Advances in Mathematics: Scientific Journal **10** (2021), no.3, 1771–1785
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.3.56

EXISTENCE AND ULAMS TYPE STABILITY FOR SYLVESTER MATRIX IMPULSIVE VOLTERRA INTEGRO-DYNAMIC SYSTEM ON TIME SCALES

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ABSTRACT. In this paper, We developed the existence and Ulam's type stability for Sylvester matrix impulsive Volterra integro-dyanamic system on time scale calculus. Banach fixed point theorem has used to established these results.Moreover, to outline the utilization of these outcomes an example is given.

1. INTRODUCTION

Integro-differential equations with impulsive matrix dynamical systems have considered important in varied applications as physics, biological systems such as heart-beats, economics, mechanical system with impact, control theory and so on. See the monograph given by [11,12]. built up the consequence of comparative system with Q(t)=0, Later Murty et.al. [13,14]. There are numerous physical problems that are characterized by unexpected changes in their states. These unexpected changes are said to be impulsive effects in the system. In the current writing these are two types of impulsive dynamical systems. Frist one is linear impulsive dynamical and second one is non-linear dynamical system. In the linear impulsive dynamical system in the span of these unexpected changes

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2020 *Mathematics Subject Classification.* 34E10,34A37,93B05,34H05.
Key words and phrases. Volterra integro-dynamic, matrix impulsive system, time scale calculus, Kronecker product.
Submitted: 03.03.2021; *Accepted:* 20.03.2021; *Published:* 30.03.2021.

is very little in examinations with the term of a whole advancement measure as like shocks and natural disasters and in non-linear impulsive dynamical system is the span of these progressions proceeds over a finite time interval and references them [4,9,10,15,16,17]. Volterra integral type equation on time scales [18].In 19th century was introducing Hyers-Ulams type stability concept and now it has been gained a lot of articles. The qualitative principal that could be significant from enhancement and mathematical factor of view is committed to the stability analysis of the solution to differential equations. Hyers-Ulams type stability for the result of the differential equations has been conveyed in parts of articles. See references [19,20].

Now, we focus our attention to study of delta differentiable existence, uniqueness and Ulma's type stability of the Volterra integro-dynamical systems with Sylvester matrix impulsive on time scales are given by

$$\begin{cases} x^{\Delta}(t) = P(t)x(t) + x(t)Q(t) + \mu(t)A(t)x(t)B(t) \\ \quad + \int_{t_0}^{t}(L_1(t,s)x(s) + x(s)L_2(t,s)) \\ \quad + F(t, X(t)), t \in \mathbb{T}_0 \backslash \{t_k\}_{k=1}^{\infty} \\ x(t_k^+) = (I + D_k)x(t_k), k = 1, 2, \ldots \\ x(t_0) = x_0 \end{cases}, \tag{1.1}$$

where $\mathbb{T}$ has the property unbounded above time scale with bounded graininess, $\mathbb{T}_0 := [t_0, \infty) \cap \mathbb{T}, t_k \in \mathbb{T}_0$ are right dense, $0 \leq t_0 \leq t_1 \leq, \ldots, \leq t_k \leq \ldots, \lim_{k\to\infty} t_k = \infty, x(t_k^-) = \lim_{h\to 0^-} x(t_k - h) and x(t_k^+) = \lim_{h\to 0^+} x(t_k + h), D_k \in M_n(\mathbb{R})$,$x(t) \in M_{n\times n}(\mathbb{R})$ is a state varuable, $F(t, X(t))$ is an $n \times n$ function and $P(t) \in C_{rd}\mathcal{R}\mathbb{M}_{n\times n}(\mathbb{R})$, $Q(t) \in C_{rd}\mathcal{R}\mathbb{M}_{n\times n}(\mathbb{R})$, $L_1(t) \in C_{rd}\mathcal{R}\mathbb{M}_{n\times n}(\mathbb{R})$,and $L_2(t) \in C_{rd}\mathcal{R}\mathbb{M}_{n\times n}(\mathbb{R})$ respectively, $X^{\Delta}(t)$ is the generalized delta derivative of X and $\mu(t)$ is a graininess function.

In section 2 and 3, we analysis basic techniques of time scales also derive basic concepts for converting given matrix valued system into a Kronecker product system by using variation of parameters. we developed existence and uniqueness stability of Volterra intrgro-dynamical system with Sylvester matrix impulsive on time scale.

