$, the allocations of the problem are obtained as follows:

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	4 [−1]	7 [−7]	3 [2]	8 [−4]	2 [2]	4
S_2	1 [6]	4 [−6]	7 [−6]	3 [1]	8 [−9]	7
S_3	7 [−3]	2 [3]	5 [5]	6 [1]	6 [−2]	9
S_4	1 [2]	5 [−7]	9 [−9]	8 [−5]	7 [−7]	2
Demand	8	3	7	2	2	

The total cost obtained by the optimality test is calculated as follows:
Total Cost = (3*2) + (2*2) + (1*6) + (3*1) + (2*3) + (5*5)
+ (6*1) + (1*2) = 58

2. Consider the following cost minimizing transportation problem with three sources and four destinations.

TABLE 2.

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	Total=34

After Applying the Weighted Arithmetic Mean method for initial basic feasible Solution with weights, the independent allocations of the problem are obtained as follows:

	D_1	D_2	D_3	D_4	Supply
S_1	19 [5] 3 3	30 2 1	50 1 2	10 [2] 4 3	7
S_2	70 1 1	30 [2] 4 2	40 [7] 3 3	60 2 1	9
S_3	40 2 2	8 [6] 4 3	70 1 1	20 [12] 3 2	18
Demand	5	8	7	14	

The transportation cost using WAM is obtained as follows:
Total Cost = (19 * 5) + (10 * 2) + (30 * 2) + (40 * 7)
+ (8 * 6) + (20 * 12) = 743

By the optimality test since $d_{w_{ij}} \leq 0$, the allocations of the problem are obtained as follows:

	D_1	D_2	D_3	D_4	Supply
S_1	19 [5]	30 $[-6]$	50 $[-6]$	10 [2]	7
S_2	70 $[-1]$	30 [2]	40 [7]	60 $[-1]$	9
S_3	40 $[0]$	8 [6]	70 $[-5]$	20 [12]	18
Demand	5	8	7	14	

The total cost obtained by optimality test is calculated as follows,
Total Cost = (19 * 5) + (10 * 2) + (30 * 2) + (40 * 7)
+ (8 * 6) + (20 * 12) = 743

3. Consider the following cost minimizing transportation problem with three sources and three destinations.

TABLE 3.

	D_1	D_2	D_3	Supply
S_1	10	12	9	40
S_2	4	5	7	50
S_3	11	8	6	60
Demand	70	50	30	Total=150

After Applying the Weighted Arithmetic Mean method for initial basic feasible Solution with weights, the independent allocations of the problem obtained as follows:

	D_1	D_2	D_3	Supply
S_1	10 $\boxed{20}$ 2 2	12 1 1	9 $\boxed{20}$ 3 1	40
S_2	4 $\boxed{50}$ 3 3	5 2 3	7 1 2	50
S_3	11 1 1	8 $\boxed{50}$ 2 2	6 $\boxed{10}$ 3 3	60
Demand	70	50	30	

The transportation cost using WAM is obtained as follows:
Total Cost = (10 * 10) + (9 * 30) + (4 * 50)
+ (11 * 10) + (8 * 50) = 1040

By the optimality test since $d_{w_{ij}} \leq 0$, the allocations of the problem are obtained as follows:

	D_1	D_2	D_3	Supply
S_1	10 $\boxed{40}$	12 [−1]	9 [−1]	40
S_2	4 $\boxed{30}$	5 $\boxed{20}$	7 [−4]	50
S_3	11 [−3]	8 $\boxed{30}$	6 $\boxed{30}$	60
Demand	70	50	30	

The total cost obtained by optimality test is calculated as follows:
Total Cost = (10 * 40) + (4 * 30) + (5 * 20) + (8 * 30)
+ (6 * 30) = 1040

4. Consider the following cost minimizing transportation problem with three sources and four destinations.

TABLE 4.

	D_1	D_2	D_3	D_4	Supply
S_1	2	2	2	1	30
S_2	10	8	5	4	70
S_3	7	6	6	8	50
Demand	40	30	40	40	Total=150

After Applying the Weighted Arithmetic Mean method for initial basic feasible Solution with weights, the independent allocations of the problem obtained as follows:

	D_1	D_2	D_3	D_4	Supply
S_1	2 [30] 1 3	2 2 3	2 3 3	1 4 3	30
S_2	10 [10] 1 1	8 [20] 2 1	5 3 2	4 [40] 4 2	70
S_3	7 2 2	6 [10] 3 2	6 [40] 4 1	8 1 1	50
Demand	40	30	40	40	

The transportation cost using WAM is obtained as follows:
Total Cost = (2 * 30) + (10 * 10) + (8 * 20) + (6 * 10)
+ (6 * 40)+ (4 * 40) = 780

By the optimality test since $d_{w_{ij}} \leq 0$, the allocations of the problem are obtained as follows:

	D_1	D_2	D_3	D_4	Supply
S_1	2 [20]	2 [0]	2 [10]	1 [0]	30
S_2	10 [−1]	8 [−1]	5 [30]	4 [40]	70
S_3	7 [20]	6 [30]	6 [−1]	8 [−5]	50
Demand	40	30	40	40	

The total cost obtained by optimality test is calculated as follows,
Total Cost = (2 * 20) + (2 * 10) + (5 * 30) + (4 * 40)
+ (7 * 20)+ (6 * 30) = 690

We provided a comparison table to compare the transportation costs obtained by various methods, including the new method for the transportation problem.

COMPARISON TABLE

TABLE 5.

NO	MATRIX	WAM (IBFS)	OPTIMALITY BY SUM OF WEIGHTS	NWCM	LCM	VAM	MODI
1	4×5	65	58	93	63	58	58
2	3×4	743	743	1015	814	779	743
3	3×3	1040	1040	1040	1060	1040	1040
4	3×4	780	690	930	790	680	680

6. DISCUSSION AND CONCLUSION

In the comparison table, the numerical consequences of the problems were discussed. The outcome of this new method is as effective as the outcomes of various transportation methods. In most of the problems, this Weighted Arithmetic Mean method directly provides the optimal solution as the initial basic feasible solution. As a consequence, this method produces better results than existing methods.

The purpose of this paper pertains to achieve the optimal solution using the new method and also it is very easy to understand and apply to the transportation problem. The Weighted Arithmetic Mean method has acquired the best (optimal) solution or the better (close to optimal) solution to the transportation problem. This new method offers a very useful concept to the decision-makers to determine the optimality that can be applied to many real-world transportation problems.

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