Advances in Mathematics: Scientific Journal **10** (2021), no.4, 1925–1936
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.10.4.7

ANALYSIS OF VISCOSITY VARIATION EFFECT IN A WIDE SLIDER BEARING HAVING AN EXPONENTIAL FILM PROFILE IN THE PRESENCE OF MHD

E. Sujatha[1], G. Lavanya, and E.P. Siva

ABSTRACT. This paper analyses the effect of viscosity variation on a porous slider bearing with an exponential film profile under the influence of a magnetic field. The modified Reynolds equation is derived based on MHD thin film lubrication theory. Expressions for pressure and load carrying capacity is derived and the same is analyzed for the effect of viscosity variation.

1. INTRODUCTION

Application of lubrication has deep roots in fields of Engineering and Medical sciences. Characteristics of slider bearings of various shapes have been analysed by Cameron [1], Pinkus and Strenclicht [2], Hamrock [3]. Various factors like roughness effect [4], thermal effects [5], inertial forces [6], turbulent flows [7] influence the nature of slider bearings and have been studied by many authors. Also various fluids like couple stress fluid [8], Ferro fluid [9], micro polar fluid [10] have been applied as lubricants for analyzing their effects. Lin et al. [11,12] studied about the stiffness and dynamics effects compared to the static

[1]*corresponding author*

2020 *Mathematics Subject Classification.* 76.
Key words and phrases. Viscosity variation, Exponential Film, Magneto hydrodynamics, Slider bearing.
Submitted: 04.03.2021; *Accepted:* 25.03.2021; *Published:* 04.04.2021.

effects of a porous slider bearing considering the magnetodynamic effects with an exponential film.

In this paper the effect of viscosity variation is analysed on a wide and porous bearing which can slide with a given velocity creating an exponential film profile, when acted upon by magnetic field. Electrical conductivity and thermal conductivity are very high in electrically conducting liquid metal fluid. This also helps in immediate conductivity of the heat generated in the bearing. A current density is developed when an external magnetic field is applied to a lubricant which can conduct electricity. This results in a production of Lorentz force upon the fluid film. These effects enhance the characteristics of the slider bearing.

2. Mathematical Formulation

Figure shows the configuration of the MHD wide porous slider bearing. Consider L to be the length of the bearing. The bearing slides in the direction of x with a velocity U. Let B_0 be the traverse magnetic field which is applied uniformly in the direction of z. For an exponential slider bearing, the film thickness $h(x,t)$ is given by the following format:

$$h(x,t) = h_m(t).\exp\left(-\frac{x}{L}\ln(r)\right), \tag{2.1}$$

$$\ln(r) = \ln\left(\frac{h_1(t)}{h_m(t)}\right), \tag{2.2}$$

and

$$.h_1(t) = d + h_m(t) \tag{2.3}$$

In the above relationship, $h_m(t), h_1(t)$ and d represents the outlet thickness when the flow is steady, the inlet thickness when the flow is steady and the shoulder height respectively. Shoulder height can be defined as the difference between inlet height and outlet height.

The concept of varying viscosity gains importance from the fact that the machineries are expected to operate over a wide range of temperature. So the lubricant taken for operation must be suitable for all such situations. The fact that the viscosity of the lubricant decreases with increase in temperature can be coupled with that of variation in viscosity over a given interval of temperature. As an accurate mathematical relation to predict the variation in viscosity with respect to temperature is not known, an empirical relation is developed for the same.

This relation is given by

$$\mu = \mu^{'} \left(\frac{h}{h_0} \right)^{\gamma}. \tag{2.4}$$

In the above expression $\mu^{'}$ represents the known viscosity when the film thickness is h_0. The parameter γ represents the viscosity variation parameter which ranges from 0 to 1. For a perfect gas, γ takes the value 1 and for perfect Newtonian fluid it takes the value 0. When applying a variation in viscosity, the assumption that, the change caused by viscosity with temperature can be interchanged with a relationship which the viscosity holds with film thickness is made. Further, the existence of a thermal equilibrium in such a situation is assumed to hold true.

