

A REVIEW OF ALGEBRAIC APPROACH ON ROUGH SETS

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ABSTRACT. Mathematics mainly concern with sets and functions. Cantor set is identified by George Cantor. After the notion of vagueness raises set concepts extended to fuzzy sets by Zadeh and an alternate idea of vagueness is implemented by Pawlak in 1982 based on boundary regions. Solving any problem algebraically gives a perfect solution. In that sense, many algebraic concepts are implemented on the fuzzy sets, rough sets etc. Focussing that in mind, in this review article, we give some study of algebraic approach on rough sets.

1. INTRODUCTION

Mathematics is predominantly agitated with sets and functions. George Cantor manifested classical set theory and stated that a collection is a set of well-defined objects which is precise. Some mathematicians aim at ambiguous (inaccurate) ideas. It is a vague concept to find young people in any particular area since it is based on the person and their age bound classification and young is inexact. In natural languages the idea which we use are not accurate. Classical logic deals with only exactness so ambiguity is essential for mathematicians and computer scientists. In classical set theory we can check whether a person can be definitely healthy or ill but in fuzzy set theory we can say that, a person is

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2020 *Mathematics Subject Classification.* 16Y60, 05C25, 08A72.

Key words and phrases. Semigroup, Semiring, Fuzzy set, Rough set.

Submitted: 12.03.2021; *Accepted:* 02.04.2021; *Published:* 04.04.2021.

healthy at 70 percent or 80 percent (in degree we can say 0.7 or 0.8). Zadeh (1965) [51] introduced the concept of fuzzy sets which is completely new approach to deal vagueness. According to Zadeh's approach fuzzy sets are defined by partial membership in contrast to crisp membership which is used in classical set theory.

Definition 1.1. [51] *For a nonempty set $A \subseteq Y$, A fuzzy set A is defined as $A = \{(y, \mu_A(y)) / y \in Y\}$ which is identified by its membership function $\mu_A(y) : Y \rightarrow [0, 1]$ and fulfilling the condition $\mu_A(y) + \gamma_A(y) = 1$ where $\gamma_A(y) = 1 - \mu_A(y)$ is the non-membership function.*

Zadeh's fuzzy set is to extend a theory which deals ambiguity and imprecision in the area of pattern recognition, communication and information. Fuzzy sets and fuzzy logics are applied in many fields like engineering, science, mathematics and social science. Rough set theory is an alternate idea to deal vagueness and it can be expressed in terms of boundary region of a set but not like fuzzy membership function. On the whole, rough set concept can be expressed in terms of approximations called as lower and upper approximation of a given set.

Pawlak [31] initiated a formal tool to process the incomplete information in the the data framework in the information systems is known as rough set theory in 1982 and [32] compared rough set and fuzzy set also described that the two notions are different. Polkowski [30] described rough sets and its mathematical foundations in his classical book.

The concept of rough set theory is the approximation space such as lower and upper approximations of a set determined by its attributes. The pair of lower and upper approximation is called rough set. In rough set theory data can be presented in the form of an information system. An information system is a pair $I = (U, A)$ where U is a non empty finite set of objects, called universal set and A is a nonempty set of fuzzy attributes defined by $\mu_a : U \rightarrow [0, 1]$, $a \in A$, is a fuzzy set. *Indiscernibility* is a core concept of rough set theory and it is defined as an equivalence between objects. Formally any set $P \subseteq A$, there is an associated equivalence relation called $P - Indiscernibility$ relation defined as follows,

$$IND(P) = \{(x, y) \in U^2 \mid \forall a \in P, \mu_a(x) = \mu_a(y)\}.$$

The partition induced by $IND(P)$ consists of equivalence classes defined by

$$[x]_p = \{y \in U \mid (x, y) \in IND(P)\}.$$

For any $X \subseteq U$, define the lower and upper approximation space respectively as $\underline{P}(X) = \{x \in U \mid [x]_p \subseteq X\}$ and $\overline{P}(X) = \{x \in U \mid [x]_p \cap X \neq \phi\}$. For every subset X of U , there is an associated rough set $RS(X) = (\underline{P}(X), \overline{P}(X))$. The boundary region of a set X with respect to R is the set difference of the upper and lower approximations. If the boundary region of a set X is empty then the set X is said to be crisp otherwise it is said to be a rough set. The definition of rough set is clearly depicted in the following figure.

