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BAYESIAN INFERENCE FOR RAYLEIGH PARETO DISTRIBUTION UNDER PROGRESSIVELY TYPE-II RIGHT CENSORED DATA

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ABSTRACT. In this paper, we consider inference problems including estimation for a Rayleigh Pareto (RP) distribution under progressively type-II right censored data. We use two approaches, the classical maximum likelihood approach and the Bayesian approach for estimating the distribution parameters and the reliability characteristics. Bayes estimators and corresponding posterior risks (PR) have been derived using different loss functions (symmetric and asymmetric). The estimators cannot be obtained explicitly, so we use the method of Monte Carlo. Finally, we use the integrated mean square error (IMSE) and the Pitman closeness criterion to compare the results of the two methods.

1. INTRODUCTION

In survival tests or reliability, it often happens that data are lost for different reasons, so we say that the data are censored. When the observation time is prefixed and the number of failures is random, we say that the data are censored of type-I. If the number of observations is prefixed for obvious cost reasons and

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the duration of life-test is random, therefore we say that the data are censored of type-II. These two censoring schemes (type-I and type-II) are the most popular.

The loss of units at points other than the final termination point may be unavoidable, as in the case of the accidental breakage of experimental units or the loss of contact with individuals under experiment. These reasons lead us to the progressive censoring. A progressive censoring scheme can be described as follows, suppose n identical units are placed on a life-testing experiment at time zero, with the corresponding lifetimes $X_1, X_2, \ldots, X_n$ being independent and identically distributed. After the first failure R_1 surviving items are removed randomly from further observation, after the second failure, R_2 surviving items are randomly removed too. This experiment stops at the time when the $m\ th$ failure is observed and the remaining $R_m = n-R_1-R_2-\ldots-R_{m-1}-m$ surviving units are withdrawn. So the set of an observed lifetime $X_{1:m:n}, X_{2:m:n}, \ldots, X_{m:m:n}$ is progressively type-II right censored sample.

It's clear that when $R_1 = R_2 = \ldots = R_{m-1} = 0$ and $R_m = n - m$, this scheme includes the conventional type-II right censoring scheme, and complete sampling scheme (no censoring) when $n = m$ and $R_1 = R_2 = \ldots = R_m = 0$. Progressive censoring have been studied by many authors. Examples are Yadav et al (2019) [12], Maurya et al (2019) [8], Valiollahi et al (2018) [10], Chadli et al (2017) [5] and Kim et al (2011) [7]. For more detailed information about this method of censoring see N. Balakrishnan and Aggarwala (2000) [3], N. Balakrishnan (2007) [2] and N. Balakrishnan and Cramer (2014) [4].
The probability density function (pdf) of progressive type-II right censored data is given by

$$f_{X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n}}(x_1, x_2, \ldots, x_m) = C \prod_{i=1}^{m} f(x_i)[1 - F(x_i)]^{R_i}, \quad (1)$$

where $C = n(n - R_1 - 1)(n - R_1 - R_2 - 2)\ldots(n - \sum_{i=1}^{m-1}(R_i + 1))$.

In this study, we consider Rayleigh Pareto distribution, it's proposed and studied recently by Al-Kadim et al (2018) [1]. The cumulative distribution function (cdf) is given by

$$F(x) = \int_0^{\frac{1}{1-F^{\$}(x)}} f^{\omega}(t)dt, \quad (2)$$

where $F^{\$}(x)$ is the cdf of Pareto distribution, $F^{\$}(x) = 1 - (\frac{\gamma}{x})^{\theta}$and $f^{\omega}(t)$ is the probability density function of the Rayleigh distribution, $f^{\omega}(t) = \frac{x}{\beta^2}\exp(-\frac{1}{2}(\frac{x^2}{\beta^2}))$. Hence, the cdf of the RP distribution is given by

$$F_{R.P}(x;\alpha,\beta,\gamma) = 1 - \exp(-\frac{1}{2\beta^2}(\frac{x}{\gamma})^{\alpha}); \alpha,\beta,\gamma > 0. \tag{3}$$

The probability density function is defined by

$$f_{R.P}(x;\alpha,\beta,\gamma) = \frac{\alpha}{2\beta^2\gamma}(\frac{x}{\gamma})^{\alpha-1}\exp\{-\frac{1}{2\beta^2}(\frac{x}{\gamma})^{\alpha}\}. \tag{4}$$

When $\alpha = 1$, the RP distribution reduces to the exponential distribution with parameter $\lambda = \frac{1}{2\beta^2\gamma}$.

When $\beta = \sqrt{1/2}$, the the RP distribution reduces to the Weibull distribution $W(x;\alpha,\gamma)$.

The Reliability and the hazard rate function are respectively given by

$$R(t) = \exp(-\frac{1}{2\beta^2}(\frac{t}{\gamma})^{\alpha}),\ t > 0, \tag{5}$$

and

$$h(t) = \frac{\alpha}{2\beta^2\gamma}(\frac{t}{\gamma})^{\alpha-1},\ t > 0. \tag{6}$$

The aim of this paper is providing the estimators of the unknown parameters and the reliability characteristics of the RP distribution based on progressive type-II censored data; by two different methods: the maximum likelihood method and the bayesian method. For the second method, we estimate the parameters, $R(t)$ and $h(t)$ under three loss functions (quadratic, entropy and Linex) using Markov chain Monte Carlo (MCMC) simulation and Metropolis-Hastings algorithm. Then, we compare the performance of these bayesian estimates with the classical maximum likelihood estimators (MLEs).