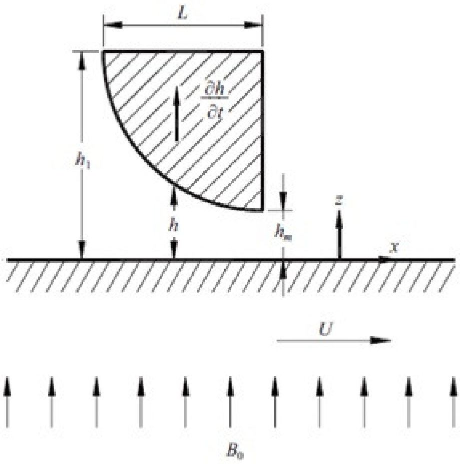

FIGURE 1. Geometry of the slider bearing with exponential film

The relevant equations that govern the MHD flow are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \text{ (Continuity Equation)} \tag{2.5}$$

$$\mu \frac{\partial^2 u}{\partial z^2} - \sigma B_0 u = \frac{\partial p}{\partial x} + \sigma B_0 E_y \text{ (Momentum equation)} \tag{2.6}$$

$$\frac{\partial p}{\partial z} = 0 \tag{2.7}$$

In the above equations σ, B_0, E_y represents the electrical conductivity, applied magnetic field and the induced electrical field respectively. The value of lubricants viscosity μ can be replaced with the expression for varying viscosity given by equation (2.4).

The velocity components of the fluid can be attained subject to the boundary conditions given by

(2.8) $u = 0,\ w = \dfrac{\partial h}{\partial t}$ on the upper surface given by $z = h$ and

(2.9) On the lower surface $z = 0$ they are given by $u = U$ and $w = 0$

Since the lower surface slides, the surface $z = 0$ experiences a velocity $u = U$.

The velocity component in the direction of x is obtained after substituting the boundary conditions as

$$\begin{aligned} u = {} & U\left\{\cosh\left(\frac{Mz}{h_{m0}}\right) - \coth\left(\frac{Mh}{h_{m0}}\right)\sinh\left(\frac{Mz}{h_{m0}}\right)\right\} \\ & - \frac{h_{m0}^2}{\mu M^2}\left(-\frac{\partial p}{\partial x} + \sigma B_0 E_y\right)\left\{\cosh\left(\frac{Mz}{h_{m0}}\right)\right. \\ & \left. -1 - \tanh\left(\frac{Mh}{h_{m0}}\right)\sinh\left(\frac{Mz}{h_{m0}}\right)\right\}, \end{aligned} \tag{2.10}$$

where h_{m0} represents the minimum film thickness at steady state condition at the outlet and M represents the Hartmann number. The expression for M is given by

$$M = B_0 h_{m0}\sqrt{\frac{\sigma}{\mu}}. \tag{2.11}$$

The paper considered here assumes that the bearing surfaces are perfectly insulated. This insulation cuts off the external circuit in the lubricant, which leads to the approximation of the electric field to be defined by the relation

$$\int_{z=0}^{h} (E_y + uB_0)dz = 0. \tag{2.12}$$

This relation also guarantees that there is no net current flow in the lubricant.