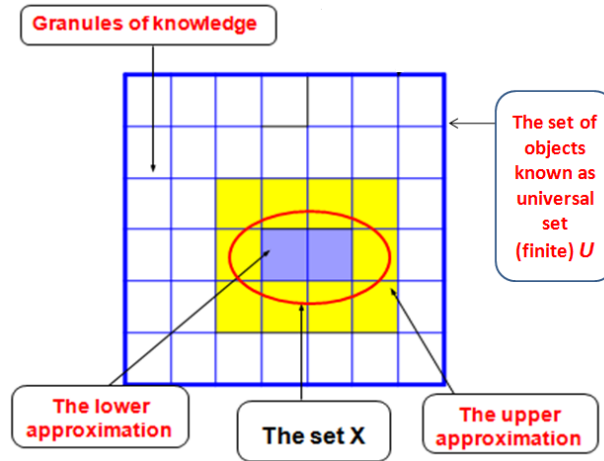


FIGURE 1. Rough Set

There are two various approaches to deal vagueness one is fuzzy set and the other is rough set. Yao [48] described a comparative study of fuzzy sets and rough sets and in 2010 [49] introduced three way decision with probabilistic rough sets based on the decision of acceptance, decision of rejection and decision of abstaining. Some remarks between rough sets and fuzzy sets has discussed [39] in 1989. Rough set also defined by using rough membership function instead of using approximations. That is, for the given universal set U which is finite and for any arbitrary subset X of U , the rough membership function is defined in such a way that

$$\mu_X^R : U \longrightarrow [0, 1]$$

and

$$\mu_X^R(x) = \frac{|X \cap R(x)|}{|R(x)|}.$$

where $R(x)$ is an equivalence class determined by an element $x \in X$.

Chouchoulas and Shen [12], Chen et al. [10], Sai et al. [40], Nasiri and Mashinchi [29] and Bisaria et al. [8] explored some applications of rough set theory in the fields like the content of data analysis, fundamental patterns in data, remove redundancies and generate decision rules etc. Zhang et al. [52] described the data mining ideas using composite rough sets. Rough set theory used in categorization and approximation of medical image segmentation, image segmentation, medical data mining and decision support systems etc. Using rough sets Chena and Wang [11] created an improved clustering algorithm. Mohabey and Ray [27] discussed C -mean clustering algorithm for colour images. Riki and Rezaei [37] described an application of rough sets in data analysis in 2014.

In 2013, Kandil et al. [21] constructed a rough set structure with respect to ideals and studied their properties. Abo-Tabl [1] described different types of Yao's approximation spaces with respect to the similarity relation and gave a comparison between the defined approximations spaces. Hassanien et al. [17] examined some applications of rough sets in medical informatics. Xiao and Zhang [47] introduced rough prime ideals and rough fuzzy prime ideals in semigroups. Kozae et al. [24] introduced four different methods of constructing rough sets and reduced the boundary region in network connectivity devices and network cables etc. Zhang and Luo [53] studied about four types of characterization of covering based generalized rough sets.

Semirings have developed in the last few decades regarding modelling and solving a rich assortment of non-classical problems. The concept of semiring was first introduced by Vandiver [44]. A significant part of the theory of rings keeps on seeming well and good when applied to arbitrary semirings specifically, the concept of algebraic structures over commutative rings can be generalized to the algebraic structures over commutative semirings. In mathematics, semirings holds an important role. The theory of semigroups and rings have considerable effect on the advancements of the theory of semirings. Semiring is defined as, a non empty set S together with two binary operations '+' and '.' is a semiring if $(S, +)$ is a commutative semigroup, $(S, .)$ is a semigroup and both distributive laws hold. In a monoid $(S, +)$, the zero element is known as neutral element

and a monoid $(S, \cdot)$ contains a unit element which is called as an absorbing element. For example, the set of all natural numbers is a semiring. Different types of semirings are listed as follows: Arithmetic semirings, Boolean semiring, Boolean like semiring, Complete semiring, Simple semiring, Viterbi semiring, Tropical semiring, Arctic semiring, Possibilistic semiring, Bottleneck semiring, Truncation semiring, Lukasiewicz semiring, Division semiring etc. Ideal plays an essential part in the structure theory of semiring and are invaluable for some reasons. Allen [2] identified Q -ideal and the quotient structure of the same and proved the fundamental theorem of homomorphism of semiring.