The rest of the article is organized as follows. In section 2, we drive the maximum likelihood estimators of the parameters α, β, γ and the reliability characteristics under progressive type-II censored data. In section 3, we discuss the bayesian estimators under different loss functions. In section 4, a simulation study is conducted for obtaining the Bayes estimators and the MLEs. A comparison between the performance of the two methods by using the closeness Pitman criterion and the IMSE is provided in section 5. Finally, a conclusion is given in Section 6.

2. Maximum Likelihood Estimation

Let $X = (X_{1:m:n}, X_{2:m:n}, \dots, X_{m:m:n})$ be the ordered m observed failures under type-II progressively censored scheme from the Rayleigh Pareto distribution, obtained from a sample of size n with the censoring scheme $R_i = (R_1, \dots, R_m)$.

The likelihood function is then given by

$$L(x \mid \alpha,\beta,\gamma) \propto \frac{\alpha^m}{\beta^{2m}\gamma^{m\alpha}} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp\{-\frac{1}{2\beta^2}\sum_{i=1}^{m}(\frac{x_i}{\gamma})^{\alpha}(1+R_i)\}. \tag{7}$$

We set $\ln L(x \mid \alpha,\beta,\gamma) = l(x \mid \alpha,\beta,\gamma)$, so the log-likelihood function is

$$\begin{aligned} l(x \mid \alpha,\beta,\gamma) &\propto m\ln\alpha - 2m\ln\beta - m\alpha\ln\gamma \\ &+ (\alpha-1)\sum_{i=1}^{m}\ln(x_i) - \frac{1}{2\beta^2}\sum_{i=1}^{m}(\frac{x_i}{\gamma})^{\alpha}(1+R_i). \end{aligned} \tag{2.1}$$

Therefore, we obtain the maximum likelihood estimators of α, β and γ by solving the following non-linear equations

$$\frac{\partial l}{\partial \alpha} = \frac{m}{\alpha} - m\ln\gamma + \sum_{i=1}^{m}\ln(x_i) - \frac{1}{2\beta^2}\sum_{i=1}^{m}(\frac{x_i}{\gamma})^{\alpha}(1+R_i)\ln(\frac{x_i}{\gamma}) = 0,$$

$$\frac{\partial l}{\partial \beta} = \frac{-2m}{\beta} + \frac{1}{\beta^3}\sum_{i=1}^{m}(\frac{x_i}{\gamma})^{\alpha}(1+R_i) = 0,$$

$$\frac{\partial l}{\partial \gamma} = -\frac{m\alpha}{\gamma} + \frac{1}{2\beta^2}\sum_{i=1}^{m} x_i^{\alpha}(\frac{\alpha}{\gamma^{\alpha+1}})(1+R_i) = 0.$$

We see that there is no analytical solution of this system then we use the numerical methods to obtain the estimators $\hat{\alpha}_{MLE}, \hat{\beta}_{MLE}$ and $\hat{\gamma}_{MLE}$.
when we obtain the parameters simultaneously, the reliability characteristics can be written as

$$\hat{R}(t)_{MLE} = \exp(-\frac{1}{2\widehat{\beta}^2}(\frac{t}{\widehat{\gamma}})^{\widehat{\alpha}}), \tag{9}$$

$$\hat{h}(t)_{MLE} = \frac{\widehat{\alpha}}{2\widehat{\beta}^2\widehat{\gamma}}(\frac{t}{\widehat{\gamma}})^{\widehat{\alpha}-1}. \tag{10}$$

3. Bayesian Estimation

In this section, using the progressive censored data, we provide the Bayes estimators of the unknown parameters, their corresponding posterior risks and the reliability characteristics under the three loss functions: quadratic loss function ($L_Q(\hat{\phi},\phi) = (\hat{\phi}-\phi)^2$), the entropy loss function ($L_E(\hat{\phi},\phi) \propto (\frac{\phi}{\hat{\phi}})^p - p\ln(\frac{\phi}{\hat{\phi}}) - 1$) and Linex loss function($L_L(\hat{\phi},\phi) \propto \exp(r(\hat{\phi}-\phi)) - r(\hat{\phi}-\phi) - 1$).

3.1. **The prior density.** Here we assume that the unknown parameters are independent. We propose for the priors of γ and α : $G(a,b)$ and $G(c,d)$ distributions, respectively with the following densities

$$\pi(\gamma) = \frac{b^a}{\Gamma(a)}\gamma^{a-1}\exp(-b\gamma),$$

$$\pi(\alpha) = \frac{d^c}{\Gamma(c)}\alpha^{c-1}\exp(-d\alpha),$$

and the non-informative prior for β

$$\pi(\beta) \propto \frac{1}{\beta}.$$

So, the prior density is

$$\pi(\alpha,\beta,\gamma) \propto \frac{b^a d^c}{\Gamma(a)\Gamma(c)}\frac{\alpha^{c-1}\gamma^{a-1}}{\beta}\exp(-d\alpha - b\gamma). \tag{11}$$

3.2. **The posterior density.** The joint posterior density function of α,β and γ is given by

$$\begin{aligned}\pi(\alpha,\beta,\gamma \mid x) &= A^{-1}\frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} \\ &\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha} + d\alpha + b\gamma\}],\end{aligned} \tag{3.1}$$

where

$$\begin{aligned} A &= \int_0^\infty\int_0^\infty\int_0^\infty \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} \\ &\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha} + d\alpha + b\gamma\}]d\alpha d\beta d\gamma.\end{aligned}$$

3.3. **The Bayes estimators.** In this subsection, we estimate the parameters of the Rayleigh Pareto distribution with the bayesian method under three loss functions (quadratic, entropy and Linex).