The incorporation of equation (2.12) in equation (2.10) gives

$$u=\frac{U}{2}\left\{1-\frac{\sinh\left(\frac{Mz}{h_{m0}}\right)}{\sinh\left(\frac{Mh}{h_{m0}}\right)}-\frac{\sinh\left(\frac{M(h-z)}{h_{m0}}\right)}{\sinh\left(\frac{Mh}{h_{m0}}\right)}\right\} \tag{2.13}$$
$$-\frac{h_{m0}^2}{2M\mu}\frac{\partial p}{\partial x}\left(\frac{\sinh\left(\frac{Mh}{h_{m0}}\right)-\sinh\left(\frac{Mz}{h_{m0}}\right)-\sinh\left(\frac{M(h-z)}{h_{m0}}\right)}{\cosh\left(\frac{Mh}{h_{m0}}\right)-1}\right).$$

The continuity equation (2.5) if integrated across the fluid thickness from $z=0$ to $z=h$ gives

$$\int_{z=0}^{h}\frac{\partial u}{\partial x}dz=-\int_{z=0}^{h}\frac{\partial w}{\partial z}dz. \tag{2.14}$$

The MHD dynamic equation is obtained by integrating the above equation (2.14) applying the boundary conditions (2.8) and (2.9),

$$\frac{1}{\mu}\frac{\partial}{\partial x}\left(g(h,m)\right)\frac{\partial p}{\partial x}=6U\frac{\partial h}{\partial x}+12\frac{\partial h}{\partial t}, \tag{2.15}$$

where

$$g(h,m)=6h_{m0}^2hM^{-2}\left(\frac{Mh}{h_{m0}}\coth\left(\frac{Mh}{2h_{m0}}\right)-2\right). \tag{2.16}$$

From equation (2.1), $h(x,t)=h_m(t).\exp(-x\ln(r))$ where $h_m(t)$ represents steady flow outlet thickness can be taken as $1+\varepsilon, \varepsilon<<1$.

The inlet – outlet ratio can be approximated by $r=\delta+1$ where δ is the profile parameter that represents the wedge effect of an exponential shaped slider bearing and $\delta=\dfrac{d}{h_{m0}}$.

Modified Darcy's law governs the flow of the lubricant in the porous matrix. The velocity component in the porous medium is given by

$$u^*=-\frac{k}{\mu}\frac{\partial p'}{\partial x}\left[1+\frac{kM^2}{mh_0^2}\right]^{-1}, \tag{2.17}$$

where p' is the pressure in the porous region, k represents the permeability in the porous medium and m, the porosity.

The conditions for the film pressure at the boundary of the bearing surface are given by $p'=0$ at $x=0$ and at $x=-1$.

Integrating equation (2.15) twice with respect to x and expressing it in non-dimensional form, the expression for pressure is obtained as

$$p^* = \frac{1}{\mu_1\left(\dfrac{h}{h_0}\right)^{\gamma}}\left[6h_m^* - \frac{12V^*}{\ln(\delta+1)}\right] g_A(x^*, h_m^*) + c(h_m^*, V^*) g_B(x^*, h_m^*), \tag{2.18}$$

where $V^* = \dfrac{dh_m^*}{dt^*}$ is the non-dimensional squeezing velocity.

$$\begin{aligned} g_A(x^*, h_m^*) &= \int_{x^*=0}^{x^*} \frac{h_e^*(x^*)}{g^*(h^*, M)} dx^* \\ g_B(x^*, h_m^*) &= \int_{x^*=0}^{x^*} \frac{1}{g^*(h^*, M)} dx^*. \end{aligned}$$

The integration function

$$\begin{aligned} c(h_m^*, V^*) &= -\frac{1}{\mu_1\left(\dfrac{h}{h_0}\right)^{\gamma}}\left[6h_m^* - \frac{12V^*}{\ln(\delta+1)}\right]\frac{g_{AM_1}(h_m^*)}{g_{BM_1}(h_m^*)} \\ g_{AM_1}(h_m^*) &= g_A(x^* = -1, h_m^*) = \int_{x^*=0}^{-1} \frac{h_e^*(x^*)}{g^*(h^*, M)} dx^* \\ g_{BM_1}(h_m^*) &= g_B(x^* = -1, h_m^*) = \int_{x^*=0}^{-1} \frac{1}{g^*(h^*, M)} dx^*. \end{aligned}$$