2. PRELIMINARIES

In 1988, Stefan Hilger introduced on the time scales calculus in his Ph.D. thesis. Binding together the continuous as well as discrete analysis of the system. Thought this paper $\mathbb{T}$ denotes the time scales calculus. For more detailed data allude the text books [6,7] and the research paper [11]. We recollection some fundamental definitions, notations and useful lemmas. The Banach space of all continuous functions $f : I \to \mathbb{R}^n$ and endowed with the norm $||f||_c = sup_{t\in I||f(t)||}$is denoted by $||.||C(I, \mathbb{R}^n)$ and let $\mathbb{R}^n$be the space of n-dimensional column vectors $x(t) = col(x_1, x_2, \ldots, x_n)$. denotes the Banach space of Lebasque integrable functions from I into $\mathbb{R}^n$ is denoted by $L^1(I, \mathbb{R}^n)$. The Banach space of piecewise continuous functions as $PC(I, \mathbb{R}^n) = x : I \to \mathbb{R}^n : x \in C((t_k, t_k + 1], \mathbb{R}^n), k = 0, 1, 2, \ldots,$ and for some $x(t_k^-)$ and $x(t_k^+)$ For our convenience notation $PC(I, \mathbb{R}^n)$ is $||x||_{PC} = sup_{t\in[a,b]} \frac{||x(t)||}{e_\Omega(t,a)}$, for some $\Omega \in \mathbb{R}^+$ Next, we define $PC_{rd}(I, \mathbb{R}^n) = \{x \in PC(I, \mathbb{R}^n)\}$ $PC_{rd}(I, \mathbb{R}^n)$ from a space the supermom norm $||x||_1 = max\{||x||_{PC}, ||x_\Delta||_PC\}$.

Definition 2.1. [6] *A nonempty closed subset of* $\mathbb{R}$ *is called a time scale. It is denoted by* $\mathbb{T}$. *We define a T interval as* $[a, b]_\mathbb{T} = \{t \in \mathbb{T} : a \le t \le b\}$ *accordingly, we define* $(a, b)_\mathbb{T}$*,*$[a, b)_\mathbb{T}$*,*$(a, b]_\mathbb{T}$ *and so on. Also, we define* $\mathbb{T}^k = \mathbb{T}\{max\mathbb{T}\}$ *if* $max\mathbb{T}$ *exists, otherwise the forward jump operator* $\sigma : \mathbb{T} \to \mathbb{T}$ *is defined by* $\sigma(t) = inf\{s \in \mathbb{T} : s > t\} \in \mathbb{T}$ *with the substitution* $inf\{\emptyset\} = sup\mathbb{T}$ *and The graininess function* $\mu(t) : \mathbb{T} \to [0, \infty) by \mu(t) = \sigma(t) - t, \forall t \in \mathbb{T}$.

Definition 2.2. [6] *The mapping x from* $\mathbb{T} \to \mathbb{R}$ *(when* τ*=sup*$\mathbb{T}$*, choose* τ *is not left scattered). The generalized delta derivative of x(t), represented by* x^Δ *(*τ*), having the nature that, for any* $\varepsilon < 0$*. There exists a nbd U(*τ*) implies*

$$|[x(\sigma(\tau)) - x(s)] - x^\Delta(\tau)[\sigma(\tau) - s]| \in \varepsilon|\sigma(\tau) - s|,$$

each s$\in U$*.*

Here x is delta derivative for every $\tau \in \mathbb{T}$; then mapping x from $\mathbb{T}$ to $\mathbb{R}$ is called as generalized derivative on time scales calculus.

Definition 2.3. [6] *The mapping H from* $\mathbb{T}^k$ *to* $\mathbb{R}$ *is know as anti-derivative of h from* $\mathbb{T}^k$ *to* $\mathbb{R}$ *only if* $h^\Delta(\tau) = H(\tau)$ *fulfilled, for all* $\tau \in \mathbb{T}^k$*. Then*

$$\int_a^t h(s)\Delta s = H(t) - H(a).$$

Definition 2.4. [7] *The regressive function x mapping from* $\mathbb{T}$ *to* $\mathbb{R}$ *is defined as*$1 + \mu(t)y(t) \neq 0$ *for all* $t \in \mathbb{T}$. *The combination of all regressive and right dense continuous function are represented as* $\mathcal{R} = \mathcal{R}(t) = \mathcal{R}(\mathbb{T}, \mathbb{R})$ *Similarly all positively regressive function are denoted as*

$$\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R}) = \{y \in \mathcal{R} : 1 + \mu(t)y(t) > 0 forallt \in \mathbb{T}\}.$$

Definition 2.5. [3] *The right dense continuous matrices M and N on* $\mathbb{T}$*, implies*

$$(M + N)^\Delta(t) = M^\Delta(t) \otimes N(t) + M(\sigma(t) \otimes N_\Delta(t)).$$

We put the vec operator to the equation (1), the it is converted into a Kronecker product dynamical system by using Kronecker product properties [3], we have

$$(2.1)\qquad \begin{cases} z^\Delta(t) = A(t)z(t) + \int_0^t G(t,s)z(s)\Delta s + f(t, Z(t)), \\ \quad t \in \bigcup_{k=0}^m (s_k, t_{k+1}]_{\mathbb{T}} \\ z(t_k^+) = (I_n \otimes R_k)z(t_k), t \in (s_k, t_{k+1}]_{\mathbb{T}},\ k = 0, 1, \dots, m \\ z(t_0) = z_0. \end{cases}$$

Here $\mathbb{T}$ is time scales.$s_k, t_k \in \mathbb{T}$ are right dense ponits with $0 = s_0 = t_0 \leq t_1 \leq t_2 \leq t_3 \leq, \dots, \leq s_m \leq t_{m+1}, \lim_{k\to\infty} t_k = \infty, z(t_k^-) = \lim_{h\to 0^-} z(t_k - h)$ and $z(t_k^+) = \lim_{h\to 0^+} z(t_k + h)$ represent the right and left limits of z(t) at $t = t_k$ in $\mathbb{T}, f(t, z(t)) = vecF(t, X(t))$ vecotor function which is piece wise rd-continuous on $\mathbb{T}.z(t) = Vecx(t)$,$A(t) = [Q^* \otimes I + I \otimes P + \mu(t)(Q^* \otimes P) \in C_{rd}\mathcal{R}(M_{n^2\times n^2}(\mathbb{R}))]$,$G(t,s) = [(L_2 \otimes I_n) + (I_n \otimes L_1)] \in C_{rd}\mathcal{R}(M_{n^2\times n^2}(\mathbb{R}))]$,$R_k = (I_n \otimes D_k) \in C_{rd}\mathcal{R}(M_{n^2\times n^2}(\mathbb{R}))]$.