Hebisch and Weinert [18], Golan [16] and Glazek [15] have given their detailed applications of semirings in their classical books. Henriksen [19] characterized a more limited class of k -ideals in semirings and these ideals have the property that if the semiring S is a ring then a subset of S is a k -ideal if and only if the subset is a ring ideal. Kuroki and Wang [22] discussed some properties of lower and upper approximations with respect to the normal subgroup. Biswas and Nanda [7] introduced the notion of rough groups and rough subgroups. Kondo [23] introduced the concepts on the structure of generalized rough sets. Liu [25] dealt the concepts of special lattice of rough algebras. Chinram [38] has introduced the concept of rough prime ideals and rough fuzzy prime ideals in gamma semigroups. Also the authors Iwinski [20] and Bonikowaski [9] have studied some algebraic properties of rough sets. Then the concept of rough fuzzy sets and fuzzy rough sets was introduced by Dubois and Prade [14]. Senthilkumar and Selvan [42] dealt fundamental theorem of set valued homomorphism between groups with respect to the lower and upper approximations of the information system and introduced the kernel of the set valued homomorphism. Sen and Adhikari [41] introduced the idea of k -ideals of semirings. Davaz [13] introduced the idea about roughness in rings and studied the relationship between rough sets, ring theory and some properties of the lower and upper approximations of rings.

Venkatalakshmi and Vasanthi [46] described some special classes of semirings. In particular they discussed the properties of viterbi semiring and positive rational domain. They proved that the semiring S is multiplicatively sub idempotent semiring. Vasanthi and Sulochana [45] described the concepts of Boolean semiring and Boolean like semiring and they have defined some additive and multiplicative structure of semirings. Sreenivasulu Reddy and Guesh

[43] introduced the concept of additive structures of simple semirings. Atani [3–6] discussed ideals in quotient of commutative semirings and introduced the idea of k - weakly primary ideals over semirings. Also, analyzed some results on ideal theory of commutative semiring and explored the idea of strong co – ideal theory in quotient semirings. In 2017, Muhammad Rameez [28] et al. generalised the roughness of fuzzy ideals of hemirings with its lower and upper approximations of the same. The authors Praba et al. [33–36] introduced rough lattice with respect to the new defined operations Δ and ∇ on the set of all rough sets $T = \{RS(X) \mid X \in U\}$ where U is a finite universal set and X is an arbitrary subset of U and explored a commutative regular rough monoid of idempotents with respect to Δ and described some of its ideals. Also, characterized the annihilators of rough semiring in 2020 and it is explored the Boolean algebraic structures induced by the set of annihilators and in the same year the authors described the principal ideals and its structures in the set of all rough sets with respect to the operation Δ and ∇ . Manimaran et al. [26] introduced an idea of regular monoid on the set of all rough sets with respect to the operation ∇ .

2. CONCLUSION

The concept of rough sets has been researched more than thirty five years. Several applications of rough sets are made in many fields. Specifically, some contributions are given by many authors in the direction of algebraic structures with the rough sets. In this paper we have explored some informations about the algebraic concepts related to rough sets and the set of all rough sets.

REFERENCES

- [1] E.A. ABO-TABL: *A Comparison of Two Kinds of Definitions of Rough Approximations Based on a Similarity Relation*, Information Sciences, **181**(12) (2011), 2587–2596.
- [2] P. J. ALLEN: *A Fundamental Theorem of Homomorphisms for Semirings*, Proceedings of the American Mathematical Society, **21**(2) (1969), 412–416.
- [3] S. E. ATANI: *The Ideal Theory in Quotient of Commutative Semirings*, Glasnik Matematicki, **42**(62) (2007), 301–308.
- [4] S. E. ATANI: *On k - Weakly Primary Ideals Over Semirings*, Sarajevo Journal of Mathematics, **3**(15) (2007), 9–13.

