Advances in Mathematics: Scientific Journal **10** (2021), no.3, 1839–1843
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.3.61

ON P_2-C-CLOSED SPACE IN BITOPOLOGICAL SPACE

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ABSTRACT. In our paper, we introduce a new concept concerning C-closed topological spaces. A pairwise C-closed bitopological space differs than what was discussed in [4], namely, $P_2 - C$-closed spaces. Also, we obtain many results concerning properties of pairwise sequential spaces namely, P_1-sequential spaces and investigate the relationship between them.

1. INTODUCTION

A bitopological space say (X, τ_1, τ_2) is typically a non-empty set X together with two topologies τ_1 and τ_2 defined on it, First, the study the study of bitopological spaces was initiated by J. C. Kelly [1] where he published in london mathematical society in 1963 and thereafter lots of papers have been proposed in order to generalize familiar topological concepts to bitopological ones.

In this paper, we introduce the concept of pairwise C-closed bitopological spaces namely, $P_2 - C$-closed spaces and we discussed some their properties.

The purpose of this research paper is to discuss the $P_2 - C$-closed spaces in the light of its hereditary properties. We also show that a pairwise sequential space, namely, P_1-sequential forms a subclass of the class of $P_2 - C$-closed spaces.

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2020 *Mathematics Subject Classification.* 22xx, 22Cxx, 22Axx.
Key words and phrases. $P_2 - C$-Closed, Bitopological spaces, C-closed spaces.
Submitted: 09.02.2021; *Accepted:* 24.02.2021; *Published:* 31.03.2021.

2. Preliminaries

Throughout this article, (X, τ) is used to denote a topological space and (X, τ_1, τ_2) to denote bitopological space. Also we use $\mathbb{N}$, $\mathbb{R}$, $t_{P_1}(X)$ to denote the set of all real numbers, natural numbers and the tightness of X with respect to $\tau_1 \cup \tau_2$ respectively. The notations of $P_1 - cl(A)$ and P_1-cluster point of A are denoting the closure of A with respect to $\tau_1 \cup \tau_2$ and cluster point of A with respect to $\tau_1 \cup \tau_2$ respectively.

3. P_2- C-closed space

Kilicman and Salleh [2] defined many concepts in bitopological space (X,τ_1,τ_2) such as P_1-open set and P_1-closed set where a set U is said to be P_1-open if U is $(\tau_1 \cup \tau_2)$-open in X and a set F is said to be P_1-closed if F is $(\tau_1 \cup \tau_2)$-closed in X. Moreover, they defined that a bitopological space (X, τ_1, τ_2) typically is said to be P_2-compact if every P_1-open cover of X has a finite subcover.

Definition 3.1. *A bitopological space (X, τ_1, τ_2) called P_2- countably compact if each countable P_1-open cover of X has a finite subcover.*

Definition 3.2. *A bitopological space say (X, τ_1, τ_2) is called $P_2 - C$-closed space if each P_2- countably compact subset of X is P_1-closed.*

Remark 3.1. *A bitopological space (X, τ_1, τ_2) is $P_2 - C$-closed if each non P_1-closed subset B of X contains a sequence that has no P_1-cluster point in B.*

Theorem 3.1. *Each supspace of P_2-C-closed spaces is P_2-C-closed.*

Proof. Let (X, τ_1, τ_2) be a $P_2 - C$-closed space and (Y,τ'_1,τ'_2) be a subspace of it. Let F be a $\acute{P}_2$- countably compact subset of Y. Now let Ũ = $\{Ui : i \in \mathbb{N}\}$ be a countable open cover for F where Ui is a P_1-*open* subset of X $\forall i \in \mathbb{N}$, then Ui $\cap Y$ is $\acute{P}_1$-*open* subset of Y $\forall i \in \mathbb{N}$ and Ő= $\{(Ui \cap Y) : i \in \mathbb{N}\}$ is also a countable $\acute{P}_1$-*open* cover for F. Since F is $\acute{P}_2$- countably compact subset of Y, there is a finite subcover $\{Ui \cap Y : i = 1, 2, \ldots, n\}$ of Ő such that $F \subseteq \bigcup_{i=1}^{n} (Ui \cap Y) \subseteq \bigcup_{i=1}^{n} Ui$. Hence, F is P_2-*countably compact* subset of X. Now, since X is P_2-C-*closed*, then F is P_1-*closed* subset of X, so X-F is a P_1-*open* subset of X. But $Y \cap (X$-$F)$ $=Y$-F is $\acute{P}_1$-*open* subset of Y, hence F is $\acute{P}_2$-closed subset of Y. □

Definition 3.3. *A bitopological space say (X,τ_1,τ_2) is called P_1-$Sequential$ if each non P_1-$closed$ subset B of X contains a sequence which is a P_1-converging sequence to a point in $X \setminus B$.*

Definition 3.4. *A bitopological space (X, τ_1, τ_2) is called P_1-$Hausdorff$ space if for each $x \neq y$ in X, there are P_1-$open$ sets W and G where $x \in W$, $y \in G$ and $W \cap G = \emptyset$.*

Definition 3.5 (2). *A space (X, τ_1, τ_2) is called a P_1-regular space if for every point $x \in X$, and each P_1-closed set A such that $x \notin A$, there are P_1-open sets G and W P_1-open sets where $x \in G$, $A \in W$, and $G \cap W = \emptyset$.*

Theorem 3.2. *Let (X, τ_1, τ_2) be a P_1-Hausdorff space, the sequence (y_n) be a convergent sequence in X, then (y_n) has one limit point exactly.*