Using Tensor product (KP) definition the matrix $P : \mathbb{T}^k \to \mathbb{R}^{n^2}$ is both the regressive as well as rd-continuous. Clearly the system 2 is known as KP system linked (1).

Definition 2.6. [7] *For* $A \in \mathcal{R}$,*the generalized exponential function on* $\mathbb{T}$ *is defined as*

$$e_A(t,s) = exp(\int_s^t \xi_\mu(t)(A(\tau))\Delta\tau), t, s \in \mathbb{T},$$

where

$$\xi_\mu(t)(A(\tau)) = \begin{cases} \frac{log(1+\mu(\tau)A)}{\mu(\tau)} if \mu(\tau) \neq 0 \\ A if \mu(\tau) \neq 0 \end{cases}$$

is a cylinder transformation.

Definition 2.7. [7] *Let* $A, B \in \mathcal{R}$*, then*

i . $A \oplus B = A + B + \mu(\tau)AB$
ii . $\ominus A = \frac{-A}{1+\mu(\tau)A}$
iii . $A \ominus B = A \oplus (\ominus B)$

Lemma 2.1. [7] *Let* $A, B \in \mathcal{R}$, *then*

a . $e_0(t,s) = 1$ *and* $e_A(t,t) = 1$.
b . $e_A(\sigma(t), s) = (1 + \mu(\tau)p(t))e_A(t,s)$.
c . $e_A(t,s)e_A(s,r) = e_A(t,r)$.
d . $e_A(t,s) = \frac{1}{e_A(s,t)} = e_{\ominus A}(s,t)$.
e . $(e_{\ominus A}(t,s))^{\Delta} = \ominus A(t)e_{\ominus A}(t,s)$

Lemma 2.2. [6] *If* $X \in PC_{rd}(\mathbb{T}, \mathbb{R}^+)$*satisfies the inequality condition. Then*

$$X(t) \leq \alpha + \int_a^t A(s)x(s)\Delta t + \sum_{a<t_k<t} \beta_k z(t_k), \forall t \in \mathbb{T},$$

then

$$X(t) \leq \alpha \prod_{a<t_k<t} (1 + \beta_k)e_A(t,a), \forall t \in \mathbb{T}.$$

We consider the linear Volterra integro-dynamical system (2.1) without impulsive, then

$$\begin{cases} z_{\Delta}(t) = A(t)z(t) + \int_{t_0}^t G(t,s)z(s)\Delta s, t \in I \\ z(t_0) = z_0 \end{cases} \tag{2.2}$$

An $n^2 \times n^2$ matrix is defined to be a real-valued function of $\emptyset(t,s)$and it is denoted by

$$\emptyset(t,s) = [z_1(t,s), z_2(t,s), \ldots, z_{n^2}(t,s)]$$

where $z_k(t,s), k = 1,2,3,\ldots,n^2$ are n^2linearly independent solution of the system (2.2). the principal matrix $\emptyset(t,s)$ is known as the transition matrix if $\emptyset(t,0) = I_{n^2 \times n^2}$ at t=0. and if $z(t) = \emptyset(t,0)z_0$is a unique solution of the system (2.2).

Lemma 2.3. *Let* $\emptyset(t,s)$ *be the transition matrix of the system (2.2), then*
i. $\emptyset(t,\tau) = \emptyset(t,s)\emptyset^{-1}(\tau,s)\emptyset^{-1}(t,\tau) = \emptyset(\tau,t)$;
ii. $\emptyset^{\Delta_t}(t,s) = A(t)\emptyset(t,s) + \int_s^t G(t,\tau)\emptyset(t,s)\Delta\tau$;
iii. $\emptyset^{\Delta_s}(t,s) = -\emptyset(t,\sigma(s))A(s) - \int_{\sigma(s)}^t G(t,\sigma(\tau))\emptyset(\tau,s)\Delta\tau$.