Integration of the film pressure leads to the estimation of non- dimensional MHD film force given by

$$\begin{aligned} F^* =& \frac{F h_{m0}^2}{UL^2B} = -\int_{x^*=0}^{-1} p^* dx^* \\ =& -\frac{1}{\mu_1\left(\dfrac{h}{h_0}\right)^{\gamma}}\left\{\left[6h_m^* - \frac{12V^*}{\ln(\delta+1)}\right] G_A(h_m^*) + c(h_m^*, V^*) G_B(h_m^*)\right\}, \end{aligned}$$

where $G_A(h_m^*) = \int\limits_{x^*=0}^{-1} g_A(x^*, h_m^*) = \int\limits_{x^*=0}^{-1}\int\limits_{x^*=0}^{x^*} \frac{h_e^*(x^*)}{g^*(h^*, M)} dx^* dx^*$ and $G_B(h_m^*) = \int\limits_{x^*=0}^{-1} g_B(x^*, h_m^*) = \int\limits_{x^*=0}^{-1}\int\limits_{x^*=0}^{x^*} \frac{1}{g^*(h^*, M)} dx^* dx^*$.

The film pressure at steady state and the work done at the steady state can be obtained by allowing the minimum film height to be constant and the squeeze velocity to be zero.

$$\begin{aligned} p_0^* &= \frac{1}{\mu_1 \left(\dfrac{h}{h_0}\right)^{\gamma}} [6h_m^*]_0 [g_A(x^*, h_m^*)]_0 + c[h_m^*]_0 [g_B(x^*, h_m^*)]_0 \\ W_0^* &= -\frac{1}{\mu_1 \left(\dfrac{h}{h_0}\right)^{\gamma}} \{[6h_m^*]_0 [G_A(h_m^*)]_0 + c[h_m^*]_0 [G_B(h_m^*)]_0\}. \end{aligned}$$

The subscript zero (0) represents the steady state.

3. Results and Discussion

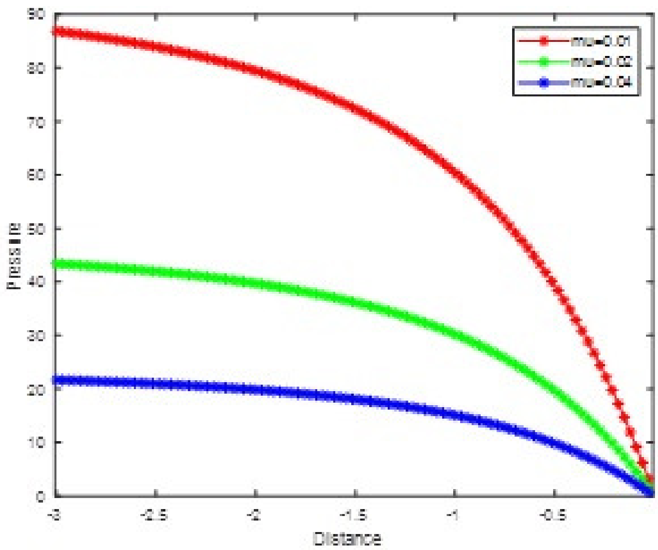

FIGURE 2. Change in pressure with respect to distance for various values of viscosity variation parameter.

Form the graph it is observed that as the values of the viscosity variation parameter increases, the pressure decreases.

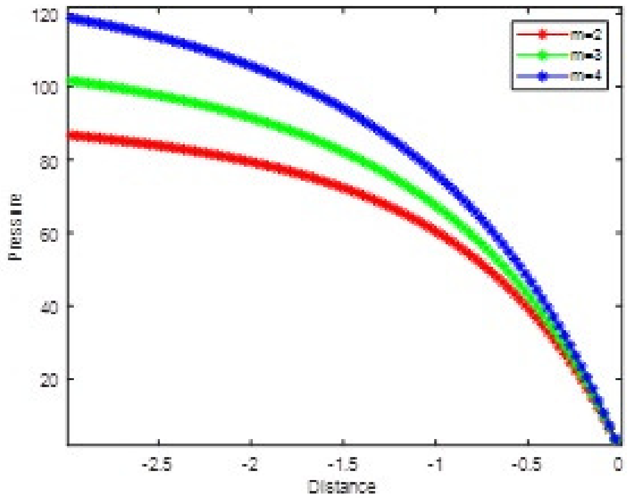

FIGURE 3. Change in pressure with respect to distance for various values of Hartmann constant (M)