- Under quadratic loss function, the bayesian estimators of α, β and γ are

$$\hat{\alpha}_{BQ} = A^{-1} \iiint \frac{\alpha^{m+c}}{\beta^{2m+1}\gamma^{\alpha m-a+1}}$$
$$\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha} + d\alpha + b\gamma\}]d\alpha d\beta d\gamma,$$
$$\hat{\beta}_{BQ} = A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m}\gamma^{\alpha m-a+1}}$$
$$\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha} + d\alpha + b\gamma\}]d\alpha d\beta d\gamma,$$
$$\hat{\gamma}_{BQ} = A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a}}$$
$$\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha} + d\alpha + b\gamma\}]d\alpha d\beta d\gamma,$$

and the corresponding posterior risks are

$$PR(\hat{\alpha}_{BQ}) = E_{\pi}(\alpha^2) - 2\hat{\alpha}_{BQ}E_{\pi}(\alpha) + \hat{\alpha}^2_{BQ},$$
$$PR(\hat{\beta}_{BQ}) = E_{\pi}(\beta^2) - 2\hat{\beta}_{BQ}E_{\pi}(\beta) + \hat{\beta}^2_{BQ},$$
$$PR(\hat{\gamma}_{BQ}) = E_{\pi}(\gamma^2) - 2\hat{\gamma}_{BQ}E_{\pi}(\gamma) + \hat{\gamma}^2_{BQ}.$$

- Under the entropy loss function, the bayesian estimators of α, β and γ are

$$\hat{\alpha}_{BE} = [A^{-1} \iiint \frac{\alpha^{m+c-p-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}}$$
$$\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha} + d\alpha + b\gamma\}]d\alpha d\beta d\gamma]^{-1/p},$$
$$\hat{\beta}_{BE} = [A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m+p+1}\gamma^{\alpha m-a+1}}$$
$$x \cdot \prod_{i=1}^{m}(x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha} + d\alpha + b\gamma\}]d\alpha d\beta d\gamma]^{-1/p},$$

$$\hat{\gamma}_{BE} = [A^{-1} \iiint \frac{\alpha^{m+c-p-1}}{\beta^{2m+1}\gamma^{\alpha m-a+p+1}} \cdot \prod_{i=1}^{m}(x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha} + d\alpha + b\gamma\}]d\alpha d\beta d\gamma]^{-1/p},$$

and the corresponding posterior risks are

$$PR(\hat{\alpha}_{BE}) = pE(\ln(\alpha) - \ln(\hat{\alpha}_{BE})),$$

$$PR(\hat{\beta}_{BE}) = pE(\ln(\beta) - \ln(\hat{\beta}_{BE})),$$

$$PR(\hat{\gamma}_{BE}) = pE(\ln(\gamma) - \ln(\hat{\gamma}_{BE})).$$

- The bayesian estimators of α, β and γ under Linex loss function are

$$\hat{\alpha}_{BL} = -\frac{1}{r}\ln(A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} \cdot \prod_{i=1}^{m}(x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha} + r\alpha + d\alpha + b\gamma\}]d\alpha d\beta d\gamma,$$

$$\hat{\beta}_{BL} = -\frac{1}{r}\ln(A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} \cdot \prod_{i=1}^{m}(x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha} + r\beta + d\alpha + b\gamma\}]d\alpha d\beta d\gamma,$$

$$\hat{\gamma}_{BL} = -\frac{1}{r}\ln(A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} \cdot \prod_{i=1}^{m}(x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha} + r\gamma + d\alpha + b\gamma\}]d\alpha d\beta d\gamma,$$

and the corresponding posterior risks are

$$PR(\hat{\alpha}_{BL}) = r(\hat{\alpha}_{BQ} - \hat{\alpha}_{BL}),$$
$$PR(\hat{\beta}_{BL}) = r(\hat{\beta}_{BQ} - \hat{\beta}_{BL}),$$
$$PR(\hat{\gamma}_{BL}) = r(\hat{\gamma}_{BQ} - \hat{\gamma}_{BL}).$$