Theorem 2.1 (Theorem[19,variation of parameters). *The solution of the system (2.1)*

$$\begin{cases} z^{\Delta}(t) = A(t)z(t) + \int_0^t G(t,s)z(s)\Delta s + H(t), t \in (0,T]_{\mathbb{T}} \\ z(t_0) = z_0 \end{cases} \tag{2.3}$$

satisfying the initial condition $z(t_0) = z_0$, is

$$z(t) = \emptyset(t,t_0)z_0 + \int_0^t \emptyset(t,\sigma(\tau))H(t)\Delta\tau,$$

Here, the principal matrix is $\emptyset(t,s)$ then the solution of

$$\emptyset^{\Delta_t}(t,s) = A(t)\emptyset(t,\sigma(s)) + \int_s^t G(t,\tau)\emptyset(\tau,s)\Delta\tau.$$

Definition 2.8. *A function $z(t) \in PC(I,\mathbb{R}^n)$is known as the solution of the system (2.1) if $z(t)$ satisfies $z(t_0) = z_0, z(t_k^+) = [I_n \otimes R_k]z(t_k), t \in (t_k,s_k]_{\mathbb{T}}, k = 1,2,\dots m$ and $z(t)$ is the solution of the following integral equations:*

$$z(t) = \emptyset(t,t_0)z_0 + \int_0^t \emptyset(t,\sigma(\tau))f(\tau,z(\tau))Delta\tau, \forall t \in (0,t_1]_{\mathbb{T}} \tag{2.4}$$

$$z(t) = \emptyset(t,s_k)[I_n \otimes R_k]z(t_k) + \int_{s_k}^t \emptyset(t,\sigma(\tau))g(\tau)\Delta\tau, \tag{2.5}$$

$\forall t \in (s_k,t_{k+1}]_{\mathbb{T}}, k = 1,2,\dots,m.$

For our convenience notations, we have

$$L = \max_{(t,s)\in I\times I} ||\emptyset(t,s)||.$$

Now, we consider the inequality conditions if $\epsilon > 0$, we have

$$\begin{cases} ||y^{\Delta}(t) - A(t)y(t) - \int_0^t G(t,s)y(s)\Delta s - f(t,y(t))|| \le \epsilon, \\ \quad t \in \bigcup_{k=0}^m (s_k,t_{k+1}]_{\mathbb{T}} \\ ||y(t_k^+) - [I_n \otimes R_k]z(t_k) \le \epsilon,\ t \in (s_k,t_k]_{\mathbb{T}}||, k = 1,2,\dots m. \end{cases} \tag{2.6}$$

Definition 2.9. [20] *Equation (2.1) is Ulam-Hyers stable if there exists a real number $\mathcal{W}_{(L_f,L_{gk},m)} > 0$ such that for $\epsilon > 0$and for each solution $y \in PC_{rd}(I,\mathbb{R})$of inequality (2.5), there exists a unique solution $z \in PC_{rd}(I,\mathbb{R})$ of equations (2.1) with*

$$||y(t) - z(t)|| \le \mathcal{W}_{(L_f,L_{gk},m)}(\epsilon), \forall t \in I.$$

Definition 2.10. [20] *Equation (2.1) is generalizes Ulam-Hyers stable if there exists a* $\mathcal{W}_{(L_f,L_{gk},m)} \in C(\mathbb{R}^+,\mathbb{R}^+), \mathcal{W}_{(L_f,L_{gk},m)}(0) = 0$ *such that for each solutions* $y \in PC_{rd}(I,\mathbb{R})$*of inequality (2.4), there exists a unique solution* $z \in PC_{rd}(I,\mathbb{R})$ *of equations (2.1) with*

$$||y(t) - z(t)|| \leq \mathcal{W}_{(L_f,L_{gk},m)}(\epsilon), \forall t \in I.$$

Remark 2.1. *Definition 2.9* $\Longrightarrow$ *Definition 2.10.*

Remark 2.2. *A function* $y \in PC(I,\mathbb{R}^{n^2})$ *is a solution of inequality (2.5) if and only if there is* $H \in PC(I,\mathbb{R}^{n^2})$ *and a sequence* $U_k, k = 1,2,3,\ldots,m,$ *such that*

(1) $||H(t)|| \leq \epsilon, \forall t \in \bigcup_{k=0}^{m}(s_k,t_{k+1})_{\mathbb{T}})$ *and* $||H_k|| \leq \epsilon, \forall k = 1,2,\ldots,m.$
(2) $y^{\Delta}(t) = A(t)y(t) + \int_0^t G(t,s)y(s)\Delta s + f(t,y(t)) + H(t), t \in (s_k,t_{k+1})_{\mathbb{T}}), k = 0,1,\ldots,m.$
(3) $y(t_k^+) = [I_n \otimes R_k]y(t_k) + H_k, t \in (s_k,t_{k+1})_{\mathbb{T}}), k = 0,1,\ldots,m.$

Indeed, by the above remarks, we have that

$$\begin{cases} y^{\Delta}(t) = A(t)y(t) + \int_0^t G(t,s)y(s)\Delta s + f(t,y(t)) + H(t), \\ \quad t \in (s_k,t_{k+1}]_{\mathbb{T}}, k = 0,1,\ldots,m. \\ y(t_k^+) = [I_n \otimes R_k]y(t_k) + H_k, \ t \in (s_k,t_{k+1}]_{\mathbb{T}}, k = 1,2,\ldots,m. \end{cases}$$

then

$$y(t) = \emptyset(t,0)z_0 + \int_0^t \emptyset(t,\sigma(\tau))(f(t,y(\tau)) + H(t))\Delta\tau, \forall t \in (0,t_1]_{\mathbb{T}}.$$
$$y(t_k^+) = [I_n \otimes R_k]y(t_k) + H_k, t \in (t_k,s_k]_{\mathbb{T}}, k = 1,2,\ldots,m.$$
$$y(t) = \emptyset(t,s_k)([I_n \otimes R_k]y(t_k) + H_k) + \int_0^t \emptyset(t,\sigma(\tau))(f(\tau,y(\tau)) + H(t))\Delta\tau.$$