As the values of the Hartmann constant which is involved with the magnetic field strength increases, the pressure developed also increases.

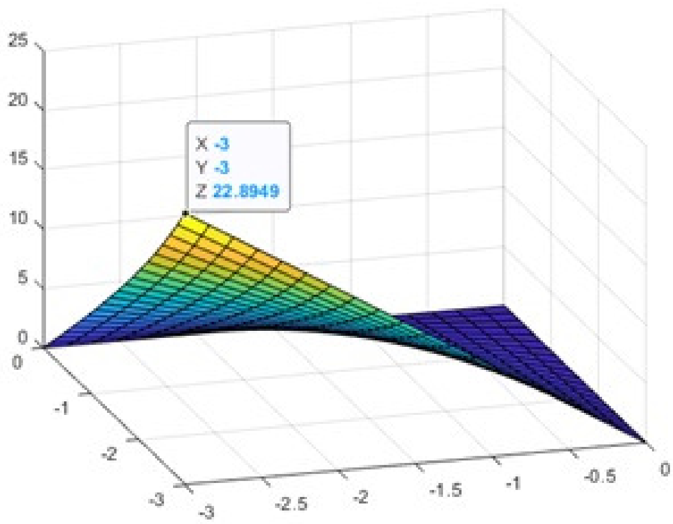

FIGURE 4. Surface plot of work done when viscosity variation parameter is 0.01.

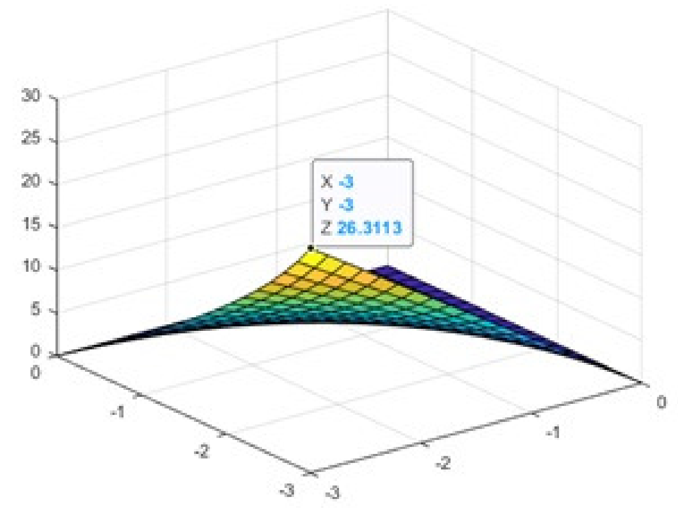

FIGURE 5. Surface plot of work done when viscosity variation parameter is 0.02.