- The bayesian estimators of the reliability and the hazard rate function under the three loss functions are given by

$$\hat{R}(t)_{BQ} = A^{-1}\iiint \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} e^{(-\frac{1}{2\beta^2}(\frac{t}{\gamma})^{\alpha})}$$
$$\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}],$$

$$\hat{h}(t)_{BQ} = A^{-1}\iiint \frac{\alpha^{m+c}}{2\beta^{2m+3}\gamma^{\alpha m-a+2}}(\frac{t}{\gamma})^{\alpha-1}$$
$$\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}],$$

$$\hat{R}(t)_{BE} = [A^{-1}\iiint \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} e^{(\frac{p}{2\beta^2}(\frac{t}{\gamma})^{\alpha})}$$
$$\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}]d\alpha d\beta d\gamma]^{-1/p},$$

$$\hat{h}(t)_{BE} = [A^{-1}\iiint \frac{\alpha^{m+c-p-1}}{2\beta^{2m-2p+1}\gamma^{\alpha m-a-p+1}}(\frac{t}{\gamma})^{\alpha-p-1}$$
$$\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}]d\alpha d\beta d\gamma]^{-1/p},$$

$$\hat{R}(t)_{BL} = -\frac{1}{r}\ln(A^{-1}\iiint e^{-r\exp(-\frac{1}{2\beta^2}(\frac{t}{\gamma})^{\alpha})}\frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}}$$
$$\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}]d\alpha d\beta d\gamma),$$

and

$$\hat{h}(t)_{BL} = -\frac{1}{r}\ln(A^{-1}\iiint e^{-r\frac{\alpha}{2\beta^2\gamma}(\frac{t}{\gamma})^{\alpha-1}}\frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}}$$
$$\cdot \prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}]d\alpha d\beta d\gamma).$$

To calculate these estimators, we use the MCMC numerical methods, such as the Metropolis-Hastings algorithm in the simulation study section.

4. Simulation Study

In this section, we perform a Monte Carlo simulation study using different sample sizes "n" (10, 30, 50), different censoring schemes R_i (see table1 1), and under different loss functions (symmetric and asymmetric). We simulated 10000 samples of the Rayleigh Pareto distribution, by assuming that $\alpha = 2$, $\beta = 0.25$ and $\gamma = 1.5$ with the hyper-parameters $a = 3, b = 2$, $c = 4$ and $d = 2$.

Table 1. Progressively Censoring Schemes.

(n, m)	R_i	Censoring scheme
(10,10)	$R_1(complete)$	(0∗ 10)=(0,0,0,0,0,0,0,0,0,0)
(10,5)	$R_2(type-II)$	(0∗ 4,5)
(10,5)	R_3	(5,0∗ 4)
(30,30)	R_4 (complete)	(0∗ 30)
(30,20)	R_5 (type-II)	(0∗ 19,10)
(30,20)	R_6	(10,0∗ 19)
(50,50)	R_7 (complete)	(0∗ 50)
(50,40)	R_8 (type-II)	(0∗ 39,10)
(50,40)	R_9	(10,0∗ 39)

4.1. **Maximum likelihood estimation.** In this study, we use the R package BB [11] which has high capacities for solving a non-linear system of equations, in order to obtain the MLEs of the parameters (α, β and γ) and the reliability characteristics $R(t)$ and $h(t)$ where $t = 0.25$ ($R(0.25) = 0.8007$, $h(0.25) = 1.7777$). The results are presented in Table 2 and Table 3.

From Table 2, we observe that the estimator of β is close to the real value more than the other estimators.

We notice also from Table 3 that the estimators of $R(t)$ and $h(t)$ are closer to the real values as the sample size n increases.

TABLE 2. Maximum Likelihood Estimation and Quadratic Errors of the RP Parameters.

(n,m)	R_i	$\widehat{\alpha}_{MLE}$	$\widehat{\beta}_{MLE}$	$\widehat{\gamma}_{MLE}$
(10,10)	R_1	1.9906(0.0081)	0.2317(0.0013)	1.6131(0.0174)
(10,5)	R_2	1.9649(0.0269)	0.2503(0.0024)	1.6492(0.0323)
(10,5)	R_3	1.8320(0.1498)	0.2200(0.0045)	1.7090(0.0775)
(30,30)	R_4	1.9990(0.0074)	0.2350(0.0005)	1.5933(0.0126)
(30,20)	R_5	1.9780(0.0070)	0.2361(0.0006)	1.6216(0.0181)
(30,20)	R_6	1.9225(0.0293)	0.1728(0.0067)	1.6725(0.0396)
(50,50)	R_7	2.0030(0.0072)	0.2365(0.0003)	1.5858(0.0113)
(50,40)	R_8	1.9918(0.0083)	0.2359(0.0003)	1.6010(0.0136)
(50,40)	R_9	2.9190(0.0116)	0.1888(0.0041)	1.6509(0.0256)

TABLE 3. MLEs with Quadratic Errors of the Reliability Function and the Hazard Rate Function

(n,m)	R_i	$\widehat{R}(t)_{mle}$	$\widehat{h}(t)_{mle}$
(10,10)	R_1	0.7823(0.0055)	1.9693(0.5413)
(10,5)	R_2	0.7998(0.0078)	1.7684(0.6533)
(10,5)	R_3	0.6900(0.0300)	2.6696(2.1197)
(30,30)	R_4	0.7949(0.0019)	1.8329(0.1516)
(30,20)	R_5	0.8021(0.0020)	1.8089(0.1921)
(30,20)	R_6	0.6329(0.0331)	3.5540(3.8524)
(50,50)	R_7	0.7978(0.0013)	1.8059(0.0921)
(50,40)	R_8	0.7959(0.0015)	1.8129(0.1053)
(50,40)	R_9	0.6815(0.0159)	2.9577(1.6362)

4.2. **Bayesian estimation.** In this subsection, we obtain the bayesian estimators of the unknown parameters and the reliability characteristics of RP distribution by using MCMC methods and Mertopolis-Hastings algorithm in R software. In Table 4, we present the Bayes estimators $\hat{\alpha}_{BQ}$, $\hat{\beta}_{BQ}$ and $\hat{\gamma}_{BQ}$ under quadratic loss function with their corresponding posterior risks. The Bayes estimators of the distribution parameters under the entropy loss function and their corresponding posterior risks where p=-0.5,1,2 are given in Table 5. And under the Linex loss function (with r=-0.5,1,2) the results are summarized in Table 6. Then, the bayesian estimators of $R(t)$ and $h(t)$ under the three loss functions are presented in Table 7.