Therefore, for $t \in (s_k,t_{k+1}]_{\mathbb{T}}, k = 1,2,\ldots,m,$ *we have*

$$||y(t) - \emptyset(t,s_k)([I_n \otimes R_k]y(t_k) - \int_{s_k}^t \emptyset(t,\sigma(\tau))f(\tau,y(\tau))\Delta\tau||,$$

$$\leq ||\emptyset(t,s_k)||||H_k|| + \int_{s_k}^t ||\emptyset(t,\sigma(\tau))||||H(k)||\Delta\tau| \leq M\epsilon(1+T).$$

Also, for $t \in [0,t_1]$*, we have*

$$||y(t) - \emptyset(t,s_k)z_0 - \int_0^t \emptyset(t,\sigma(\tau))f(\tau,y(\tau))\Delta\tau|| \leq M\epsilon T.$$

Similarly, for $t \in (s_k, t_{k+1}]_{\mathrm{T}}, k = 1, 2, \ldots, m,$ *we have*

$$||y(t_k^+) - [I_n \otimes R_k]y(t_k)|| \leq \epsilon.$$

For Ulam's-type stability of the system (2.1), we need the following conditions.

C1*: The non-linear function* $f : J_1 \times \mathbb{R}^{n^2} \to \mathbb{R}^{n^2}, J_1 = \bigcup_{k=0}^{m}(s_k, t_{k+1}]_{\mathbb{T}}$ *is continuous and there exists a* $\oplus$ *ve constant such that*

$$||f(t, z) - f(t, y)|| \leq M_f||z - y||, \forall z, y \in \mathbb{R}^{n^2}, t \in J_1.$$

Also, there exista $\oplus$ *ve constant* L_f *such that* $||f(t, z)|| \leq L_f, \forall t \in J_1$ *and* $z \in \mathbb{R}^{n^2}$.

C2*: The non-linear function* $g_k : I_k \times \mathbb{R}^{n^2} \to \mathbb{R}^{n^2}$*, are continuous and there exists a* $\oplus$ *ve constant such that* $L_{g_k}, k = 1, 2, \ldots, m,$ *such that*

$$||([I_n \otimes R_k]z(t_k) - ([I_n \otimes R_k]y(t_k)|| \leq M_{g_k}||z - y||, \forall z, y \in \mathbb{R}^{n^2}, t \in I_k, k = 1, 2, \ldots, m.$$

Also, there exista $\oplus$ *ve constant* L_g *such that* $||([I_n \otimes R_k]z(t_k)|| \leq L_g, \forall t \in I_k$ *and* $z \in \mathbb{R}^{n^2}$.

3. Existence and Ulam's type stability

Now, we developed the existence and Ulam's type stability for the system (2.1) by using Banach fixed points theorem.

Theorem 3.1. *If the conditions (C1) - (C2) are satisfied, then system (2.1) has a unique solution.*

Proof. Let$\mathcal{D} \subseteq PC$ such that $\mathcal{D} = \{z \in PC(I, \mathbb{R}^{n^2}) : ||z||_{PC} \leq \gamma\}$, where $\gamma = \max(LL_g + LL_fT, L||z_0|| + LL_fT, L_g)$.

Now, the operator $\mathcal{G} : \mathcal{D} \to \mathcal{D}$, we have

$$(\mathcal{G}z)(t) = \emptyset(t, 0)z_0 + \int_0^t \emptyset(t, \sigma(\tau))f(\tau, z(\tau))\Delta\tau, \forall t \in [0, t_1]. \tag{3.1}$$

$$(\mathcal{G}z)(t) = g_k(t, \emptyset(t_k, s_{k-1})([I_n \otimes R_k]z(t_k) + \int_{s_{k-1}}^t \emptyset(t_k, \sigma(\tau))f(\tau, z(\tau))\Delta\tau), \tag{3.2}$$

$\forall t \in (t_k, s_k]_{\mathbb{T}}, k = 1, 2, \ldots, m.$

$$(\mathcal{G}z)(t) = \emptyset(t, s_k)([I_n \otimes R_k]z(t_k) + \int_{s_k}^t \emptyset(t, \sigma(\tau))f(\tau, z(\tau))\Delta\tau), \forall t \in (t_k, s_k]_{\mathbb{T}}, \tag{3.3}$$

$k = 1, 2, \ldots, m.$

We have to prove $\mathcal{G}:\mathcal{D}\to\mathcal{D}$ by using the Banach fixed points theorem. Now, for $\forall\ t\in(t_k,s_{k+1}]_{\mathbb{T}}, k=1,2,\dots,m,$ and $z\in\mathcal{D}$, we have

$$\begin{aligned}||(\mathcal{G}z)(t)|| &\leq ||\emptyset(t,s_k)||||([I_n\otimes R_k]z(t_k)||+\int_{s_k}^{t}||\emptyset(t,\sigma(\tau))||||f(\tau,z(\tau))||\Delta\tau)\\ &\leq LL_g+LL_f(t-s_k).\end{aligned}$$