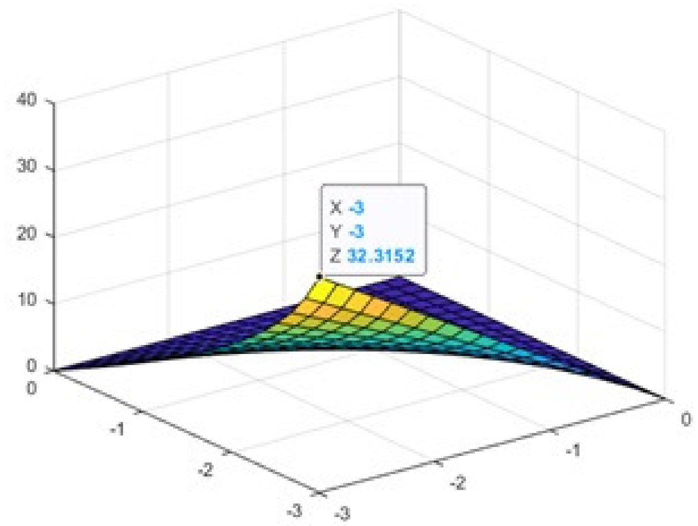

FIGURE 6. Surface plot of work done when viscosity variation parameter is 0.03.

Comparing figures 4, 5 and 6 we observe that for a fixed Hartmann constant M=5, the work done at steady state varies with viscosity variation parameter.

With an increase in the value of the viscosity variation parameter, the work done also increases, showing a direct relationship between these two parameters.

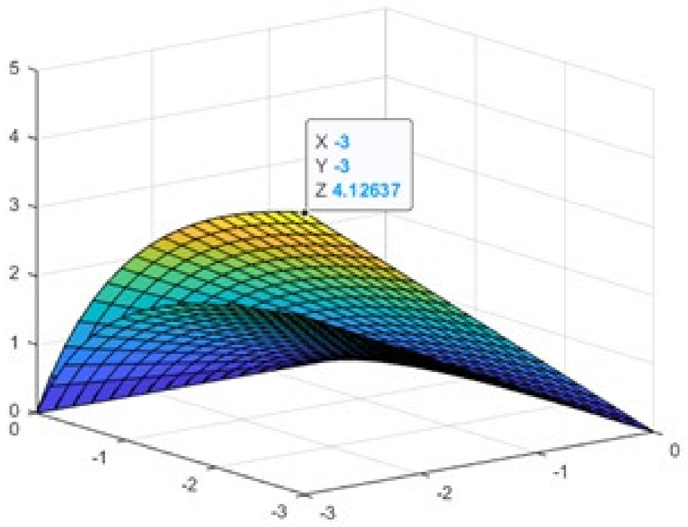

FIGURE 7. Surface plot of work done with Hartmann constant M=3 and viscosity variation parameter $\mu = 0.04$.

Figure 7 and 8 displays the surface plot when the viscosity variation parameter μ is kept fixed at 0.04 and the Hartmann constant representing the magnetic field varies from 3 to 5. With an increase in the Hartmann constant, the work done by the system also increases.

4. CONCLUSION

This paper analyses the effect of viscosity variation parameter on the exponential film bearing coupled with the effect of MHD. The results show that the pressure decreases with increase in the viscosity variation parameter and the work done increase with increase in viscosity variation parameter.

REFERENCES

[1] A CAMERON: *The principles of lubrication theory*, Longmans, London, 1966.

[2] O PINKUS, B STERNLICHT: *Theory of hydrodynamic lubrication*, McGraw-Hill, New York, 1981.