It's noticed from Table 4 that the best estimators of α, β and γ are in the case of n=10.

From Table 5, it's observed that p=-0.5 gives the best estimators of the distribution parameters under entropy loss function, and from Table 6, r=-0.5 gives the best estimators of the parameters under the Linex loss function.

We remark from the Table 7, that the good results for the reliability function R(t) are under the Linex loss function. However, for the hazard rate function h(t), the estimators under the entropy loss function perform better than the other loss functions.

From the results in Table 8, we find that the bayesian estimation of the reliability characteristics perform better than the classical maximum likelihood approach.

TABLE 4. Bayesian Estimation under Quadratic Loss Function with Posterior Risks between brackets.

(n,m)	R	$\widehat{\alpha}_{BQ}$	$\widehat{\beta}_{BQ}$	$\widehat{\gamma}_{BQ}$
(10,10)	R_1	2.0203(0.0735)	0.2773(0.0504)	1.5223(0.0200)
(10,5)	R_2	2.0109(0.0011)	0.3064(0.0029)	1.5239(0.0012)
(10,5)	R_3	2.0247(0.0014)	0.3132(0.0033)	1.5357(0.0016)
(30,30)	R_4	2.0887(0.5475)	0.3492(0.3185)	1.5917(0.1378)
(30,20)	R_5	2.0207(0.0017)	0.3162(0.0028)	1.5337(0.0008)
(30,20)	R_6	1.9808(0.0016)	0.2938(0.0028)	1.4988(0.0008)
(50,50)	R_7	1.8324(0.8387)	0.5716(0.4744)	1.5113(0.2129)
(50,40)	R_8	2.0382(0.0062)	0.3232(0.0072)	1.5482(0.0026)
(50,40)	R_9	2.0196(0.0061)	0.2941(0.0071)	1.5266(0.0026)

TABLE 5. Bayesian Estimation under Entropy Loss Function with PR between brackets

(n, m)	R_i	$Parameters$	$p = -0.5$	$p = 1$	$p = 2$
$(10, 10)$	R_1	$\hat{\alpha}_{BE}$	$1.9323(0.0036)$	$1.8818(0.0192)$	$1.8313(0.0927)$
		$\hat{\beta}_{BE}$	$0.3682(0.0154)$	$0.3415(0.0441)$	$0.3319(0.1456)$
		$\hat{\gamma}_{BE}$	$1.4940(0.0014)$	$1.4795(0.0068)$	$1.4670(0.0306)$
$(10, 5)$	R_2	$\hat{\alpha}_{BE}$	$2.0152(2.94e-05)$	$2.0149(0.0001)$	$2.01467(0.0004)$
		$\hat{\beta}_{BE}$	$0.2986(7.17e-04)$	$0.2973(0.0027)$	$0.2965(0.0107)$
		$\hat{\gamma}_{BE}$	$1.5249(3.10e-05)$	$1.5246(0.0001)$	$1.5244(0.0004)$
$(10, 5)$	R_3	$\hat{\alpha}_{BE}$	$2.0208(3.16e-05)$	$2.0204(0.0001)$	$2.0201(0.0005)$
		$\hat{\beta}_{BE}$	$0.3016(8.33e-04)$	$0.3002(0.0031)$	$0.2993(0.0124)$
		$\hat{\gamma}_{BE}$	$1.5297(3.74e-05)$	$1.5294(0.0001)$	$1.5291(0.0005)$
$(30, 30)$	R_4	$\hat{\alpha}_{BE}$	$1.4146(1.50e-02)$	$1.2967(0.0570)$	$1.2347(0.2119)$
		$\hat{\beta}_{BE}$	$0.6883(2.88e-02)$	$0.5745(0.1230)$	$0.5135(0.4705)$
		$\hat{\gamma}_{BE}$	$1.2335(5.89e-03)$	$1.1915(0.0228)$	$1.1665(0.0881)$
$(30, 20)$	R_5	$\hat{\alpha}_{BE}$	$2.0003(6.98e-05)$	$1.9993(0.0003)$	$1.9984(0.0016)$
		$\hat{\beta}_{BE}$	$0.2923(9.91e-04)$	$0.2907(0.0034)$	$0.2898(0.0131)$
		$\hat{\gamma}_{BE}$	$1.5125(3.94e-05)$	$1.5122(0.0001)$	$1.5119(0.0007)$
$(30, 20)$	R_6	$\hat{\alpha}_{BE}$	$2.0020(6.76e-05)$	$2.0010(0.0003)$	$2.0001(0.0016)$
		$\hat{\beta}_{BE}$	$0.2925(9.27e-04)$	$0.2910(0.0032)$	$0.2901(0.0123)$
		$\hat{\gamma}_{BE}$	$1.5138(3.88e-05)$	$1.5134(0.0001)$	$1.5131(0.0007)$
$(50, 50)$	R_7	$\hat{\alpha}_{BE}$	$1.1489(9.79e-03)$	$1.0928(0.0304)$	$1.0699(0.1032)$
		$\hat{\beta}_{BE}$	$0.8794(1.49e-02)$	$0.7837(0.0852)$	$0.6967(0.4060)$
		$\hat{\gamma}_{BE}$	$1.0943(4.44e-03)$	$1.0684(0.0149)$	$1.0560(0.0533)$
$(50, 40)$	R_8	$\hat{\alpha}_{BE}$	$2.0105(0.0002)$	$2.0061(0.0016)$	$2.0012(0.0081)$
		$\hat{\beta}_{BE}$	$0.3115(0.0021)$	$0.3080(0.0068)$	$0.3063(0.0244)$
		$\hat{\gamma}_{BE}$	$1.5259(0.0001)$	$1.5245(0.0006)$	$1.5233(0.0028)$
$(50, 40)$	R_9	$\hat{\alpha}_{BE}$	$2.0118(0.0002)$	$2.0075(0.0015)$	$2.0027(0.0079)$
		$\hat{\beta}_{BE}$	$0.3117(0.0021)$	$0.3083(0.0067)$	$0.3066(0.0241)$
		$\hat{\gamma}_{BE}$	$1.5269(0.0001)$	$1.5255(0.0006)$	$1.5244(0.0027)$