Therefore,

$$||(\mathcal{G}z)(t)||_{PC}\leq LL_g+LL_fT. \tag{3.4}$$

Now, for $t\in(0,t_1,]$ and $z\in\mathcal{D}$, we have

$$\begin{aligned}||(\mathcal{G}z)(t)|| &\leq ||\emptyset(t,0)||||z_0||+\int_0^t||\emptyset(t,\sigma(\tau))||||f(\tau,z(\tau))||\Delta\tau,\\ &\leq L||z_0||+LL_ft.\end{aligned}$$

Therefore,

$$||(\mathcal{G}z)(t)||_{PC}\leq L||z_0||+LL_fT. \tag{3.5}$$

Similarly, for $t\in(t_k,s_k]_{\mathbb{T}}, k=1,2,\dots,m$; and $z\in\mathcal{D}$, we have

$$||(\mathcal{G}z)(t)||_{PC}\leq L_g. \tag{3.6}$$

Subsequent to summing the inequalities (3.4) -(3.6), we have

$$||(\mathcal{G}z)(t)||_{PC}\leq\gamma.$$

Therefore, $\mathcal{G}:\mathcal{D}\to\mathcal{D}$, for any $z,y\in\mathcal{D}, t\in(t_k,s_k]_{\mathbb{T}}, k=1,2,\dots,m,$ we get

$$\begin{aligned}&||(\mathcal{G}z)(t)-(\mathcal{G}y)(t)||\leq||\emptyset(t,s_k)||||[I_n\otimes R_k]z(t_k)-[I_n\otimes R_k]y(t_k)||\\ &+\int_{s_k}^{t}||\emptyset(t,\sigma(\tau))||||f(\tau,z(\tau))-f(\tau,y(\tau))||\Delta\tau\\ &\leq LM_{g_k}||z(t_k^+)-y(t_k^+)||+LM_f\int_{s_k}^{t}||z(\tau)-y(\tau)||\Delta\tau\\ &\leq LM_{g_k}e_\Omega(t_k,s_k)||z-y||_{PC}+LM_f||z-y||_{PC}\int_{s_k}^{t}e_\Omega(\tau,s_k)\Delta\tau\\ &\leq LM_{g_k}e_\Omega(t_k,s_k)||z-y||_{PC}+\frac{LM_fe_\Omega(\tau,s_k)||z-y||_{PC}}{\Omega}.\end{aligned}$$

Therefore,

$$||(\mathcal{G}z)(t)-(\mathcal{G}y)(t)||_{PC}\leq[\frac{LM_{g_k}}{e_\Omega(\tau,s_k)}+\frac{LM_f}{\Omega}]||z-y||_{PC}. \tag{3.7}$$

Next, any $z, y \in \mathcal{D}, t \in ([0, t_1]$, we get

$$||(\mathcal{G}z)(t) - (\mathcal{G}y)(t)|| \leq \int_0^t ||\emptyset(t, \sigma(\tau))|| ||f(\tau, z(\tau)) - f(\tau, y(\tau))||\Delta\tau)$$
$$\leq LM_f||z - y||_{PC} \int_0^t e_\Omega(\tau, 0)\Delta\tau \leq \frac{LM_f e_\Omega(\tau, 0)||z - y||_{PC}}{\Omega}).$$

Therefore,

$$||(\mathcal{G}z)(t) - (\mathcal{G}y)(t)|| \leq \frac{LM_f}{\Omega}||z - y||_{PC}. \tag{3.8}$$

Similarly, for $t \in (t_k, s_k]_\mathbb{T}, k = 1, 2, \ldots, m$, we have

$$||(\mathcal{G}z)(t) - (\mathcal{G}y)(t)||$$
$$\leq M_{g_k}(||\emptyset(t_k, s_{k-1})|| ||[I_n \otimes R_k]z(t_k) - [I_n \otimes R_k]y(t_k)||$$
$$+ \int_{s_k}^t ||\emptyset(t_k, \sigma(\tau))|| ||f(\tau, z(\tau)) - f(\tau, y(\tau))||\Delta\tau)$$
$$\leq LM_{g_k}(M_{g_k}||z(t_k^+) - y(t_k^+)|| + M_f \int_{s_{k-1}}^t ||z(\tau) - y(\tau)||\Delta\tau)$$
$$\leq LM_{g_k}(M_{g_k} e_\Omega(t_{k-1}, t_k))||z - y||_{PC} + M_f||z - y||_{PC} \int_{s_{k-1}}^t e_\Omega(\tau, t_k)\Delta\tau)$$
$$\leq LM_{g_k}(M_{g_k} e_\Omega(t_{k-1}, s_k))||z - y||_{PC} + \frac{M_f(1 - e_\Omega(s_{k-1}, t_k))||z - y||_{PC}}{\Omega}))$$

Therefore,

$$||(\mathcal{G}z)(t) - (\mathcal{G}y)(t)||_{PC} \leq [\frac{LM_{g_k}^2}{e_\Omega(t_k, t_{k-1})} + \frac{LM_f M_{g_k}}{\Omega}]||z - y||_{PC}. \tag{3.9}$$

Subsequent to summing the inequalities (3.7) -(3.9), we get

$$||(\mathcal{G}z)(t) - (\mathcal{G}y)(t)||_{PC} \leq M_F||z - y||_{PC},$$

where $M_F = \max_{1 \leq k \leq m}(\frac{LM_{g_k}}{e_\Omega(t_k, s_k)} + \frac{LM_f}{\Omega}, \frac{LM_{g_k}^2}{e_\Omega(t_k, t_{k-1})} + \frac{LM_f M_{g_k}}{\Omega})$.