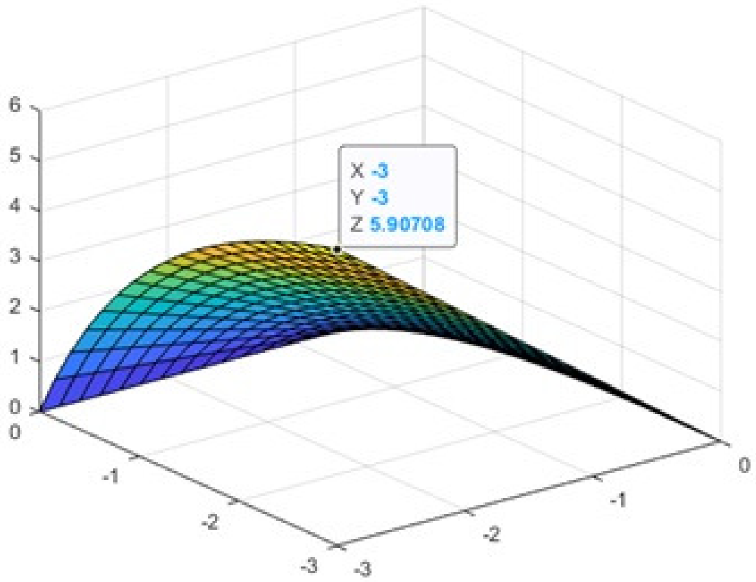

FIGURE 8. Surface plot of work done with Hartmann constant $M = 5$ and viscosity variation parameter $\mu = 0.04$

[3] B.J HAMROCK: *Fundamentals of fluid film thickness*, McGraw-Hill, New York, 1994.

[4] P.I ANDHARIA, J.L GUPTA, G.M DEHERI: *Effect of surface roughness of hydrodynamic lubrication of slider bearings*, Tribology Transactions, **44** (2001), 291–297.

[5] Z.S SAFAR: *Thermal and Inertia Effects in Turbulent Flow Thrust Bearing*, Proc. I. Mech. E., **22** (1980), 43–45.

[6] C.M. RODKIEWICZ AND M.I ANWAR: *Inertia and Convective Effect in Hydrodynamic Lubrication of a Slider Bearing* ASME J. Lubric. Technol., **93** (1971), 313–315.

[7] C.M. TAYLOR, D. DOWSON: *Turbulent Lubrication Theory-Application to Design*, ASME J. Lubric.Technol., **96** (1974), 36–46.

[8] N.B. NADUVINAMANI, SYEDA TASNEEM FATHIMA, P.S. HIREMATH: *Hydrodynamic lubrication of rough slider bearings with couple stress fluids,* Tribology International, **36** (2003), 949–959.

[9] RAJESH C. SHAH, M.M. PARSANIA: *Comparative Study of Parallel Plate Slider Bearing with Other Slider Bearings Using Magnetic Fluid as Lubricant*, American Journal of Mathematics and Statistics, **3**(4) (2013), 179–189.

[10] SIDDANGOUDA, T.V. BIRADAR, N.B. NADUVINAMANI: *Combined effects of micropolarity and surface roughness on the hydrodynamic lubrication of slider bearings*, J. Braz. Soc. Mech. Sci. Eng., **36** (2014), 45–58.

[11] JAW-REN LIN AND RONG-FANG LU: *Dynamic characteristics for magneto-hydrodynamic wide slider bearings with an exponential film profile*, Journal of Marine Science and Technology, **18**(2) (2010), 268–276.

[12] JAW-REN LIN AND CHI-REN HUNG: *Analysis of dynamic characteristics for wide slider bearings with an exponential film profile*, Journal of Marine Science and Technology, **12**(3) (2004), 217–221.

DEPARTMENT OF MATHEMATICS, COLLEGE OF ENGINEERING AND TECHNOLOGY
FACULTY OF ENGINEERING AND TECHNOLOGY, SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
SRM NAGAR, KATTANKULATHUR-603203, KANCHIPURAM, CHENNAI, TN, INDIA
Email address: `sujathae@srmist.edu.in`

DEPARTMENT OF MATHEMATICS, COLLEGE OF ENGINEERING AND TECHNOLOGY
FACULTY OF ENGINEERING AND TECHNOLOGY, SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
SRM NAGAR, KATTANKULATHUR-603203, KANCHIPURAM, CHENNAI, TN, INDIA
Email address: `lavanyag@srmist.edu.in`

DEPARTMENT OF MATHEMATICS, COLLEGE OF ENGINEERING AND TECHNOLOGY
FACULTY OF ENGINEERING AND TECHNOLOGY, SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
SRM NAGAR, KATTANKULATHUR-603203, KANCHIPURAM, CHENNAI, TN, INDIA
Email address: `sivae@srmist.edu.in`