TABLE 6. Bayesian Estimation with PR between brackets under Linex Loss Function.

(n,m)	R_i	$parameters$	$r=-0.5$	$r=1$	$r=2$
$(10,10)$	R_1	$\hat{\alpha}_{BL}$	$1.9594(0.0077)$	$1.8980(0.0459)$	$1.8258(0.2363)$
		$\hat{\beta}_{BL}$	$0.3915(0.0044)$	$0.3686(0.0140)$	$0.3583(0.0486)$
		$\hat{\gamma}_{BL}$	$1.5025(0.0023)$	$1.4864(0.0114)$	$1.4717(0.0524)$
$(10,5)$	R_2	$\hat{\alpha}_{BL}$	$2.0156(1.18e-04)$	$2.0149(0.0004)$	$2.0144(0.0019)$
		$\hat{\beta}_{BL}$	$0.2992(6.93e-05)$	$0.2987(0.0002)$	$0.2985(0.0010)$
		$\hat{\gamma}_{BL}$	$1.5251(7.32e-05)$	$1.5247(0.0002)$	$1.5244(0.0011)$
$(10,5)$	R_3	$\hat{\alpha}_{BL}$	$2.0211(1.29e-04)$	$2.0204(0.0005)$	$2.0198(0.0020)$
		$\hat{\beta}_{BL}$	$0.3023(8.28e-05)$	$0.3018(0.0003)$	$0.3015(0.0012)$
		$\hat{\gamma}_{BL}$	$1.5300(8.92e-05)$	$1.5295(0.0003)$	$1.5291(0.0013)$
$(30,30)$	R_4	$\hat{\alpha}_{BL}$	$1.5217(3.20e-02)$	$1.3405(0.1170)$	$1.2519(0.4114)$
		$\hat{\beta}_{BL}$	$0.7476(1.14e-02)$	$0.6775(0.0472)$	$0.6317(0.1861)$
		$\hat{\gamma}_{BL}$	$1.2670(9.39e-03)$	$1.2122(0.0360)$	$1.1800(0.1364)$
$(30,20)$	R_5	$\hat{\alpha}_{BL}$	$2.0009(1.99e-04)$	$1.9995(0.0010)$	$1.9980(0.0051)$
		$\hat{\beta}_{BL}$	$0.2932(1.30e-04)$	$0.2925(0.0004)$	$0.2921(0.0016)$
		$\hat{\gamma}_{BL}$	$1.5128(7.91e-05)$	$1.5123(0.0003)$	$1.5119(0.0014)$
$(30,20)$	R_6	$\hat{\alpha}_{BL}$	$2.0026(1.88e-04)$	$2.0012(0.0009)$	$1.9997(0.0050)$
		$\hat{\beta}_{BL}$	$0.2933(1.23e-04)$	$0.2927(0.0004)$	$0.2923(0.0015)$
		$\hat{\gamma}_{BL}$	$1.5141(7.75e-05)$	$1.5136(0.0003)$	$1.5132(0.0014)$
$(50,50)$	R_7	$\hat{\alpha}_{BL}$	$1.2202(2.24e-02)$	$1.1138(0.0615)$	$1.0784(0.1938)$
		$\hat{\beta}_{BL}$	$0.9119(6.05e-03)$	$0.8698(0.0300)$	$0.8318(0.1360)$
		$\hat{\gamma}_{BL}$	$1.1205(7.68e-03)$	$1.0807(0.0244)$	$1.0630(0.0842)$
$(50,40)$	R_8	$\hat{\alpha}_{BL}$	$2.0129(6.80e-04)$	$2.0074(0.0041)$	$1.9998(0.0234)$
		$\hat{\beta}_{BL}$	$0.3140(4.41e-04)$	$0.3118(0.0014)$	$0.3107(0.0049)$
		$\hat{\gamma}_{BL}$	$1.5267(2.33e-04)$	$1.5251(0.0011)$	$1.5237(0.0051)$
$(50,40)$	R_9	$\hat{\alpha}_{BL}$	$2.0141(6.56e-04)$	$2.0088(0.0039)$	$2.0014(0.0228)$
		$\hat{\beta}_{BL}$	$0.3142(4.30e-04)$	$0.3120(0.0013)$	$0.3109(0.0048)$
		$\hat{\gamma}_{BL}$	$1.5277(2.28e-04)$	$1.5261(0.0010)$	$1.5247(0.0050)$

TABLE 7. Bayesian Estimation and the Quadratic Errors of the Reliability and Hazard Rate Functions.