Proof. Suppose the contrary, Then $y_n \to y$ and $y_n \to x$ for some $y \neq x$, then there are disjoint P_1-$open$ sets G and W with $y \in G$, $x \in W$. Hence, there exists $N_G \in \mathbb{N}$ so that $y_n \in G$ for every $n>N_G$ and $N_W \in \mathbb{N}$such that $y_n \in W$ for every $n>N_W$, choose $N = max\{N_G, N_W\}$, Thus, there exists $N \in\mathbb{N}$ such that $y_n \in G$, $y_n \in W$ for each $n \geq M$. But $G \cap W = \emptyset$, which is naturally contradiction. □

Theorem 3.3. *If (X, τ_1, τ_2) is P_1-Hausdorff, P_1- sequential space, then X is a P_2-C-closed.*

Proof. Let B be non P_1-closed subset of a space X. Now, since X is P_1- sequential, there exist a sequence (y_n) which is P_1-converging to a point in $X \setminus B$ say y, by uniqueness of limit point of the sequence in P_1-Hausdorff space, we conclude that a seq (y_n) has no P_1-cluster points in B. So, we get the result. □

Theorem 3.4. *If X is P_1-Hausdorff, and each P_2-countably compact subset of X is P_1- sequential, then X is a P_2-C-closed space.*

Proof. let B be P_2-countably compact subset of X, and assume that B is not P_1-closed in X, then there exists $y \in P_1$-cl$(B) \setminus B$, let $C = B \cup \{y\}$, then C is also P_2-countably compact. Now, B is not P_1-closed in C, since C is P_1- sequential, then there exists a sequence (y_n) in B such that y_n P_1-convergent to $C \setminus B = \{y\}$. Therefore, there exists sequence y_n in B has no P_1-$cluster\ points$ in a set B. This is naturally contradiction. □

Definition 3.6. *A subset B of a space (X, τ_1, τ_2) is called P_1-dence in X if P_1-cl$(B) = X$. X is P_1- separable if there is a countable set B which is P_1-dence in X.*

Definition 3.7. *In the space (X, τ_1, τ_2), the density of X with respect to $\tau_1 \cup \tau_2$, which is denoted by $d_{P_1}(X)$, is the least cardinality of a P_1-dence subset of X.*

Definition 3.8. *A bitopological space (X, τ_1, τ_2) is called P_1-left-separated if there is a well-ordering "<" on a space X such that for every $y \in X$, the set $\{z \in X : y \leq z\}$ is P_1-open supset of X.*

Definition 3.9. *A family μ of non-empty subsets of a space X is $irreducible$ if there are no members of μ are contained in the union of another members of μ. A space X is called P_1-$irreducible$ if each P_1-open cover of a space X has an $irreducible$ P_1-open refinement which covers X.*

Corollary 3.1. *Every P_2-countably compact and P_1-irreducible spaces is P_2-compact.*

Theorem 3.5. *Every P_1-Hausdorff, P_1-left-separated space X is a P_2-C-closed space.*

Proof. We claim that each P_2-countably compact subset of X is P_1-closed by showing that it is P_2-compact. Since each subset of P_1-left-separated is P_1-left-separated, using the previous corollary, it is sufficient to show that X is P_1-irreducible, so Let $<$ be a well-ordering on X, then $\forall\ y \in X$, the set $H_y = \{z \in X : y \leq z\}$ is P_1- open supset of X. Suppose that μ be a P_1- open cover of X. Choose $y_0 = min_{<}X$ and $W_0 = G_0 \in \mu$ such that $y_0 \in G_0$. Assume that for every $\beta < \alpha$, we have $y_\beta \in X$, $G_\beta \in \mu$ and W_β is P_1-open subset of X such that $y_\beta = min_{<}(X \setminus \cup_{\ \beta^* < \beta}\ W_{\beta^*})$ and $y_\beta \in W_\beta \subseteq G_\beta \cap\ H_{y_\beta}$. Then, we have $\sigma_\alpha = \{W_\beta : \beta{<}\alpha\}$ is P_1-irreducible open refinement of μ. Now, if $X \setminus \cup\sigma_\alpha \neq \emptyset$, take $y_\alpha = min_{<}\ (X \setminus \cup\sigma_\alpha)$. Choose $G\alpha \in \mu \ni y_\alpha \in$ Gα, and let Wα=G$\alpha\cap$ H$_{y\alpha}$. then $\sigma_{\alpha+1} = \sigma_\alpha \cup\ W_\alpha$ is P_1- irreducible. Continue such steps and let δ be the smallest ordinal such that $\cup\ \sigma_\delta = X$, then σ_δ is required P_1-irreducible refinement of μ. Thus, X is P_1-irreducible. □

Corollary 3.2. *Every bitopological space includes a P_1-left-separated, P_1-dence subspace.*

Possible applications of this study can be found in problems of $[3,\ 4,\ 5]$ and $[6]$.

REFERENCES

[1] C. KELLY: *Bitopological spaces*, Proc. London Math. Soc. **13, 1963.**

[2] A. KILIÇMAN, Z. SALLEH: *A note on pairwise continuous mappings and bitopological spaces*, **Eur. J. Pure Appl. Math, 2 (2009), 325-337.**

[3] N. OMAR, H. HDEIB: *On pairwise C-closed space in bitopological space*, **Journal of Semigroup Theory and Applications, (2019).**

[4] E. ALMUHUR, M. ALLABADI: *Pairwise Strongly Lindelöf, Pairwise Nearly, Almost and Weakly Lindelöf Bitopological Spaces*, **WSEAS Transactions on Mathematics, (2021).**

[5] M. ALLABADI: *Planar of special idealization rings*. **WSEAS Transactions on Mathematics. 19 (2020), 606-609.**

[6] M. ALLABADI: *Eulerian graph of some special idealization rings*.

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