Hence, the system (2.1) has a uniquely solution by Banach fixed points theorem. □

Theorem 3.2. *If the conditions (C1) - (C2) are satisfied, then the system (2.1) is Hyer-Ulam's type stable.*

Proof. We consider the $y \in PC(I, \mathbb{R}^{n^2})$be the solution of inequality (2.5) and $z \in PC(I, \mathbb{R}^{n^2})$ be a unique solution of the system (2).

Therefore by Lemma 2.9. if $z(t_0) = z_0$,

$$z(t) = \emptyset(t,0)z_0 + \int_0^t \emptyset(t,\sigma(\tau))f(\tau,z(\tau))\Delta\tau, \forall t \in [0,t_1].$$

If $z(t_k^+) = [I_n \otimes R_k]z(t_k), k = 1,2,\ldots,m,$

$$z(t) = \emptyset(t,s_k)[I_n \otimes R_k]z(t_k) + \int_{s_k}^t \emptyset(t,\sigma(\tau))f(\tau,z(\tau))\Delta\tau, \tag{3.10}$$

$\forall t \in [s_k, t_{k+1}]_{\mathbb{T}}$, $k = 1,2,\ldots,m$. Now, for $t \in [s_k, t_{k+1}]_{\mathbb{T}}, k = 1,2,\ldots,m,$ we have

$$\begin{aligned}
||y(t) - z(t)|| &\le ||y(t) - \emptyset(t,s_k)[I_n \otimes R_k]z(t_k) \\
&- \int_{s_k}^t \emptyset(t,\sigma(\tau))f(\tau,z(\tau))\Delta\tau|| \\
&\le ||y(t) - \emptyset(t,s_k)[I_n \otimes R_k]z(t_k) - \int_{s_k}^t \emptyset(t,\sigma(\tau))f(\tau,z(\tau))\Delta\tau|| \\
&+ ||\emptyset(t,s_k)([I_n \otimes R_k]y(t_k) - [I_n \otimes R_k]z(t_k)) \\
&+ \int_{s_k}^t \emptyset(t,\sigma(\tau))(f(\tau,y(\tau)) - f(\tau,z(\tau)))\Delta\tau|| \\
&\le L\epsilon(1+T) + LM_{g_k}||y(t_k^+) - z(t_k^+)|| + LM_f \int_{s_k}^t ||y(\tau) - z(\tau)||\Delta\tau \\
&\le L\epsilon(1+T) + \sum_{k=1}^m LM_{g_k}||y(t_k^+) - z(t_k^+)|| + LM_f \int_{s_k}^t ||y(\tau) - z(\tau)||\Delta\tau.
\end{aligned} \tag{3.11}$$

Similarly, for $t \in [0, t_1]$, we have

$$\begin{aligned}
||y(t) - z(t)|| &\le ||y(t) - \emptyset(t,0)z_0 - \int_0^t \emptyset(t,\sigma(\tau))f(\tau,z(\tau))\Delta\tau|| \\
&\le ||y(t) - \emptyset(t,0)y_0 - \int_0^t \emptyset(t,\sigma(\tau))f(\tau,z(\tau))\Delta\tau|| \\
&+ ||\int_0^t \emptyset(t,\sigma(\tau))(f(\tau,y(\tau)) - f(\tau,z(\tau)))\Delta\tau|| \\
&\le L\epsilon T + LM_f \int_0^t ||y(\tau) - z(\tau)||\Delta\tau \\
&\le L\epsilon(1+T) + \sum_{k=1}^m LM_{g_k}||y(t_k^+) - z(t_k^+)|| + LM_f \int_0^t ||y(\tau) - z(\tau)||\Delta\tau.
\end{aligned} \tag{3.12}$$

Similarly, for $t \in [s_k, t_{k+1}]_{\mathbb{T}}, k = 1, 2, \ldots, m,$ we can easily find that

$$
\begin{aligned}
&||y(t) - z(t)|| \le ||y(t) - [I_n \otimes R_k]z(t_k)|| \\
&\le ||y(t) - [I_n \otimes R_k]y(t_k)|| + ||[I_n \otimes R_k]y(t_k) - [I_n \otimes R_k]z(t_k)|| \\
&\le \epsilon + M_{g_k}||y(t_k^+) - z(t_k^+)|| \qquad (3.13)\\
&\le L\epsilon(1+T) + \sum_{k=1}^{m} LM_{g_k}||y(t_k^+) - z(t_k^+)|| + LM_f \int_0^t ||y(\tau) - z(\tau)||\Delta\tau.
\end{aligned}
$$

From the inequality conditions (3.11) -(3.13), we have

$$
\begin{aligned}
||y(t) - z(t)|| \le L\epsilon(1+T) &+ \sum_{k=1}^{m} LM_{g_k}||y(t_k^+) - z(t_k^+)|| \\
&+ LM_f \int_0^t ||y(\tau) - z(\tau)||\Delta\tau.
\end{aligned} \tag{3.14}
$$