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(n,m)	R_i	$\widehat{R}(t)_{BQ}$	$\widehat{h}(t)_{BQ}$	$\widehat{R}(t)_{BE}$	$\widehat{h}(t)_{BE}$	$\widehat{R}(t)_{BL}$	$\widehat{h}(t)_{BL}$
(10,10)	R_1	0.8402 (0.0068)	1.4081 (0.6727)	0.8751 (0.0001)	1.0056 (0.0174)	0.8758 (0.0001)	1.0697 (0.0156)
(10,5)	R_2	0.8611 (0.0030)	1.2052 (0.2619)	0.8507 $(8.28e-05)$	1.2976 $(3.54e-03)$	0.8510 $(6.14e-05)$	1.3156 $(4.77e-03)$
(10,5)	R_3	0.8685 (0.0036)	1.1442 (0.3110)	0.8552 $(1.02e-04)$	1.2551 $(4.84e-03)$	0.8556 $(7.72e-05)$	1.2780 $(5.96e-03)$
(30,30)	R_4	0.9163 (0.0091)	0.7286 (1.3834)	0.8942 $(6.17e-05)$	0.6054 $(9.17e-03)$	0.8944 $(4.98e-05)$	0.6283 $(5.13e-03)$
(30,20)	R_5	0.8730 (0.0021)	1.0986 (0.1891)	0.8406 $(8.63e-05)$	1.3862 $(3.14e-03)$	0.8409262 $(6.21e-05)$	1.4047 $(5.13e-03)$
(30,20)	R_6	0.8326 (0.0021)	1.4601 (0.1959)	0.8415 $(8.77e-05)$	1.3779 $(3.21e-03)$	0.8418 $(6.32e-05)$	1.3967 $(5.20e-03)$
(50,50)	R_7	0.9301 (0.0081)	0.4527 (1.5825)	0.8892 $(4.02e-05)$	0.5198 $(1.74e-03)$	0.8893 $(3.30e-05)$	0.5233 $(7.11e-04)$
(50,40)	R_8	0.8855 (0.0046)	0.9906 (0.4083)	0.8628 (0.0001)	1.1755 (0.0058)	0.8632 $(8.93e-05)$	1.2042 $(7.70e-03)$
(50,40)	R_9	0.8521 (0.0047)	1.2975 (0.4160)	0.8635 (0.0001)	1.1699 (0.0059)	0.8639 $(9.19e-05)$	1.1994 $(7.90e-03)$

TABLE 8. MLEs and Bayes Estimators of the Reliability Characteristics with Quadratic Errors between brackets.

(n,m)	R_i	$\widehat{R}(t)_{mle}$	$\widehat{h}(t)_{mle}$	$\widehat{R}(t)_{BL}$	$\widehat{h}(t)_{BE}$
(10,10)	R_1	0.7969(0.0040)	1.9197(0.5254)	0.8758(0.0001)	1.0056(0.0174)
(10,5)	R_2	0.7994(0.0096)	1.8688(1.3082)	0.8510(6.14e-05)	1.2976(3.54e-03)
(10,5)	R_3	0.6855(0.0367)	3.2914(8.4523)	0.8556(7.72e-05)	1.2551(4.84e-03)
(30,30)	R_4	0.8077(0.0011)	1.7795(0.1240)	0.8944(4.98e-05)	0.6054(9.17e-03)
(30,20)	R_5	0.8021(0.0020)	1.8089(0.1921)	0.8409262(6.21e-05)	1.3862(3.14e-03)
(30,20)	R_6	0.6451(0.0301)	3.5749(4.1357)	0.8418(6.32e-05)	1.3779(3.21e-03)
(50,50)	R_7	0.8101(0.0007)	1.7490(0.0694)	0.8893(3.30e-05)	0.5198(1.74e-03)
(50,40)	R_8	0.8074(0.0008)	1.7800(0.0883)	0.8632(8.93e-05)	1.1755(0.0058)
(50,40)	R_9	0.8075(0.0008)	1.7797(0.0896)	0.8639(9.19e-05)	1.1699(0.0059)

5. Comparison Study for the Parameter Estimators

In this section, we compare the performance of the two methods for different censoring schemes, by using the Pitman closeness criterion [6, 9] and the integrated mean square error defined (respectively) as follow.

Definition 5.1. *Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be two different estimators of a parameter θ, we say that $\hat{\theta}_1$ is Pitman closer estimate than $\hat{\theta}_2$ if, for all values of the $\theta \in \Theta$*

$$P_\theta(\left|\hat{\theta}_1 - \theta\right| < \left|\hat{\theta}_2 - \theta\right|) > \frac{1}{2}.$$

Definition 5.2. *The integrated mean square error is defined as*

$$IMSE = \frac{1}{N}\sum_{i=1}^{N}(\hat{\theta}_i - \theta)^2.$$

For this comparison, we select the best bayesian estimators of α, β and γ under the three loss function (quadratic, entropy (p=-0.5), and Linex (r=-0.5)) to compare it with the MLEs.