- [5] R. E. ATANI, S. E. ATANI: *Ideal Theory in Commutative Semirings*, Buletinul Academiei de Stiinte a Republicii Moldova. Matematica, **57**(2) (2008), 14–23.
- [6] S. E. ATANI, S. D. P. HESARI, M. KHORAMDEL: *Strong Co-ideal Theory in Quotients of Semirings*, Journal of Advanced Research in Pure Mathematics, **5**(3) (2013), 19–32.
- [7] R. BISWAS, S. NANDA: *Rough groups and Rough Subgroups*, Bulletin of the Polish Academy of Sciences Mathematics, **42**(1994), 251–254.
- [8] J. BISARIA, N. SRIVASTAVA, K.R. PARADASANI: *A Rough Set Model for Sequential Pattern Mining with Constraints*, International Journal of Computer Network Security, **1** (2) (2009), 16–22.
- [9] Z. BONIKOWASKI: *Algebraic Structures of Rough Sets*, Rough sets, Fuzzy sets and knowledge discovery, Springer London, (1994), 242–247.
- [10] D. CHEN, W. C. DU, X. W. CHAO, R. ZHU: *A Rough Set Based Hierarchical Clustering Algorithm for Categorical Data*, International Journal of Information Technology, **12**(3) (2006), 149–159.
- [11] C. B. CHENA, L. Y. WANG: *Rough Set-Based Clustering with Refinement Using Shannon's Entropy Theory*, Computers and Mathematics with Applications, **52**(10–11) (2006), 1563–1576.
- [12] A. CHOUCHOULAS, Q. SHEN: *Rough Set-Aided Keyword Reduction for Text Categorisation*, Applied artificial intelligence, **15**(2001), 843–873.
- [13] B. DAVAZ: *Roughness in Rings*, Information Sciences, **164**(2004), 147–163.
- [14] D. DUBOIS, H. PRADE: *Rough fuzzy sets and Fuzzy rough sets*, International Journal of General Systems, **17**(2-3) (1990), 191–209.
- [15] K. GLAZEK: *A Guide to Literature on Semirings and Their Applications in Mathematics and Information Sciences with Complete Bibliography*, Springer Science and Business Media, Berlin.
- [16] J. S. GOLAN: *Semirings and Their Applications*, Kluwer, Dordrecht.
- [17] A. E. HASSANIEN, A. ABRAHAM, J. F. PETERS, G. SCHAEFER: *Rough Sets in Medical Informatics Applications*, Applications of Soft Computing, Springer, Berlin Heidelberg, 23–30.
- [18] U. HEBISCH, H. J. WEINERT: *Semirings - Algebraic Theory and Applications in Computer Science*, World Scientific, Singapore, (1998).
- [19] M. HENRIKSEN: *Ideals in Semirings with Commutative Addition*, Notices of the American Mathematical Society, **6**(1) (1958), 321.
- [20] T.B. IWINSKI: *Algebraic Approach to Rough Sets*, Bulletin of the Polish Academy of Sciences Mathematics, **35**(1987), 673–683.
- [21] A. KANDIL, M. M. YAKOUT, A. ZAKARIA: *Generalized Rough Sets Via Ideals*, Annals of Fuzzy Mathematics and Informatics, **5**(3) (2013), 525–532.
- [22] N. KUROKI, P. P. WANG: *The Lower and Upper Approximations in a Fuzzy Group*, Information Sciences, **90**, (1996), 203–220.