Now, we set $||y(t) - z(t)|| = \zeta(t)$, then

$$\zeta(t) \le L\epsilon(1+T) + \sum_{k=1}^{m} LM_{g_k}\zeta(t_k) + LM_f \int_0^t \zeta(\tau)\Delta\tau.$$

By Lemma 2.9, we can find that

$$
\begin{aligned}
||y(t) - z(t)|| &\le L\epsilon(1+T) + \prod_{k=1}^{m}(1 + LM_{g_k})e_\beta(T, t_0) \\
&\le \mathcal{W}_{(M_f, M_{g_k}, m)}\epsilon, t \in I,
\end{aligned}
$$

where $\mathcal{W}_{(M_f, M_{g_k}, m)} = L\ (1+T) + \prod_{k=1}^{m}(1 + LM_{g_k})e_\beta(T, t_0) > 0$ and $\beta = LM_f$. Then, the system (2) is Hyer-Ulam's type stable.

Additionally, if we put $\mathcal{W}_{(M_f, M_{g_k}, m)}(\epsilon) = \mathcal{W}_{(M_f, M_{g_k}, m)}\epsilon, \mathcal{W}_{(M_f, M_{g_k}, m)}(0) = 0.$ Therefore, the system (2) is generalized Ulam-Hyers stable. □

Example 1. *Consider the following matrices with impulsive on* $\mathbb{T}$*,*$(0, 3/5, 4/5, 1 \in \mathbb{T})$*,*

$$P = \begin{pmatrix} -1/2 & 0 \\ 0 & -1/2 \end{pmatrix}, Q = \begin{pmatrix} -1/2 & 0 \\ 0 & -1/2 \end{pmatrix}, L_1 = \begin{pmatrix} -4/5 & 0 \\ 0 & -4/5 \end{pmatrix}, L_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$F(t, x(t)) = \frac{1}{e^{t+4}} \begin{pmatrix} cosx_{11}(t) & 0 \\ 0 & cosx_{22}(t), \end{pmatrix} D_k = \frac{1}{e^{t^2+2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} and X_0 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

The Volterra integro-dynamical systems with Sylvester matrix impulsive on time scales are

$$\begin{cases} z^{\Delta}(t) = A(t)z(t) + \int_0^t G(t,s)z(s)\Delta s + f(t, Z(t)), t \in (0,1]_{\mathbb{T}} \\ z(t_k^+) = (I_n \otimes R_k)z(t_k), t \in (0,1]_{\mathbb{T}} \\ z(t_0) = 1 \end{cases},$$

where $A(t) = [-1 + \mu(t)/4]I_4$*,* $G(t,s) = \frac{-4}{5}I_4$ *and* $(I_n \otimes R_k) = \frac{e^{t^2+2}+1}{e^{t^2+2}}I_4$*,*

$$f(t, z(t)) = \frac{1}{e^{t+4}} \begin{pmatrix} cos z_{11}(t) \\ 0 \\ 0 \\ cos z_{22}(t) \end{pmatrix}.$$

Then, $\forall [0,1]\mathbb{T} \in \mathbb{R}$*, we have* $||f(t,z) - f(t,y)|| \leq \frac{1}{e^4}||z - y||$ *and* $||(I_n \otimes R_k)y(t_k^+) - (I_n \otimes R_k)z(t_k^+)|| \leq \frac{e^2+1}{e^2}||z - y||$ *Hence, the conditions* $C_1 - C_2$ *are holds with* $L = e^{-1} - 5/4e^{-4/5} + 1/4$ $M_f = \frac{1}{e^4}$*,* $M_g = \frac{1}{e^4}$*.,Also fort*$_0 = 0, s_1 = 3/5, t_1 = 4/5, t_2 = \mathtt{T} = 1, \Omega = 25$

$$M_F = \max_{1 \leq k \leq 0}(\frac{LM_{g_k}}{e_{\Omega}(t_k, s_k)} + \frac{LM_f}{\Omega}, \frac{LM_{g_k}^2}{e_{\Omega}(t_k, t_{k-1})} + \frac{LM_f M_{g_k}}{\Omega}) = 0.00047 < 1$$

holds. Thus, from Theorem 3.1 has a Ulam Hyer's stable solution which is unique.

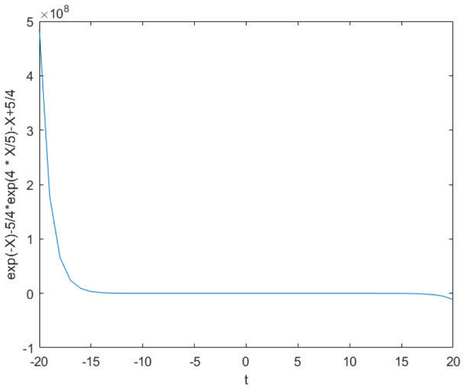

FIGURE 1. Ulam hyer's stability

4. Conclusion

We investigated positive non-linear functional analysis and also, we have flourishing developed the existence and Ulam's type stability for a Sylvester matrix impulsive Volterra integro-dynamical system (2.1) by using Banach fixed point theorem on $\mathbb{T}$. To illustrate the application of obtained results, we have given an example.

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