In Table 9, we compare the MLEs with the Bayes estimators by using the Pitman criterion, so we say that the bayesian estimator is better than the MLEs when the Pitman probability is greater than 0.5. In term of this criterion, we notice that the bayesian estimators of α and γ seem to be more efficient than the maximum likelihood method, especially when the data are progressive censored.

In Table 10, we compare the bayesian estimators with the MLEs by using IMSE criterion. It's observed from the results that the bayesian method perform well, but in the complet data (R_1, R_4 and R_7) the MLEs are better than the bayesian estimators.

Table 9: Comparison of MLE and Bayesian Estimation under the Three Loss –Functions using Pitman Criterion.

n	m	R	Parametres	quadratic	Entropy (p=-0.5)	Linex (a=-0.5)
10	10	R_1	α	0.6043	0.6055	0.5982
			β	0.1982	0.2187	0.1880
			γ	0.7548	0.7566	0.7520

10	5	R_2	α	0.7297	0.7304	0.7227
			β	0.3036	0.3526	0.2778
			γ	0.9614	0.9629	0.9596
10	5	R_3	α	0.7405	0.7445	0.7289
			β	0.4822	0.5125	0.4624
			γ	0.9592	0.9599	0.9579
30	30	R_4	α	0.2175	0.2182	0.2108
			β	0.0135	0.0160	0.0125
			γ	0.2932	0.2952	0.2927
30	20	R_5	α	0.8199	0.8092	0.8311
			β	0.2209	0.2649	0.1999
			γ	0.9826	0.9827	0.9823
30	20	R_6	α	0.7827	0.7684	0.7969
			β	0.8208	0.8518	0.7980
			γ	0.9932	0.9932	0.9928
50	50	R_7	α	0.0834	0.0840	0.0826
			β	$9e-04$	$9e-04$	$8e-04$
			γ	0.1148	0.1164	0.1136
50	40	R_8	α	0.7817	0.7816	0.7770
			β	0.1061	0.1275	0.0938
			γ	0.8645	0.8705	0.8563
50	40	R_9	α	0.7727	0.7697	0.7700
			β	0.5346	0.5775	0.5056
			γ	0.9782	0.9793	0.9776

Table 10: Comparison of the MLE and the Bayesian Estimation using the IMSE.

n	m	R	Parametres	MLE	quadratic	Entropy (p=-0.5)	Linex (a=-0.5)
10	10	R_1	α	0.0088	0.0759	0.0778	0.0735
			β	0.0013	0.0520	0.0498	0.0536
			γ	0.0176	0.0204	0.0206	0.0230

10	5	R_2	α	0.0241	0.0012	0.0012	0.0011
			β	0.0023	0.0029	0.0024	0.0033
			γ	0.0311	0.0012	0.0011	0.0012
10	5	R_3	α	0.1445	0.0015	0.0015	0.0015
			β	0.0044	0.0034	0.0029	0.0037
			γ	0.0741	0.0016	0.0015	0.0017
30	30	R_4	α	0.0075	0.5412	0.5440	0.5375
			β	0.0005	0.3150	0.3125	0.3169
			γ	0.0128	0.1363	0.1366	0.1359
30	20	R_5	α	0.0072	0.0015	0.0017	0.0013
			β	0.0006	0.0026	0.0021	0.0031
			γ	0.0183	0.0007	0.0007	0.0007
30	20	R_6	α	0.0270	0.0016	0.0018	0.0015
			β	0.0068	0.0027	0.0022	0.0031
			γ	0.0385	0.0008	0.0008	0.0008
50	50	R_7	α	0.0073	0.8052	0.8067	0.8033
			β	0.0003	0.4584	0.4571	0.4594
			γ	0.0114	0.2018	0.2019	0.2016
50	40	R_8	α	0.0082	0.0045	0.0046	0.0043
			β	0.0003	0.0062	0.0054	0.0068
			γ	0.0136	0.0022	0.0022	0.0023
50	40	R_9	α	0.0114	0.0053	0.0056	0.0051
			β	0.0041	0.0067	0.0059	0.0073
			γ	0.0255	0.0024	0.0024	0.0025

6. Conclusion

In this paper, we studied the Rayleigh Pareto parameters and reliability characteristics with bayesian and maximum likelihood methods under the progressive type-II censoring. We used numerical methods to obtain the Bayes estimators and MLEs, because they are not in their explicit form. The bayesian estimators are obtained under three loss functions by the use of MCMC method. It has been checked that we can obtain the results under complete data ($R_1 =$

$R_2 = \ldots = R_m = 0$) and type-II censoring ($R_1 = R_2 = \ldots = R_{m-1} = 0$ and $R_m = n - m$) from the progressively censored scheme. For the reliability characteristics, we found that the method of Bayes perform better than the classical maximum likelihood method. For the parameters of the RP distribution, we used two criterions (Pitman and IMSE) to compare the performance of the two approaches and we showed that the bayesian estimation under progressive censored data is more efficient than the maximum likelihood estimation.

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