- [23] M. KONDO: *On The Structure of Generalized Rough Sets*, Information Sciences, 176 (2006), 586–600.
- [24] A. M. KOZAE, S. A. EL-SHEIKH, E. H. ALY, M. HOSNY: *Rough Sets and its Applications in a Computer Network*, Annals of Fuzzy Mathematics and Informatics, 6(3) (2013), 605–624.
- [25] Y. LIU: *Special Lattice of Rough Algebras*, Applied Mathematics, 2(2011), 1522–1524.
- [26] A. MANIMARAN, B.PRABA, V. M. CHANDRASEKARAN: *Regular Rough ∇ Monoid of Idempotents*, International Journal of Applied Engineering and Research, 9(16) (2014), 3469–3479.
- [27] A. MOHABEY, A. K. RAY: *Fusion of Rough Set Theoretic Approximations and FCM for Color Image Segmentation*, Systems, Man, and Cybernetics, IEEE International Conference, 2(2000), 1529– 1534.
- [28] M. RAMEEZ, M.I. ALI, A. EJAZ: *Generalized Roughness in $(\epsilon, \epsilon \vee q)$ -Fuzzy Ideals of Hemirings*, Kuwait Journal of Science, 44(3) (2017), 34–43.
- [29] J. H. NASIRI, M. MASHINCHI: *Rough Set and Data analysis in Decision Tables*, Journal of uncertain systems, 3(3) (2009), 232–240.
- [30] L. POLKOWSKI: *Rough Sets: Mathematical foundations*, Advances in Soft Computing, Springer-Verlag Berlin Heidelberg, Newyork, (2002).
- [31] Z. PAWLAK: *Rough Sets*, International Journal of Computer and Information Sciences, 11(1982), 341–356.
- [32] Z. PAWLAK: *Rough Sets and Fuzzy Sets*, Fuzzy Sets and Systems, 17(1985), 99–102.
- [33] B. PRABA, R. MOHAN: *Rough Lattice*, International Journal of Fuzzy Mathematics and System, 3(2) (2013), 135–151.
- [34] B. PRABA, V. M. CHANDRASEKARAN, A. MANIMARAN: *A Commutative Regular Monoid on Rough Sets*, Italian Journal of Pure and Applied Mathematics, 31 (2013), 307–318.
- [35] B. PRABA, R. SARANYA, G. GOMATHI: *A Characterization of the Annihilators of a Rough Semiring*, International Journal of Recent Technology and Engineering, 8 (5) (2020), 3625–3628.
- [36] B. PRABA, A. MANIMARAN, I. MUCHTADI-ALAMSYAH: *A Note on Principal Rough Ideals of a Rough Monoid*, Advances in Mathematics: Scientific Journal, 9(9) (2020), 6855–6861.
- [37] M. RIKI, H. REZAEI: *Introduction of Rough Sets Theory and Application in Data Analysis*, Journal of Mathematics and Computer Science, 9(2014), 25–32.
- [38] R. CHINRAM: *Rough Prime Ideals and Rough Fuzzy Prime Ideals in Gamma Semigroups*, Korean Mathematical Society, 24(3) (2009), 341–351.
- [39] M. WYGRALAK: *Rough Sets and Fuzzy Sets - Some Remarks on Interrelations*, Fuzzy Sets and Systems, 29(1989), 241–243.
- [40] Y. SAI, P. NIE, R. XU, J. HUANG: *A Rough Set Approach to Mining Concise Rules from Inconsistent Data*, IEEE international conference on granular computing, 10(12) (2006), 333–336.

- [41] M. K. SEN, M. R. ADHIKARI: *On k - Ideals of Semirings*, International Journal of Mathematical Science, **15**(2) (1992), 347–350.
- [42] G. SENTHILKUMAR, V. SELVAN: *A Fundamental Theorem of Set Valued Homomorphism of Groups*, International Journal of Contemporary Mathematical Sciences, **7**(41) (2012), 2013–2025.
- [43] P. SREENIVASULU REDDY, Y. T. GUESH: *Simple Semirings*, International Journal of Engineering Inventions, **2**(7) (2013), 16–19.
- [44] H. S. VANDIVER: *Note on a Simple Type of Algebra in Which the Cancellation Law of Addition Does Not Hold*, Bulletin of the American Mathematical Society, **40**(12) (1934), 914–920.
- [45] T. VASANTHI, N. SULOCHANA: *On the Additive and Multiplicative Structure of Semirings*, Annals of Pure and Applied Mathematics, **3**(1) (2013), 78–84.
- [46] C. VENKATALAKSHMI, T. VASANTHI: *Some Special Classes of Semirings*, International Journal of Applied Information Systems, **6**(8) (2014), 27–29.
- [47] Q. M. XIAO, Z. L. ZHANG: *Rough Prime Ideals and Rough Fuzzy Prime Ideals in Semi-groups*, Information Sciences, **176**(6) (2006), 725–733.
- [48] Y. Y. YAO: *A Comparative Study of Fuzzy Sets and Rough Sets*, Information Sciences, **109**(1–4) (1998), 227–242.
- [49] Y. Y. YAO: *Three Way Decisions with Probabilistic Rough Sets*, Information Sciences, **180**(3) (2010), 341–353.
- [50] Y. LIU: *Special Lattice of Rough Algebras*, Applied Mathematics, **2**(2011), 1522–1524.
- [51] L.A. ZADEH: *Fuzzy Sets*, Information and Control, **8**(1965), 338–353.
- [52] J. ZHANG, T. LI, H. CHEN: *Composite Rough Sets for Dynamic Data Mining*, Information Sciences, **257**(2014), 81–100.
- [53] Y. L. ZHANG, M. K. LUO: *On Minimization of Axiom Sets Characterizing Covering-Based Approximation Operators*, Information Sciences, **181**(14), 3032–3042.

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